**Activity 4.5.3 Geometric Means**

In **Activity 4.5.2**, you proved that an altitude to the hypotenuse of a right triangle from the opposite vertex creates two new triangles that are similar to each other and the original triangle.

$ΔABC$ is a right triangle with altitude $\overbar{DC}$.

$∠C$ is a right angle.

**Part I:**

1. By the Right Triangle Similarity Theorem we know that $⊿\\_\\_\\_\\_\\_\\_\\_≅⊿\\_\\_\\_\\_\\_\\_\\_≅⊿\\_\\_\\_\\_\\_\\_\\_$.

2. Using the two smaller triangles we can set up the following proportion

$\frac{\\_\\_}{AD}=\frac{BD}{\\_\\_}$ which results in $AD⋅BD=\\_\\_\\_\\_\\_\\_\\_\\_\\_$

When a the same number appears twice in a proportion, in the numerator of one ratio and the denominator of the other ration, it is called the **geometric mean** of the other two numbers.

3. In the example above, \_\_\_\_\_\_ is the geometric mean of *AD* and *BD.*

**If an altitude is drawn to the hypotenuse of a right triangle from the opposite vertex, then the altitude to the hypotenuse is the geometric mean between the segments on the hypotenuse.**

**4.** In the figure at the right,$h^{2}=\\_\\_\\_⋅$

****$h$ is the geometric mean between \_\_\_ and \_\_\_\_\_.

5. Find the geometric mean between

1. 20 and 5
2. 10 and 90
3. 28 and 15

**Part II:**

6. Using $ΔACB$ and $ΔCDB$, set up the following proportion

$\frac{BC}{AB}=\frac{BD}{\\_\\_}$ which results in $AB⋅BD=\\_\\_\\_\\_\\_\\_\\_\\_\\_$

7. Using $ΔACB$ and $ΔADC$, set up the following proportion

$\frac{AC}{AB}=\frac{\\_\\_}{AC}$ which results in $AC⋅AC=\\_\\_\\_\\_\\_\\_\\_\\_\\_$

**If an altitude is drawn to the hypotenuse of a right triangle from the opposite vertex, then a leg of the given right triangle is the geometric mean between the hypotenuse of the triangle and the segment on the hypotenuse that is adjacent to the chosen leg.**



8. In the figure *c* = *x* + *y.* Find

$b^{2}=\\_\\_\\_\\_\\_⋅\\_\\_\\_\\_\\_\\_$ and $a^{2}=\\_\\_\\_\\_\\_⋅\\_\\_\\_\\_\\_\\_$

9. Find the geometric mean between *c* and *x*: \_\_\_\_\_\_\_\_\_\_\_\_

**Part III:** Exercises

Given $ΔNOM$ is a right triangle with right angle $∠MON$ and altitude $\overbar{PO}$, determine the missing measurement in each problem.

1. If *MP* = 7 and *PN* = 28, find *OP*.
2. If *OP* = 8 and *NP* = 6, find *MP*.
3. If *MN* = 12 and *MP* = 4, find *MO*.
4. If *NO* = 7 and *MN* = 12, find *NP*.