**Activity 3.2.4 The Triangle Inequality Theorem**

Study the proof and answer all questions in **bold.**

Prove the **Triangle Inequality Theorem:** The sum of the lengths of any two sides of a triangle is greater than the third side.

Given ∆*ABC*.

Prove: (1) *BA + AC > BC*

** (2) *AB + BC > AC*

 (3) *BC + CA > BA*

We will prove (1) since the proofs of (2) and (3) would be similar.

*Step 1*. Construct a circle with center *A* passing though point *C*.

**The circle will intersect line** $\overleftrightarrow{BA}$ **in how many points?**

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Let *D* be the point where this circle intersects ray $\rightharpoonaccent{BA}$ with point *A* between *B* and *D*.



*Step 2*. Join points *C* and *D*.

**Explain why *AD = AC \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_***



*Step 3.* In ∆*ADC* m$∠ $*ADC* = m $∠ $*DCA.*

**What theorem is used here? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

*Step 4*. m $∠$*DCB* > m $∠$*DCA.* **Why?\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

Therefore, m $∠$*DCB* > m $∠$ *ADC.* **Why? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

(Notice that $∠$*BDC* is another name for $∠$*ADC.*)

*Step 5.*In ∆*BCD* we have m$∠$*DCB* > m $∠$ *BDC.*

Therefore *BD > BC* .

**What theorem is used here? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

*Step 6. BD = BA + AD = BA + AC*.

**Explain \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

Step 7. From Steps 5 and 6 we conclude that *BA + AD > BC*.

**Explain** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_