**Activity 1.1.5 The Rationale For Only Evaluating the Objective Function at Corner Points**

In this activity we will investigate the rationale for only evaluating the objective function at the corner points of a feasible region to find the optimal value of the objective function. We will do this by revisiting the peacekeeping problem. In doing so, we will also check that our final answer makes sense in terms of the original problem. (This is the seventh and final step to solving a linear programming problem.) Lastly, we will apply all seven steps to a new problem situation and examine an efficient method for locating the feasible region.

**Why We Only Evaluate the Objective Function at the Corner Points**

In **Activity 1.1.4** you graphed several lines representing force equations in the peacekeeping problems feasible region. In this problem the objective function is $F=x+y$, where *x* represents the number of Hummer units and *y* represents the number of Apache helicopters.

Our goal is to define a method that will always identify the optimal solution within the feasible region.

While making a table of the coordinates of some of the points in the feasible region and evaluating the objective function at those points provides insight into the problem, this process will not let us know if we have found the maximum or minimum value of the objective function. The table below of points in the feasible region and their objective function values can provide us the insight we need to understand why we only need check the corner points in the objective function.

|  |  |  |
| --- | --- | --- |
| *x* | *y* | $$F=x+y$$ |
| 0 | 6 | *F* = 0 + 6 = 6 |
| 2 | 4 | *F* = 2 + 4 = 6 |
| 4 | 6 | *F* = 4 + 6 = 10 |
| 6 | 8 | *F* = 6 + 8 = 14 |

Each of the points in the table belongs to a particular force line. The points (0, 6) and (2, 4) are on the same force line $x+y=6$ so they produce the same force value of 6. This is the case for all force lines. All points ($x,y)$ that satisfy a specific force line will have the same force value.

From the graph we see that force line intersects points on the edges and in the interior of the feasible region*. Since the points on each force line are always going to produce the same force value, we can ignore the interior points and only focus on the points on the edge (i.e. points on boundary lines).*

For example, since the point (0,6) yields a force of 6, there is no need to test the interior point (2, 4), since this interior point is on the same line as (0, 6). The graph below shows how the force line $x+y=6$ intersects the feasible region.

**Feasible Region and Force Line** $x+y=6$



In the same manner, the points (8,0) and (2,6) both deliver the same force since they are on the same force line $x+y=8$. Again, rather than checking all points on this line, we can just check the boundary point (8,0) on this force line.

**Feasible Region and Force Line** $x+y=8$



So we know that we only need to examine the points that are on the perimeter of the feasible-region polygon. However that is still way too many points to check. (The perimeter contains an infinite number of points!) Fortunately, we can leverage an important feature about force lines.

Force lines have the special property that they are parallel to each other. Since the coordinates of any point on a given force line always produce the same force, the greatest force will occur on the force line that intersects the feasible region with the greatest force. Using the edge of a ruler as a model of a force line, move the ruler in parallel fashion so that the *F* value gets larger and until it only touches the feasible region at one point. What happens? If you were to continue to move the ruler in parallel fashion the force line would no longer intersect the feasible region. The point we find that the unique force line intersects the feasible region and yields the largest force value is special point. It is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ point. For the peacekeeping problem, the force line with the greatest force is $x+y=16$ and it intersects the corner point (6,10). The graph below shows this optimal solution and several force lines.

**Feasible Region with Optimal Solution (6,10)**



For our peacekeeping problem, the point (6, 10) yields the largest force value, *F* = 16. Graphically, this is where the force line $x+y=16$ intersects the feasible region at exactly one point. For force values greater than 0 but less than 16, force lines intersect the feasible region in an infinite number of points. For force values greater than 16, the force lines do not intersect the feasible region at all.

We do not usually graph the objective function *F*, because we know that a maximum or minimum value of the objective function must occur at a corner point. Instead, we merely find the corner points and evaluate the objective function at each point. If there are a lot of corner points (due to more constraints), we can use the table feature in our calculator to evaluate the objective function.

The final and last step in solving a linear programming problem is to check that the solution makes sense in the original problem. This is a necessary step when solving real-world problems. We must verify that the optimal solution satisfies the constraints, that we have evaluated the objective function correctly, and that we have included appropriate units in our answer.

**Step 7:** Check that the solution makes sense in terms of the original problem and include appropriate units

1. For the peacekeeping problem, verify that the solution (6,10) makes sense. What does this solution mean?

**The Seven Steps for Solving a Linear Programing Problem**

Let’s recap the seven-step procedure for solving a linear programming problem.

1. Define variables for unknowns in the problem
2. Used defined variables to write constraints for the problem
3. Graph the system of inequalities to obtain the feasible region
4. On the graph, determine and label the corner points of the feasible region
5. Define the objective function
6. Evaluate the objective function at all corner points
7. Check that the solution makes sense in terms of the original problem and include appropriate units

**Saying No to World Hunger**

Let’s apply the linear programming procedure to a new problem situation.

The *Say No to World Hunger* committee has decided to have a fundraiser next month. The idea for the fundraiser is to make and sell t-shirts with tie-dyed designs. There are two designs being offered: a child’s face and a custom design. No more than 100 shirts can be made and production may not cost over $560. It costs $5 to make a child face tie-dye design and $7 to make a custom tie-dye design. The profit for making and selling a t-shirt with a child face design is $3.50 per shirt; the profit for a custom design t-shirt is $5.00 per shirt. Find the maximum possible profit from the fundraiser.

1. Identify the variables for this problem.
2. Write four inequalities that constrain the problem. Remember to include the non-negativity constraints.
3. Graph the inequality constraints on the coordinate plane below. Label and scale the axes. Identify and shade in the feasible region.



1. Find the equations of the constraint lines and the coordinates of the corner points of the feasible region. Reminder: all your corner points are intersections of pairs of the constraint lines. Since we have the constraints *x* ≥ 0 and *y* ≥ 0, *x* = 0 and *y* = 0 are the equations of two of our constraint lines.
2. Write the objective function.
3. Enter the corner points in the table below and find the objective function value for each corner point. Identify the corner point that maximizes the objective function.

|  |  |  |
| --- | --- | --- |
| *x* | *y* | *Objective Function Value* |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

1. State the solution and verify that it makes sense. Summarize your result in a statement using a complete sentence.