**Activity 4.4.2 The Side Splitting Theorem and Its Converse**

1. In Activity 4.4.1 you discovered the **Side-Splitting Conjecture: If a line is parallel to one side of a triangle, then it divides the other two sides proportionally.**



In this figure, line $\overleftrightarrow{DE}$ is parallel to side $\overbar{BC}$ of ∆*ABC*.

1. Write a proportion based on the Side Splitting Conjecture.

$\frac{}{}=\frac{}{}$.
2. Use your proportion to solve for *x.*
3. Since $\overleftrightarrow{DE}$ || $\overbar{BC}$ we know that m$∠$ *ABC* = m $∠$*ADE*. State the theorem that allows us to draw that conclusion.
4. We also know that $∠$ *A* is an interior angle of both ∆ \_\_\_\_\_\_ and ∆ \_\_\_\_\_\_\_\_\_.
5. Therefore these two triangles are similar. State the theorem that allows us to draw that conclusion.
6. Because the two triangles are similar, corresponding sides are proportional, that is

$\frac{AD}{AB}=\frac{}{AC}$.
7. Now substitute the known values and *x* to get this proportion: $\frac{6}{}=\frac{}{x+8}$.
8. Solve this proportion for *x* and compare your result with the value you found in part (b).
9. **Finding proportions:**

In the figure at the right, $\overbar{PQ}$ || $\overbar{KL}$.

a. According to the Side Splitting Conjecture, which of these proportions must be true? (There may be more than one correct answer.)

 (1) $\frac{JP}{PK}=\frac{JQ}{QL}$ (2) $\frac{JP}{PK}=\frac{JL}{QL}$ (3) $\frac{JP}{QL}=\frac{JQ}{PK}$ (4) $\frac{PK}{JP}=\frac{QL}{JQ}$ (5) $\frac{PK}{JP}=\frac{JQ}{QL}$

b. The lengths of segments $\overbar{JP}$, $\overbar{PK}$, $\overbar{JQ}$, and $\overbar{QL}$ are given on the figure. Show that these numbers satisfy the proportion or proportions you found in question (a).

c. As we did in question 1, we can prove that ∆*PJQ* ~ ∆*KJL.* Write a proportion showing that all three pairs of corresponding sides of the two triangles are proportional.

d. Using the numbers given on the diagram, substitute values for *JK*, *JP*, *JQ*, and *JL* in the proportion in part (c).

e. Fill in the blanks:

 Start with the proportion $\frac{20}{8}=\frac{50}{20}$
 Subtract 1 on both sides of the equation $\frac{20}{8}-1=\frac{50}{20}-1$

 Find common denominators $\frac{20}{8}-\frac{}{8}=\frac{50}{20}-\frac{}{20}$
 Write each side of the equation as a single fraction $\frac{}{8}=\frac{}{20}$

f. What do you notice about the last proportion in question (e)?

1. **Adding and subtracting lengths of segments:**

Sides $\overbar{AB}$ and $\overbar{AC}$ are each split into two segments.
Fill in the blanks to make each statement true:

*BD* + *DA* = \_\_\_\_\_\_ *AB* – \_\_\_\_ = *BD*

*CE* + \_\_\_ = *AC* \_\_\_ – *AE* = *CE*

4. Complete this proof of the **Side Splitting Theorem:** If a line is parallel to one side of a triangle, then it divides the other two sides proportionally.

Given: ∆*ABC* with $\overleftrightarrow{DE}$ || $\overbar{BC}$.

Prove: $\frac{BD}{AD}=\frac{CE}{AE}$

*Step 1*. Prove that $∠ADE$ $≅$ $∠ABC.$

*Step 2*. Prove that ∆*ABC* ~ ∆ *ADE*

*Step 3*. Because corresponding sides of similar triangles are proportional, $\frac{AB}{AD}=\frac{AC}{AE}$.

Now subtract 1 from both sides of the proportion to show that $\frac{BD}{AD}=\frac{CE}{AE}$.

****5. Complete this proof of the **Side Splitting Converse:** If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

Given ∆*ABC* with $\frac{BD}{AD}=\frac{CE}{AE}$

Prove: $\overleftrightarrow{DE}$ || $\overbar{BC}$.

*Step 1*. Given that $\frac{BD}{AD}=\frac{CE}{AE}$, add 1 to both sides of the proportion to show that $\frac{AB}{AD}=\frac{AC}{AE}$.

*Step 2*. Prove that ∆*ABC* ~ ∆ *ADE*

*Step 3.* Prove that $∠ADE$ $≅$ $∠ABC$

*Step 4.* Use the result in step 3 to prove that $\overleftrightarrow{DE}$ || $\overbar{BC}$.