**Activity 3.6.4 Converses of Quadrilateral Theorems**

In Activity 3.6.3 it was shown that quadrilaterals may be represented by variable coordinates. This allows us to prove theorems in general using coordinate methods.

Recall the coordinate representations for Quadrilateral, Trapezoid, Parallelogram, and Rectangle.



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Quadrilateral with vertices Trapezoid with vertices
P(0,0), Q(*a*,0), R(*d*,*e*) and S(*b*,*c*) P(0,0), Q(*a*,0), R(*d*,*c*) and S(*b*,*c*)

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Parallelogram with vertices Rectangle with vertices

P(0,0), Q(*a*,0), R(*a* + *b*, *c*) and S(*b*,*c*) P(0,0), Q(*a*,0), R(*a*,*c*) and S(0,*c*)

1. Prove the **Parallelogram Diagonals Converse**: If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Given: Quadrilateral *PQRS* with *T* the midpoint of both diagonals $\overbar{PR}$ and $\overbar{QS}$.

Prove: *PQRS* is a parallelogram.

Proof: Because *T* is the midpoint of $\overbar{PR}$ its coordinates are ( \_\_\_\_, \_\_\_\_\_)

Because *T* is the midpoint of $\overbar{QS} $its coordinates are ( \_\_\_\_, \_\_\_\_\_)

 Because the *y*-coordinates are equal, we have *e* = \_\_\_\_\_\_\_

 Because the *x*-coordinates are equal we have *d* = \_\_\_\_\_\_\_

 The coordinates of *PQRS* are P(0, 0) Q(*a,*0), R(\_\_\_, \_\_\_), and S(*b*, *c*)

 Therefore, *PQRS* is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

2. Prove the **One Pair Congruent and Parallel Theorem:** If one pair of sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

Given: Quadrilateral *PQRS* with $\overbar{PQ}$|| $\overbar{RS }$and *PQ* = *RS*

Prove: *PQRS* is a parallelogram.

Proof: The slope of $\overbar{PQ}$ = \_\_\_\_\_\_\_\_\_\_\_\_\_

 The slope of $\overbar{RS }$ =

 Because the slopes of parallel lines are the same this means that e – c = \_\_\_\_\_\_\_

 Therefore e = \_\_\_\_\_\_\_

 Use the distance formula to find *PQ* = \_\_\_\_\_\_\_ and *RS* = \_\_\_\_\_\_\_\_\_\_\_.

 Because *PQ* = *RS* we can solve for *d* in terms of *a* and *b.* Show how in the space below:
 (Hint: Assume *a* >0 and *d* > *b.*)

 The coordinates of *PQRS* are P(0, 0) Q(*a,*0), R(\_\_\_, \_\_\_), and S(*b*, *c*)

 Therefore, *PQRS* is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

3. Prove the **Rectangle Diagonals Converse:**  If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Given: Parallelogram *PQRS* with *PR = QS*.

Prove: *PQRS* is a rectangle.

Proof: Using the distance formula we find *PR = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*and *QS* = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

 Now show that *b* = 0.

 The coordinates of *PQRS* are P(0, 0) Q(*a,*0), R(\_\_\_, \_\_\_), and S(*\_\_\_, \_\_\_*)

 Therefore, *PQRS* is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

4. Prove the **Rhombus Diagonals Converse:** If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

Given: Parallelogram *PQRS* with $\overbar{PR}$$⊥$$\overbar{QS}$.

Prove: *PQRS* is a rhombus.

Hint 1: In order show that *PQRS* is a rhombus we need to show that *PQ* = *QR* = *RS* = *SP.* This means that we will need to show that $\\_\\_\\_\\_^{2}=b^{2}+c^{2}$.

Hint 2: Find the slopes of $\overbar{PR}$ and $\overbar{QS}$. Since the slopes of perpendicular lines are opposite reciprocals, their product = \_\_\_\_\_\_. Use this fact to get the result you need from Hint 1.

Now write a proof in the space below: