**Activity 3.3.6 That Sums it Up!**

**Triangle Numbers:** From the early days of mathematics, Pythagoras was interested in arrays of dots that have geometric shapes. A pattern of one such array is show below.

Figure: 1 2 3 4

Number: 1 3 6 10

The numbers 1, 3, 6, and 10 are called triangle numbers because that number of dots can be arranged in the shape of a triangle with each successive row containing one more dot than the previous row.

1. How many dots are in the next three shapes in the pattern? Explain how you figured out the pattern.

2. How many dots would be in the 10th figure?

3. Pythagoras found a way to calculate the number of dots in the nth figure. Here’s how he figured out the number of dots in, say, the 5th triangle number. First he put all the dots is a right triangle starting with one dot in a row and working down to 5 dots in a row.

4. Then he repeated the number of dots, but only starting with the row of five dots and working down to a row of one dot, and he put them next to each other as shown below.



5. How many dots are in the rectangular array?

6. How many dots are therefore in the triangle?

7. Use this approach to create a formula that calculates the number of dots in the nth triangle number.

8. Therefore, the sum 1 + 2 + 3 + ••• + n =

9. In a class of 20 students, the teacher lined up all the students and then asked the first person to go down the row, shake everyone’s hand and sit down. Then he asked the next person to do the same thing with the rest of the line. The students continued to shake hands until the next to last person shook the last person’s hand. How many handshakes were there in total?

**Square Numbers:** Another geometric array that fascinated Pythagoras was the square number. Pythagoras noticed that the square numbers could also be written as a sum of whole numbers. Look at how Pythagoras counted his square numbers and see if you can find what sum each square number is.

Figure: 1 2 3 4

Number: 1 4 9 16

10. What are the next three square numbers?

11. How can I write each square number as a sum?

12. What does the sum: 1 + 3 + 5 + ••• + 99 equal?

13. The sum of the first n odd numbers equals .