**Unit 2: Investigation 4 (2 Days)**

**Congruent Triangles by SSS**

***CCSS: G-CO 8***

**Overview**

The third triangle congruence theorem will be proved using transformations as well as two previously proved theorems, SAS Triangle Congruence and the Isosceles Triangle Theorems.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Explain how transformations are used to form a kite given two triangles that meet the SSS criterion
* Explain the role of isosceles triangles in proving the SSS Triangle Congruence Theorem
* Apply the SSS Triangle Congruence Theorem to show that two triangles are congruent and that corresponding parts are also congruent.

**Assessment Strategies: How Will They Show What They Know?**

* + The **Journal Entry** asks students to explain the proof of the theorem.
	+ **Exit Slip 2.4** asks students to identify pairs of congruent triangles and the theorem that justifies their congruence.

**Background Information for Teachers**

As with the SAS and ASA triangle congruence properties, the Common Core State Standards call for using transformations to prove these as theorems rather than making them postulates. The sequence of theorems in this course is similar to the one used by Euclid in the *Elements.* Euclid first proved SAS as his Proposition 4 in Book 1 and then used that to prove the Isosceles Triangle Theorem as his Proposition 5. His Proposition 8 was SSS triangle congruence, which relied on the “superposition” of one triangle on top of another, which as we have seen is more accurately described by a series of isometries. Here we use an approach that is different from his in that we place the two triangles so that they lie on opposite sides of the line containing a common side, thus forming a kite. Variations on this approach have been used various textbooks over the years (e.g. Milne, 1899; Welchons Krikenberger and Pearson, 1958. Coxford and Usiskin, 1968;).

**Launch Notes**

Start with a video on kite making or kite flying. For example

<https://www.youtube.com/watch?v=pe2PddwZJAI>

[www.youtube.com/watch?v=5X8wwHpqeU0](http://www.youtube.com/watch?v=5X8wwHpqeU0)

Examine various types of kites that are flown. Find that most of them have a line of symmetry. Discuss the simplest form of kite with four sides. Show how a common English word then is given a precise mathematical meaning. Define kite as a quadrilateral with two pairs of consecutive sides that are congruent. If necessary, discuss the meaning of consecutive. After discussion with the class you may want to specify that the pairs must not overlap, that is a quadrilateral with three congruent sides is not necessarily a kite.

**Teaching Strategies**

In **Activity 2.4.1 Making Kites** four pairs of congruent triangles are given. Students cut them out to place them together to make kites and other figures. The key question to answer for each pair is “how many was can a kite be formed?”

The first pair contains acute scalene triangles, with which there are three ways to make a kite and 3 ways to make a parallelogram

The Second pair contains right scalene triangles with which there is one way to make a kite, two ways to make an isosceles triangle, and three ways to make a parallelogram.

The third pair contains obtuse scalene triangles. There is one way to make a kite and two ways to make a non-convex figure called a dart and three ways to make a parallelogram. (The vocabulary convex, non-convex, and dart, will emerge from the discussion of this example)

The fourth pair contains acute isosceles triangles. Only three possible figures can be made. If the triangles share their bases we have a rhombus, which is a special type of kite as well as a parallelogram. It they share legs we can get a kite or a parallelogram which is not a rhombus. Relationships among the special quadrilaterals will be studied in more detail in Unit 3.

**Group Activity**

There are several advantages to using **Activity 2.4.1** with groups of four. First you can save time by having each member assigned to cut out only one pair of figures. If you run them off on paper of different colors, it will be clear who is responsible for each pair. Also you will need less paper and fewer scissors. More importantly by working a group students will exchange ideas and answer the question themselves. Each group may then present their findings to the class as a whole.

One conclusion that you will want students to draw from this experience is that given two cut-out congruent triangles there is always at least one way to make a kite. This activity helps student understand the figure that will be formed during the proof of the SSS Triangle Congruence Theorem.

You may extend student understanding of kites with **Activity 2.4.2 Kites in the Coordinate Plane.** Given four vertices in the coordinate plane students will use the distance formula to determine whether or not the figure is a kite. If it is they will identify the line of symmetry and test to see if the diagonals are perpendicular to each other. This activity helps keep their coordinate geometry skills alive. This may be assigned for homework the first day.

**Activity 2.4.3 The SSS Triangle Congruence Theorem** is the essential activity of this investigation. This may be the major in-class activity for the second day. Many details of the proof are subtle and may be glossed over. For example, students should be able to conclude from the diagram that m $∠$*ACE’’* = m $∠F''CE'' $ + m $∠$*­­­­ F’‘CA* without necessarily questioning the justification for this step. You may introduce the Angle Addition Postulate if your class decides it needs it.

You may want to point out that in the proof in Activity 2.4.3 it is assumed that *F’’* and *C* lie on opposite side of $\overleftrightarrow{AB}$. If that is not the case, one additional transformation (reflection over $\overleftrightarrow{AB}$) is needed to form the kite.

**Differentiated Instruction (For Learners Needing More Help)**

Some students may have trouble seeing the component parts of the figure in Activity 2.4.3. Have them draw the two congruent triangles (∆*ABC* and ∆*AF’’C*) separately. Then have then draw the two isosceles triangles (∆*CAF*’’ and ∆*CBF*’) separately. They can then focus on the individual triangles as they go through the steps of the proof; e.g. to show that $m∠ACF''=m∠AF^{''}C$ they should look at ∆*CAF’’.*

**Differentiated Instruction (Enrichment):** In Activity 2.4.1 students may discover that if the triangles are obtuse they can make a kite only if they match the longest sides with each other. Some students may try to prove the theorem when a pair of shorter sides is matched so that the figure formed is a dart rather than a kite. That proof will also use two pairs of isosceles triangles, but the measures of the angles at *C* and *F’’* will found by subtracting rather than adding the pairs of congruent angles in the triangles.

**Journal Entry.** Explain how a kite is used to prove the SSS Congruence Theorem. Look for students to explain that one triangle together with the image of the other can share a side and form a kite. The kite is then divided into two isosceles triangles.

**Activity 2.4.4 Rigid Structures** demonstrates an application congruence.In this activity students will discover why triangles are used in constructing rigid structures whereas polygons with more than 3 fixed sides may collapse. This is directly related to the SSS Congruence Theorem since once three sides of a triangle are determined the angle are fixed. There are two methods for demonstrating this, one using dynamic geometry software and the other hands-on materials. This activity may be used for individual or group work or as a classroom demonstration by the teacher. You may also want to remind students of Activity 1.3.2 from the Connecticut Core Algebra 1 curriculum in which they constructed bridge trusses. Note the role triangles play in making the structures rigid.

In **Activity 2.4.5 Using the SSS Triangle Congruence Theorem** students are given pairs of triangles with the measures of certain angles and sides. They will be asked if the triangles can be proven congruent and if so which theorem they would use. In addition they will be given some problems in which after a pair of triangles is shown to be congruent students will be asked to show that a given pair of corresponding parts are congruent. All or part of this activity may be assigned as homework.

The **Exit Slip** for this investigation asks students to identify pairs of congruent triangles and the theorem that guarantees their congruence.

**Closure Notes**

Ask students to summarize the three ways they have learned to prove that two triangles are congruent. Ask them to speculate about whether these combinations of congruent parts will also guarantee that triangles are congruent: AAS, SSA, AAA.

**Vocabulary**

consecutive (sides, angles)

convex (polygon)

dart

kite

non-convex (polygon)

**Resources and Materials**

Scissors (Activity 2.4.1)

Graph Paper (Activity 2.4.2)

Sticks or straws cut to different lengths (Activity 2.4.3)

Paper strips with punched holes and fasteners (Activity 2.4.3 and 2.4.4)

Dynamic Geometry Software (Activity 2.4.4)