**Activity 1.1.3 Peacekeeping Problem: Finding the Objective Function**

In **Activity 1.1.1** we found the best team solutions to the peacekeeping problem by using a guess-check-and-revise method. Then we found the best class solution. In this activity we will continue exploring linear programming, so we can find the best solution. (We may have found it already, but we do not know we have.)

Your teacher has made a master class transparency from the group transparencies for the weight-constraint. Examine the transparency.

* Do you notice a pattern?
* Where are all the solutions with respect to the boundary line?
* Where are all the nonsolutions with respect to the boundary line?

We will now return to groups and examine the graph of the solutions to the area-constraint inequality. Each team recorder should plot on the team transparency the solutions and nonsolutions from the tables of the group members. Again, color green the ordered pairs that satisfy the area-constraint inequality; color red the points that represent the nonsolutions.

1. What pattern emerges when looking at the graphs of the area-constraint inequalities? Explain.
2. Where are the solutions of the inequality with respect to the boundary line?
3. Is it necessary to find a lot of solutions of $1200x+1800y\leq 25200$ in order to graph this inequality? How many solutions do we need to find assuming we have plotted the boundary line?

Using one of the team graphs as the **master class** graph, pool the class data by having each group recorder plot their solutions and nonsolutions on the **class** graph.

1. You may not have been able to make a conjecture about the location – with respect to the boundary line – of points that satisfy the weight-constraint inequality based solely on your individual points or your group. Now, based on the points of the entire class, make a conjecture about the location of points that satisfy the inequality.
2. Is it necessary to find a lot of solutions of $15x+6y\leq 150$ in order to graph this inequality? Explain.
3. How many solutions do we need to find, assuming we have plotted the boundary line?
4. When graphing an inequality such as $x+2y\leq 5$, how could we get a quick graph of the solution set?

**Graphing All Constraint Inequalities Simultaneously**

We have explored the constraint inequalities and their graphs individually. We will now graph all constraint inequalities on the same axes and find their common solution. This is the third step in solving a linear programming problem.

When we graph all the inequalities on the same axes, the region that contains the points that satisfy all the constraints simultaneously is called the *feasible region.* The feasible region is the *solution* of a set of inequalities.

Let’s revisit the peacekeeping problem.

The United Nations (UN) needs to move Hummers units (one Hummer unit has 5 Hummer vehicles) and Apache helicopters for ground and air support to a region in the world for peacekeeping purposes. A ship is used to transport these vehicles and can accommodate a total equipment weight of 150 tons. Each Hummer unit weighs 15 tons and each Apache helicopter weighs 6 tons. Each Hummer unit takes up an area of 1,200 square feet and each Apache helicopter take up an area of 1,800 square feet. The ship has a total of 25,200 square feet of area to use for transport. What combination of Hummer units and Apache helicopters will yield a maximum force? Force is defined as the total number of Hummer units and Apache helicopters.

**In Steps 1 & 2 we defined the variables and created constraint inequalities:**

* Let *x* represent the number of Hummer units to be transported.
* Let *y* represent the number of Apache helicopters to be transported.
* Weight constraint: $15x+6y\leq 150;$ Area constraint: $1200x+1800y\leq 25200$
* Implied constraints: $x\geq 0, y\geq 0$

**Step 3:** Graph the system of inequalities to obtain the feasible region.

1. Look carefully at the overlay of the graph of the area-constraint inequality and graph of the weight-constraint-inequality. You should see a region in the shape of a quadrilateral that appears to only have green points. Some parts of the first quadrant will have only red points; other parts will have red and green points.

The feasible region is the quadrilateral region that only has green points. Why?

The graph below shows the boundary lines and the four parts of the first quadrant defined above. Region B contains all the possible solutions of the peacekeeping problem. Region B contains all the ordered pairs that satisfy the inequalities: $15x+6y\leq 150$, $1200x+1800y\leq 25200$, $x\geq 0$ and $y\geq 0$.

1. The feasible region is usually shaded. Lightly shade in Region B.

**Graph of Feasible Region**

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**Corner Points**

So far we have completed the following steps:

**Step 1:** Clearly define each variable to be used in the constraints.

**Step 2:** Develop all constraints. Recall that many real world problems must include the constraints of *x* > 0 and *y* > 0.

**Step 3:** Graph the system of inequalities to obtain the feasible region. Be sure to label the graph and shade the feasible region.

The four vertices of Region B are very important. (We will explore why in the next lesson.) Region B is bounded by four boundary lines which we derived from problem constraints.

Finding the coordinates where the boundary lines intersect is the fourth step in solving a linear programming problem.

1. To find the intersection of $15x+6y=150$ and $1200x+1800y=25200, $remember you can use the equivalent system: area constraint line, $2x+3y=42$ and weight constraint line, $5x+2y=50$. Why?

**Step 4:** On the graph, determine and label the corner points.

1. Find the point where the boundary lines $x=0$ and $y=0$ intersect.
2. Find the point where the boundary lines $1200x+1800y\leq 25200$ and $x=0$ intersect. These lines intersect at the *y*-intercept of the constraint line.
3. Find the point where the boundary lines $15x+6y\leq 150$, and $y=0$ intersect. These lines intersect at the *x*-intercept of the constraint line.
4. Find the point where the boundary lines $1200x+1800y\leq 25200$ and $15x+6y\leq 150$ intersect. To find this corner point you must solve a system of linear inequalities.
5. Plot the corner points and label them with their coordinates on the graph on the previous page.

**The Objective Function**

Now that we have identified the feasible region (the region with all the points that satisfy all the constraints) and the corner points, we now need to select the best (optimal) solution. This is the fifth step in the process.

**Step 5:** Define the objective function.

In this step we need to determine the *objective function* - a function that describes the quantity that needs to be maximized or minimized. For the peacekeeping problem, our objective is to maximize force – the total number of Hummer units and Apache helicopters that we can transport. The objective function must be defined in terms of the variables that we established: the number of Hummer units, *x*, and number of Apache helicopters, *y*.

In general, when we have a linear function that must be optimized (minimized or maximized), whose variables are subject to linear constraints (linear inequalities), we have a *linear programming problem*. An objective function is said to be linear when the graph of the solutions of $f\left(x,y\right)=c$ forms a line. Solving a linear programming problem means finding the optimal (maximum or minimum) value of the objective function.

Our peacekeeping problem meets these conditions. Let’s now define the objective function.

1. Reread the problem to create the objective function. Focus on the part of the problem statement that discusses maximizing the force.

**Evaluating the Objective Function**

Of the many points in the feasible region, the corner points are the ones that can maximize (or minimize) the objective function. (This will be explained in the next lesson.) The sixth step in the linear programming procedure is to evaluate the objective function at the corner points to find the optimal solution.

**Step 6:** Evaluate the objective function at all corner points.

1. Evaluate the objective function from Question 15 at the corner points (0,0), (0,14), (10,0) and (6,10). What is the maximum force for the peacekeeping problem?

**Section 7: Six of the Seven Linear Programming Steps**

So far we have completed the following steps:

1. Clearly define each variable to be used in the constraints.
2. Develop all constraints. Recall that many real world problems must include the constraints of *x* > 0 and *y* > 0.
3. Graph the system of inequalities to obtain the feasible region. Be sure to label your graph and shade the feasible region.
4. On the graph, find and label the corner points with their coordinates.
5. Define the objective function.
6. Now evaluate the objective function $f=ax+by$ at all corner points. Make a table or use the table feature of your graphing calculator. What is the maximum or minimum value?

Next we will examine the rationale behind Step 6 and add the important, mysterious Step 7.