**(+) Activity 4.3.6 – Graphing Rational Functions V**

**Section I (+)**

Sketch the graphs of the following functions. For each graph, identify the vertical and horizontal asymptotes, holes, zeros, *x*-intercept(s), *y*-intercept, and domain. If something does not exist, state so.

1.  $f\left(x\right)=\frac{x - 1}{x^{2} + 2x - 3}$

V.A.:

H.A.:

Hole:

Zeros:

x-intercept(s):

y-intercept:

 Domain:

1. $g\left(x\right)=\frac{x-1}{x^{2}+5x-6}$

V.A.:

H.A.:

Hole:

Zeros:

x-intercept(s):

y-intercept:

Domain:

1. $k\left(x\right)=\frac{x^{2 }+ 3x - 10}{x + 5}$

V.A.:

H.A.:

Hole:

Zeros:

x-intercept(s):

y-intercept:

Domain:

1. $h\left(x\right)=\frac{x^{4}+x^{2}+1}{x^{2}-1}$

V.A.:

H.A.:

Hole:

Zeros:

x-intercept(s):

y-intercept:

Domain:

**Asymptote or Hole?**

To determine whether the graph of a rational function has a vertical asymptote or a hole at a restriction, proceed as follows:

1. Factor the numerator and denominator of the original rational function *f*. Identify any restrictions on *f*.
2. Reduce the rational function to lowest terms, naming the new function *g*. Identify any restrictions on the function *g*.
3. Those restrictions of *f* that remain restrictions of the function *g* will be used to define the vertical asymptotes of the graph of *f*.
4. Those restrictions of *f* that are no longer restrictions of the function *g* will be the x-coordinates of the “holes” of the graph of *f*.
5. To determine the coordinates of the “holes”, substitute each restriction of *f* that is not a restriction of *g* into the function *g* to determine the *y*-value of the hole.
6. Write the equation of a rational function *g*(*x*) who has a zero at 0, vertical asymptotes at

*x* = 2 and *x* = -1, and a horizontal asymptote at *y* = 3.

1. Write the equation of a rational function *f*(*x*) who has zeroes at 3 and -4, a hole at

*x* = -2, a vertical asymptote at *x* = 5, and a horizontal asymptote at *y* = 4.

Name the vertical asymptotes, horizontal asymptotes and holes in the graphs of the following equations. **Do not use a calculator**.

1. $f\left(x\right)=\frac{3(x - 2)(x + 4)}{(x + 3)(x + 4)}$ V.A. H.A.

Hole

1. $f\left(x\right)=\frac{(x - 4)^{2}}{(x + 4)(2x - 3)(\frac{1}{4 }x + 8)}$ V.A.

H.A. Hole

**Section II (+)**

If the numerator's degree is one degree greater than the denominator’s degree, you have a *slant asymptote* of the form $y=mx+b$. You will need to use long division to find the equation of the slant asymptote.

**Example:** Find the slant asymptote of the rational function $f\left(x\right)=\frac{-x^{2} + 3x + 1}{x^{ }- 2}$.

**Solution***:* Perform long division.

 -*x* + 1

 *x* – 2 | -*x*2 + 3*x* + 1

 -*x*2  + 2*x*

 *x* + 1

 *x* – 2

 3

Ignore the remainder and use only the polynomial part. So, $y=-x+1$ is the slant asymptote (S.A.) as shown above with the dotted line.

Graph the following functions and identify the features listed below.

1. $f\left(x\right)=\frac{x^{2}+ 1}{x}$

V.A.:

S.A.:

Hole:

Zero(s):

x-intercept(s):

y-intercept:

Domain:

1. $f\left(x\right)=\frac{x^{2} - x - 12}{x - 2}$

V.A.:

S.A.:

Hole:

Zero(s):

x-intercept(s):

y-intercept:

Domain:

1. $f\left(x\right)=\frac{3x^{2} + 2}{x - 1}$

V.A.:

S.A.:

Hole:

Zero(s):

x-intercept(s):

y-intercept:

Domain: