**Activity 4.3.2 AA & SAS Similarity Theorems**

**AA Similarity Theorem**

Prove the **AA Similarity Theorem**: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Given: ∆*ABC* and ∆*DEF* with m$∠$*ACB* = m$∠$*DFE* and m$∠$*ABC* = m$∠$*DEF*

Prove ∆*ABC*$ \~ $∆*DEF.*

1. If *C* and *F* do not coincide then translate ∆*ABC* by the vector from \_\_\_\_ to \_\_\_\_ and name the image ∆*A’B’C’*. Sketch the translated image.



2. If *B’* does not lie on $\overleftrightarrow{EF}$, then rotate ∆*A’B’C’* about point *F* or *C’*
 through $∠$\_\_\_\_\_\_. Name the image ∆*A’’B’’C’’*. Sketch the rotated image.



3. $\overbar{C''B''}$will lie on \_\_\_\_ and $\overbar{C^{''}A^{''}}$ will lie on \_\_\_\_ since m$∠$*ACB* = m$∠$*DFE*.

4. Since m$∠$*A’’B’’C’’* = m$∠$\_\_\_\_\_\_\_ then $\\_\\_\\_\\_\\_\\_\\_ ||\\_\\_\\_\\_\\_\\_$ because when \_\_\_\_\_\_\_\_\_\_\_\_\_ angles formed by two lines are a transversal are congruent, the two lines are parallel.

5. Dilate ∆*A’’B’’C’’* using center point \_\_\_\_\_\_ by factor $\frac{}{B"C"}$. Point *B’’’* will then coincide with point \_\_\_\_.



6. By our dilation postulate:

1. $\overleftrightarrow{B’’’A’’’}$ || $\overleftrightarrow{B’'A'’}$(Dilations map a line not through the center onto a parallel line.) Therefore, through point *B’’’* (that is, point *E*) both $\overleftrightarrow{ED}$ and $\overleftrightarrow{B’’’A’’’}$ are parallel to $\overleftrightarrow{B’’A’}$’. Since there is only one line through a given point parallel to a given line, this means that lines \_\_\_\_\_ and \_\_\_\_\_ coincide.
2. $\overleftrightarrow{C’’A’’}$will map onto $\overleftrightarrow{C’’'A’’}'$ (Dilations map a line through the center onto itself). Since two lines ($\overleftrightarrow{B’’’A’’’}$) and $\overleftrightarrow{FA’’’}$ intersect in at most one point. points \_\_\_\_\_ and \_\_\_\_ coincide.

7. Since ∆*A’’’B’’’C’’’* coincides with ∆*DEF*, ∆*DEF* is the image of ∆*ABC* under a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ transformation. This means that ∆*ABC* and ∆*DEF* are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

This completes the proof.

**SAS Similarity Theorem**

Prove the **SAS Similarity Theorem:** If two sides of one triangle are proportional to two sides of another triangle and the angles included by these sides are congruent, then the triangles are similar.



Given ∆*ABC* and ∆*DEF*
with m$∠$*ACB* = m$∠$*DFE* and $\frac{DF}{AC}=\frac{EF}{BC}$.

Prove ∆*ABC*$ \~ $∆*DEF*.

1. If Points *C* and *F* do not coincide then translate ∆*ABC* by
the vector from \_\_\_\_ to \_\_\_\_.
Draw the translated image and call it ∆*A’B’C’*.



2. If *\_\_\_\_*does not lie on $\overleftrightarrow{EF}, $then rotate ∆*A’B’C’* about point \_\_\_\_\_ through $∠$\_\_\_\_\_\_. Draw the image of this rotation and call it ∆*A’’B’’C’’*.

3. Segment \_\_\_\_\_\_ will lie on $\overbar{FE}$ and segment \_\_\_\_\_ will lie on $\overbar{FD}$ since m$∠\\_\\_\\_\\_\\_\\_\\_\\_$ = m$∠$*DFE*.

4. Dilate ∆\_\_\_\_\_\_\_\_\_ using center point *F*(*C’’*) by factor $\frac{DF}{A"C"}$. Point *A*’’’ will then coincide with Point *D* and since $\frac{}{AC}=\frac{EF}{}$ then Point *B*’’’ will coincide with Point \_\_\_\_\_\_.



5. Side $\overbar{A’’’B’’}$’ must coincide with side $\overbar{DE}$ because only \_\_\_\_\_ \_\_\_\_\_\_\_\_ may pass through two points.

6. Since ∆*DEF* is the image of ∆*ABC* under a \_\_\_\_\_\_\_\_\_\_\_\_\_ transformation then $ $∆*ABC*$ \~ $∆*DEF*. This completes the proof.

7. Suppose after step 2 in either of the above proofs, we had a situation like this:



What additional transformation would be needed to complete the proof?