**Activity 4.4.5 Dividing a Line Segment in the Coordinate Plane**



In Unit 1 you learned the Midpoint Formula to find the midpoint of any line segment in the coordinate plane.

1. Segment $\overbar{AB}$ has endpoints *A* (–2, 8) and *B* (10, 2).

a. Write the Midpoint Formula. The midpoint of the segment from (*x*1, *y*1) to (*x*2, *y*2) has coordinates (\_\_\_\_\_\_\_, \_\_\_\_\_\_)

b. Let *M* be the midpoint of $\overbar{AB}$. Find the coordinates of *M* (\_\_\_\_\_\_, \_\_\_\_\_\_)

2. Another way to determine the location of *M* is to think of traveling along the segment halfway from *A* to *B.*

1. Find the vector from *A* to *B* = [\_\_\_, \_\_\_]. This indicates that we are moving \_\_\_ units in the horizontal direction and \_\_\_\_ units in the vertical direction.
2. Take half of each component of the vector in (a): [ \_\_\_, \_\_\_ ].
Now move \_\_\_\_ units in the horizontal direction and \_\_\_\_ units in the vertical direction. Starting at point *A*(–2, 8) we will end up with the point (\_\_\_\_, \_\_\_\_)
3. Compare your answers in questions 1b and 2b. What do you notice?
4. Here is another way to look at what we just did. We found a point *M* on $\overbar{AB}$ such that the ratio of its distance from *A* to its distance to *B* is 1 to 1. Check to see that this is true.

a. Find the distance *MA* = \_\_\_\_\_

b. Find the distance *MB = \_\_\_\_\_*

c. Find the ratio $\frac{MA}{MB}= \\_\\_\\_\\_\\_\\_$

4. Now let’s find a point *N* that is $\frac{1}{3}$ the distance from point *A* (–2, 8) to point *B* (10, 2).

a. The vector from *A* to *B* = [\_\_\_, \_\_]. Take $\frac{1}{3}$ of each component: [\_\_\_, \_\_\_].

b. Now move \_\_\_\_ units in the horizontal direction and \_\_\_\_ units in the vertical direction.
Starting at point *A*(–2, 8) we will end up with the point *N* (\_\_\_\_, \_\_\_\_)

c. Find the distance *NA* = \_\_\_\_\_

d. Find the distance *NB = \_\_\_\_\_*

e. Find the ratio $\frac{NA}{NB}= \\_\\_\\_\\_\\_$

f. We can say that the ratio of the distance from *N* to *A* and the distance from *N* to *B* is in the ratio of \_\_\_\_ to \_\_\_\_.

**Consider this problem:**

**Find Point *M* on the line segment below such that point *M* divides** $\overbar{AB}$ **into the ratio 2 to 3. That is, Point *M* is** $\frac{2}{5}$ **the way from Point *A* (3, 1) to Point *B* (13, 6) or Point *M* is** $\frac{3}{5}$ **the way from Point *B* to point *A* along** $\overbar{AB}$**.**



5. We want to partition $\overbar{AB}$ into two parts; one worth 2 units and the other worth 3 units. These five units will make up the whole. You might begin by dividing the horizontal distance (the “run”) into 5 congruent units.

*A*(3, 1) *B*(13, 6)

Horizontal distance = \_\_\_\_-\_\_\_\_= 10

$\frac{Horizontal distance }{\\_\\_\\_  pieces}=$ \_\_\_ units per 1/5

6. Now divide the vertical distance (the “rise”) into 5 congruent units.

*A*(3, 1) *B*(13, 6)

Vertical distance = \_\_\_-\_\_\_= 5

$\frac{Vertical distance }{\\_\\_\\_  pieces}=$ \_\_\_ unit per 1/5

7. Now that we know the number of spaces for 1/5 of the segment in both vertical and horizontal directions from point *A*, determine 2/5 or 2 of the $\frac{1}{5}$ units in BOTH the vertical and horizontal directions.

a. 2/5 in the horizontal direction:

 2$×$\_\_\_ units per 1/5 = \_\_\_ spaces in the horizontal direction from A

b. 2/5 in the vertical direction:

 2$×$\_\_\_units per 1/5 = \_\_\_ spaces in the vertical direction from A



8. Now move from point *A*, in both the horizontal and vertical directions, the number of spaces you determined in question 7.

1. This can also be thought of as adding the horizontal distance to which coordinate?
2. This can also be thought of as adding the vertical distance to which coordinate?
3. The coordinates of point *P*: (\_\_\_\_, \_\_\_\_\_)



**Now let’s generalize. Given segment** $\overbar{AB}$**, *A*(*x*1, *y*1) and *B*(*x*2, *y*2), determine the coordinates of point *P* such that point *P* divides** $\overbar{AB}$ **into the ratio *m/n*.**

9. Since we want point *P* to partition $\overbar{AB}$ into the ratio $\frac{m}{n}$ , then we need to determine how much is $\frac{1}{m+n}$. You might begin by dividing the horizontal distance of the segment into *m+n* congruent units.

*A*$(x\_{1}, y\_{1})$ *B*$(x\_{2}, y\_{2})$

Horizontal distance = \_\_\_\_\_–\_\_\_\_\_

$\frac{Horizontal distance }{\\_\\_\\_\\_  pieces}=$ $\frac{\\_\\_\\_}{\\_\\_\\_}$ units per 1/(*m*+*n*)

10. Now divide the vertical distance of the segment into n congruent units.

 $A(x\_{1}, y\_{1})$ *B*$(x\_{2}, y\_{2})$

Vertical distance = \_\_\_\_\_-\_\_\_\_\_

$\frac{Vertical distance }{\\_\\_\\_\\_\\_\\_\\_  pieces}=$ \_\_\_units per 1/(*m*+*n*)

11. Now that we know the number of spaces for 1/(*m*+*n*) units we can determine how many spaces for *m*/(*m+n*) in both the vertical and horizontal directions

a. *m*$×$\_\_\_units per 1/(*m+n*) = \_\_\_spaces in the horizontal direction from *A.*

 b. *m*$×$\_\_\_units per 1/(*m+n*) = \_\_\_spaces in the vertical direction from *A.*

12. Now move from point *A*, in both the horizontal and vertical directions, the number of spaces you determined in question #7.

1. For the horizontal distance this can found by

 \_\_\_+ $x\_{1}$ = \_\_\_

1. For the vertical distance this can found by

\_\_\_ + $y\_{1}$ = \_\_\_

13. Given segment $\overbar{AB}$, *A*(*x*1, *y*1) and *B*(*x*2, *y*2), the coordinates of point *P* such that point *P* divides $\overbar{AB}$ into the ratio *m* to *n* are ($\\_\\_\\_\\_\\_\\_,\\_\\_\\_\\_\\_\\_$ ).

14. Suppose the coordinates are *A*(*–*2, 8) and *B*(10, 2) and that *m* = 1 and *n* = 1. Use the formula your found in question 14 to find the coordinates of *P*, the point that divides$\overbar{ AB}$ into the ratio 1to 1. Compare your result with what you found in questions 1 and 2.

15. Suppose the coordinates are *A*(*–*2, 8) and *B*(10, 2) and that *m* = 1 and *n* = 2. Use the formula your found in question 14 to find the coordinates of *P*, the point that divides$\overbar{ AB}$ into the ratio 1to 2. Compare your result with what you found in question 4.

16. Suppose the coordinates are *A*(3, 1) and *B*(13, 6) and that *m* = 2 and *n* = 3. Use the formula your found in question 14 to find the coordinates of *P*, the point that divides$\overbar{ AB}$ into the ratio 2to 3. Compare your result with what you found in question 8.

17. Suppose the coordinates are *A*(3, 1) and *B*(13, 6) and that *m* = 3 and *n* = 2. Use the formula your found in question 14 to find the coordinates of *P*, the point that divides$\overbar{ AB}$ into the ratio 3to 2. How does this point compare with the one you found in questions 8 and 13?

 a. Which one is closer to *A*?

 b. Which one is closer to *B*?

c. Do these results make sense? Explain.