**Unit 4 Similarity and Trigonometry**

**(4-5 Weeks)**

**UNIT PLAN**

This unit builds on students’ experiences with transformations and proving triangles congruent. Students continue to discover a pattern, make a conjecture, and then prove the theorem. This unit is also designed to help students to strengthen their understanding of proof. Students will use techniques learned in Unit 2 to prove triangles congruent to now prove triangles similar. These new theorems are then used to prove additional theorems. Right triangle trigonometry is then developed from similarity and used in the analysis of special right triangles and applications to real world problems.

In this unit students will continue using dynamic geometry software to explore geometric properties and make conjectures. As in previous units, some activities are written for GeoGebra, but there are also “low tech” activities based on hands-on materials or explorations in the coordinate plane.

**Investigation 1 Dilations** is a formal approach to understanding dilations as compared to their exposure in middle school. Students begin by constructing dilations using compass and straightedge. Students then use dynamic geometry software to continue to explore various examples of dilations. This helps them to formalize a dilation postulate to use later in various proofs as well as to explore properties of dilations.

**Investigation 2 Similar Figures** introduces students to similarity transformations as the composition of isometries and dilations and to the definition of similarity as it applies to polygons and circles. They then focus on similarity transformations on triangles. Students will end the unit wondering if there are “shortcuts” for proving triangles are similar as there are for proving triangles congruent.

**Investigation 3 Proving Similar Triangles.** Through hands-on investigations in groups, students discover shortcuts for proving similar triangles. A whole class discussion leads the class to develop three conjectures—the AA Similarity Conjecture, the SAS Similarity Conjecture, and the SSS Similarity Conjecture. These conjectures are then proved and used as theorems in subsequent problems.

**Investigation 4** **Parallel Lines in Triangles.** In this investigation students will discover and prove the Side Splitting Theorem and its converse. They will apply their side-splitting theorem in a construction to divide a line segment into *n* congruent parts and look at a corresponding problem in coordinate geometry.

Much like earlier investigations, **Investigation 5 Similarity in Right Triangles**,has students discovering through hands-on exploration that the altitude to the hypotenuse of a right triangle creates two new triangles that are similar to each other and the original triangle. Students then make a conjecture and prove the Right Triangle Similarity Theorem. Extending their study of similarity in right triangles, students prove the Pythagorean theorem and its converse. Students will be reminded that they have already seen an informal (dissection) proof of the theorem in Unit 1 and that there are over 300 known proofs of this famous theorem.

**Investigation 6 Right Triangle Trigonometry** begins by building on Investigation 5 by having students explore the ratio of sides, relative to a given angle, in various similar right triangles. The initial dynamic geometry software activity leads students to develop definitions for the sine, cosine, and tangent ratios. They examine the relationship between sine and cosine as co-functions and learn how to solve for the measure of an acute angle in a right triangle using inverse trigonometric functions on a calculator. Students then apply these ratios to various real world application problems to deepen their understanding.

The unit culminates in **Investigation 7 Special Right Triangles.** In this investigation, students will use isometric and square dot paper to learn the relationships that exist among the sides in both the 45-45-90 triangles and the 30-60-90 triangles. For students to learn these relationships they will need to look at patterns that exist among the similar triangles. To reveal this pattern students need to simplify square roots, something they have not done up to this point. Thus we begin investigation 7 with an activity that teaches students how to simplify square roots. After learning this skill and the relationships among the sides in special right triangles, then students connect these ideas to right triangle trigonometry. Students then solve real-world problems involving these concepts.

**Investigation 8 Indirect Measurement** is a one-day lesson designed to introduce students to the performance task. It is not needed prior to the End-of-Unit Assessment and may be given later in the year if the performance task is delayed until the weather improves. If the performance task is not given, then this becomes an optional activity.

In the **Performance Task** students will apply trigonometry to find the height of an inaccessible building or structure using indirect measurement. This task may be delayed until a time of year when students are able to go outdoors.

**A note on technology:** GeoGebra is used in 6 of the 32 activities in this unit. When it is not available to all students, the teacher may do part of the activity as a classroom demonstration. Hands-on tools including compasses, straightedges and protractors are used in 9 activities. Scientific or graphing calculators (with trigonometric functions) are require for the four activities in Investigation 6.

**Essential Questions**

* How is the image under a dilation related to its pre-image?
* What are similarity transformations?
* How is congruence a special case of similarity?
* How are the trigonometric ratios related to similarity?

**Enduring Understandings**

The concept of similarity enables us to explore geometric relationships and apply trigonometric ratios to solve real world problems.

**Unit Contents**

Investigation 1 Properties of Dilations (2 days)

Investigation 2 Similar Figures (2 days)

Investigation 3 Proving Similar Triangles (3 days)

Investigation 4 Parallel Lines in Triangles (2 days)

Investigation 5 Similarity in Right Triangles (2 days)

Investigation 6 Right Triangle Trigonometry (3 days)
Investigation 7 Special Right Triangles (2 days)

Investigation 8 Indirect Measurement (1 day, optional)
Performance Task (1-2 days)

Review (1 day)

End of Unit Test (1 day)

**Common Core Standards**

*Mathematical Practices #1 and #3* *describe a classroom environment that encourages thinking mathematically and are critical for quality teaching and learning. Practices in bold are to be emphasized in the unit.*

1. Make sense of problems and persevere in solving them.

2. **Reason abstractly and quantitatively.**

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. **Use appropriate tools strategically.**

6. Attend to precision.

7. **Look for and make use of structure.**

8. Look for and express regularity in repeated reasoning.

**Content Standards Overview**

* Understand similarity in terms of similarity transformations
* Prove theorems involving similarity
* Define trigonometric ratios and solve problems involving right triangles.

**Content Standards**

* G-SRT.A.1.A A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
* G-SRT.A.1.A The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
* G-SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
* G-SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
* G-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
* G-SRT.B.4 Prove theorems about triangles. Theorems include: *a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
* G-GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
* G-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.\*
* G-SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
* G-SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.

**Assessment Strategies**

**Performance Task**

Students will work outside using indirect measurement to determine the height of a large object on or visible from the school campus. A version has been provided that uses the Newington Clock Tower as an example. However, it is suggested that the context be changed with each individual school’s example.

**Other Evidence (Formative and Summative Assessments)**

* Exit slips
* Class work
* Homework assignments
* Math journals
* End of Unit test
* Performance Task

**Postulates and Theorems**

**(4-1) Dilation Postulate:** Dilations preserve angle measure and betweenness. They map parallel lines onto parallel lines and midpoints onto midpoints. The length of the image of a segment is the length of the segment times the scale factor. If a line passes through the center of dilation it is mapped onto itself. If a line *l* does not pass through the center of dilation it is mapped onto a line *l’* parallel to *l.*

**(4-2) Triangle Similarity Theorem:** If two triangles are similar then pairs of corresponding sides have the same ratio and pairs of corresponding angles are congruent.

**(4-2) Circle Similarity Theorem:** All circles are similar to each other.

**(4-3) AA Similarity Theorem:** : If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

**(4-3) Equilateral Triangle Similarity Theorem:** All equilateral triangles are similar to each other.

**(4-3) SAS Similarity Theorem:** If two sides of one triangle are proportional to two sides of another triangle and the angles included by these sides are congruent, then the triangles are similar.

**(4-3) SSS Similarity Theorem:** If three sides of one triangle are proportional to three sides of another triangle, then the two triangles are similar.

**(4-4) Side Splitting Theorem:** If a line parallel to one side of a triangle, then it divides the other sides proportionally.

**(4-4) Side Splitting Converse:** If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

**(4-4) Construction:** To divide a given line segment into *n* congruent parts.

**(4-4) Construction:** To find the point on a directed line segment between two given points that partitions the segment in a given ratio.

**(4-5) Right Triangle Similarity Theorem:** The altitude to the hypotenuse of a right triangle forms two triangles that are similar to the whole triangle.

**(4-5) Pythagorean Theorem:** In a right triangle the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

**(4-5) Converse of Pythagorean Theorem:** If *a*2 + *b*2 = *c*2, where *a*, *b*, and *c* are the lengths of the sides of a triangle, then the triangle is a right triangle with *c* as the length of the hypotenuse.

**Abbreviations**

Corresponding Angles of Similar Triangles are Congruent (CASTC)

Corresponding Sides of Similar Triangles are in Proportion (CSSTP)

sine (sin)

cosine (cos)

tangent (tan)

inverse sine (sin–1)

inverse cosine (cos–1)

inverse tangent (tan–1)

**Vocabulary**

adjacent (leg)

angle of descent

angle of depression

angle of elevation

altitude

center of dilation

clinometer

composition (of transformations)

cosine

dilation

directed line Segment

division (of a segment)

geometric mean
hypotenuse

indirect measurement

inverse cosine

inverse sine

inverse tangent

leg (of right triangle)

opposite (leg)

mean proportional

partitioning

proportion

right triangle

scale factor

similar figures

similarity transformation

simplified square root

sine

special right triangle

 30o-60o-90o triangle

45o-45o-90o triangle

tangent (trigonometric ratio)