**Unit 4: The Rational and Power Functions**

**UNIT OVERVIEW**

22.5 days

This unit provides a study of rational functions, building on the students’ familiarity with other function families (linear in algebra 1, quadratic in algebra 1 and unit 2 of algebra 2, polynomial in algebra 2 and exponential in algebra 1). Rational functions are introduced in the context of a science experiment that discovers the Inverse Square Law (for Light) and hence requires the study a new function family and a review of integral exponents. Students will later use data that they have researched on the internet and then model, using the power regression on a grapher or other available technology. This will provide the motivation to review and extend their earlier work in unit 7 of algebra 1 with rational exponents and the work done in unit one of this course with radical notation. A second modeling example will require a negative rational exponent. Modeling will require students to problem solve, research, and use a discovery approach to learning key features and behavior of simple rational functions. Students will explore the graphs of the power family and the roles of the parameters *a* and *p* in f(x) = *a*x*p* . Selected members of the rational family will be explored so that end behavior can be determined, vertical asymptotes and location of holes can be determined and students can determine if a graph will cross its horizontal asymptote. Students will also add, subtract, multiply and divide rational expressions so they can see that rational expressions form a system analogous to the rational numbers. They will solve rational equations.

By the conclusion of the unit they will see that polynomial functions are a subset of the rational function family and that some rational functions are also members of the power family. The power family permits students to look back upon linear functions in the guise of direct variation and the power family and simple rational family in the guise of indirect variation and then to extend both to other variation situations, for example, the area of a circle is directly related to the square of the radius. The interrelationship between multiple representations verbal (words), numerical or tabular, graphical, and symbolic (equations) will continue to be emphasized. Comparisons and contrasts with all functions studied to date will continue to be made and by the end of the unit students should see that the polynomial function family is built on linear and quadratic functions, and that the rational family is built on the polynomial family. Students will continue to be asked to defend their statements, to formulate definitions and to use precise mathematical language and appropriate symbols.

**Investigation 1** introduces students to the rational function family using a science experiment and later data from a treadmill can be modeled by a rational function. The indirect variation family also belongs to the power family, providing an opportunity to study the entire power family. Then direct variation is reviewed and extended to situations where one variable varies directly (and later indirectly) to the power of another variable. Work with integral and more generally rational exponents is reviewed briefly. Students should have completed Activities 7.2.2- 7.2.4 in Algebra 1. If unit 7 was omitted these activities can be completed in this investigation before the review exercises provided in activities 4.1.2 and 4.1.6 are assigned.

**Investigation 2** has students model data they have researched and use the power regression on their grapher to study the roles of the parameters, *a* and *p* in f(x) = *a* x*p* and how changes in *a* and *p* affect the graphs of the defining equations. They compare and contrast the power family with the exponential and polynomial families, by equation, graph and in selected instances by table.

**Investigation 3** has students examine more complex rational expressions and graph rational functions with more complicated defining equations. Reducing, combining rational expressions with like denominators, and dividing by a monomial will be studied. Factoring denominators to determine excluded values, equations of vertical asymptotes and holes (+) will also be studied. Dividing by a binomial (studied in unit 3) will be used to determine if the graphs of selected rational functions have an oblique asymptote (+).

**Investigation 4** has students build upon the work done in investigation 3 and extend to include adding expressions with unlike denominators and multiplying and dividing rational expressions. Students will not only manipulate expressions symbolically but also using a context, model situations with a rational expressions.

**Investigation 5** hasstudents solve rational equations and they have to be sensitive to the possibility of extraneous roots. Students will solve word problems that need a rational equation. They will also solve equations from all families studied to date.

**Essential Questions**

* What is the advantage of using a function or an equation to model a real world relationship?
* What is the advantage of using a graphical representation of a function to model a real world relationship?
* Why do some mathematical models have limitations when used to model a real world situation?
* How do the mathematical models you have studied so far differ from one another? How are they similar?
* When you are deciding upon a mathematical model to use, what factors must you consider?

**Enduring Understandings**

* The arithmetic of rational expressions uses the same rules that as the arithmetic of rational numbers.
* In solving rational equations it is possible that nonequivalent equations may be generated making the check is essential.

**Unit Understandings**

* Some functions have vertical asymptotes and unique behavior near one
* Why do some functions have horizontal asymptotes?
* What is the significance of a vertical or horizontal asymptote especially in a contextual setting?
* Why can rational equations have extraneous roots?
* How does inverse variation differ from direct variation?
* How does a power function differ from polynomial functions with only one term?

**Unit Contents**

Investigation 1: Indirect Variation Functions (4 days)

Investigation 2: Modeling with Power Functions(4 days)

Investigation 3: Graphs of Rational Functions(4 days)

Investigation 4: Operations on Rational Expressions (3 days)

Investigation 5: Rational Equations (3 days)

Performance Task: Eliminating Alcohol for the Human Body(1.5 days)

Mid Unit Assessment (1 day)

Review for Unit Test (1 day)

End of Unit Test (1 day)

**Common Core Standards**

*Mathematical Practices #1 and #3* *describe a classroom environment that encourages thinking mathematically and are critical for quality teaching and learning. Practices in bold are to be emphasized in the unit.*

**1. Make sense of problems and persevere in solving them.**

2. Reason abstractly and quantitatively.

**3. Construct viable arguments and critique the reasoning of others.**

**4. Model with mathematics.**

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

**Common Core State Standards**

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

A.SSE.1b Interpret complicated expressions by viewing one or more parts as a single entity.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship.

F.IF.5Relate the domain of a function to its graph and where applicable, to the quantitative relationship it describes.

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F.IF.7d(+)Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, by table, or verbally)

F.BF.3Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k ( both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable in a modeling context.

A.APR.1bRewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for more complicated examples, a computer algebra system.

A.APR.6Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for more complicated examples, a computer algebra system.

A.APR.7(+)Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous roots may arise.

A.REI.11b Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equations f(x) = g(x); find the solutions approximately, e.g. using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

**Assessment Strategies**

**Performance Task**

The Unit 4 Performance Task focuses on the graphs of rational functions with an emphasis on the interpretation of horizontal and vertical asymptotes in the context of elimination of alcohol from the body. Students will need to be able to algebraically manipulate rational functions, solve rational equations, find domains in context, and determine the equation of an inverse function. The mathematical model of how alcohol is eliminated from the body as well as many of the insightful questions posed in the Performance Task are being used with the kind permission of James Sandefer and Rosalie Dance who developed the material with an NSF grant and published their module in the Consortium 66 HIMAP pullout section in 1998.

**Other Evidence (Formative and Summative Assessments)**

* Exit slips
* Class work
* Homework assignments
* Math journals
* Unit 4 Mid-unit assessment
* Unit 4 End of Unit assessment

**Vocabulary**

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| Amount of light called power or luminosityAsymptote-- vertical and horizontalAreaBrightnessCombined VariationContinuousDiscontinuousDomainDegreeDirect variation or varies directly or directly proportionalDistanceExcluded ValuesExtraneous solutionEquivalent expressionsEnd BehaviorFactorsfactor/or constant of proportionalityHorizontal asymptoteIndirect (Inverse) Variation or inverselyInverse square law Joint variation | OutlierproportionalParameterPower regressionPower function familyProportionRational functionRational EquationRangeRatioScaling factor or stretch /compression SolutionSolveSquares illuminatedValues excluded from the domainVertical asymptote |