**Teacher Launch 2.2 Notes for Crumpled Paper Toss**

To motivate the desire to find the solution to a quadratic equation, ask 4 volunteers to come to the front of the room to illustrate 4 instances of projectile motion –all from a height of a hand stretched overhead: drop a crumpled paper ball, toss the ball straight up, toss the ball at a 75° angle to the floor, and toss the ball parallel to the floor, and ask the class to

a) sketch the graph of the trajectory of the projectile that is vertical position y as a function of horizontal position x, and

b) sketch a graph of the height of the object as a function of time,

c) estimate the parameters for the function

h(t) = -.5 g(t2) +vy0 (t) +h0

where h is the height of the projectile above ground,

t is the time elapsed since the projectile was released,

g is the standard acceleration due to gravity, (use 9.8 m/s2, or 32 ft/s2)

vy0 is the initial velocity in the vertical direction,

h0 is the initial height of the object.

d) sketch a horizontal line to indicate various heights and write an equation to solve that determines when the projectile will be at a given height for either function.

In addition to providing fodder for quadratic equations, this launch serves two other purposes: you want students to be able to see that a formula tells a story, and that it is great fun to be able to read all that is packed into a formula. Another goal is to help students distinguish between the function that gives the trajectory or path of a projectile and the function that tells the height as a function of time. By the conclusion of the launch, students should be able to distinguish between the function for the vertical distance of a projectile as a function of time and the function that models the trajectory of the projectile, i.e., vertical distance as a function of horizontal distance. They should be able to write the function for h(t) with the given parameters for standdard gravity, initial velocity and initial height, and they should be able to write an equation that when solved would tell when the projectile attains a given height – including a height of zero that is ground level.

If you choose, you can bring use a motion detector to find the equation for height as a function of time, and work with more accurate numbers for the parameters. You could also have students take a time-lapse video with a video camera. They may have this feature on their tablet or cell phone.

Have volunteer #1 hold a piece of crumpled paper above his head and drop it from about 2 meters high. Ask the class to estimate the initial height? (about 2 meters): Initial velocity? (0 meters per sec); Ignoring the minor effects of air resistance, what is the only force acting on the paper? (the acceleration due to gravity that is 9.8meters per second every second).

Tell students that Galileo demonstrated the formula for the free fall of an object when he did the experiment in 1589 when he dropped cannon balls of differing weights from the Leaning Tower of Pisa to see if the heavier one hit the ground sooner than the lighter one did. Ask students to vote: will the heavier ball hit the ground before the lighter ball? Yes or no. Don’t give away the answer, yet, but challenge students to think about it or research it. By the end of the launch, students may recognize that since the only force acting on a projectile is gravity, the projectiles will hit the ground at the same time. They may notice that the formula for h(t) does not require information about the weight of the projectile.

Distribute the **Investigation 2.2 Student Launch Sheet** entitled: **Launch 2.2 Crumpled Paper Toss** for students to record the information for each scenario of paper tossing. Tell the students the formula for the height of a projectile as a function of time – projectile motion problems are defined to be those for which the only force acting on the projectile is gravity, and the projectile stays close enough to the earth’s surface so that gravity is constant. The formula for the height of a projectile as a function of time is: h(t) = $-\frac{1}{2}$g(t2) +vy0 (t) +h0 . Be sure to define the variables and parameters. This information is on the student sheet **Launch 2.2 Crumpled Paper.**  Have students fill in the values of each parameter to find the formula for this first scenario: h(t)=-4.9t2+0t+2 , sketch its graph and make a table of values for t = 0, .1, .2, .3 and .4 ... Note that the coefficient of the squared term is negative, indicating that gravity decreases the height of the projectile, and the larger t is, the more effect the force of gravity has. The end behavior of the function is due to the effect of gravity. Observe that the drop in height between the first tenth of a second is less than drop in height for the second tenth of a second. Observe that the first differences are not positive, so h(t) is not linear. Now take the second differences, use units of measure. Note the second differences, the average acceleration for each tenth of a second, are constant – in fact the constant is -9.8 m/s2. One characterization of a second degree polynomial is that its second differences are constant.

Students may notice that the slope of h(t) is negative and the parabola is concave down, so the velocity is “more and more negative,” but the ball is falling faster and faster. Explain that the speed , the absolute value of velocity, at which the ball falls is increasing, but the velocity itself – the slope – is indeed a negative number that has greater and greater magnitude. Therefore, the velocity is decreasing, and function is indeed concave down (not linear) Note that the slope ( average velocity) between the points listed in the table show that velocity is decreasing. (-.49, -1.47, -2.45, -3.43m/s…).

Now have volunteer #2 toss the projectile straight up. The projectile should leave the person’s hand at about 2 meters high, and attain a maximum height of about 3 meters. Tossing a projectile straight up (at a 90 degree angle to the ground) at about 4.5 meters per second will give you a maximum height of about 3 meters at about .46 seconds after the projectile is released. The projectile will hit the floor in a bit less than 1 second.

First have students sketch the graph of trajectory of the projectile – the path that it follows – the variable for the horizontal position is on the x axis and the vertical position on the y axis. The function for the trajectory or path is vertical height as a function of horizontal distance from the starting point. Next have them graph the vertical position function: height as a function of time. Since the force of gravity is 9.8 m/s2, we have h(t) = -9.8/2 t2 + 4.5 t + 2 . Students can experiment with changing the parameter vy0 on their graphing calculator to observe how the change in the initial velocity of the projectile in the vertical direction affects the maximum height the projectile obtains.

Ask students how they would alter the value for the initial velocity in the vertical direction if the projectile were tossed downward at 4.5 meters per second instead of upward.

Student #3 should toss the crumpled paper with roughly the same gentle toss, but this time with a bit of a horizontal trajectory that forms about a 75 degree angle with the floor. Historically, the study of projectile motion became more important during the Renaissance, when Galileo lived, because canons were used more and more for warfare.

Tell students to assume that the initial velocity is the same as that of volunteer #2: about 4.5 meters per second. Will the projectile reach the same height? Help student see that now some of the energy from the toss is directed horizontally, it is not all directed vertically. Hopefully they see that at this angle, the initial velocity in the vertical direction will be greater than that in the horizontal direction. You can tell them that the vy0 = 4.3 m/s and vx0 = 1.2 m/s (I wouldn’t focus on how the vertical and horizontal components of a vector are found, but you can show interested students how at a later time using right triangle trigonometry. For a projectile that has an initial velocity of 4.5 m/s and an initial trajectory of 75° , the hypotenuse of the right triangle is 4.5 and the angle with its vertex at the origin and initial ray on the positive x axis is 75°. Use sine to find the initial vertical velocity : sin(75°)= vy0/4.5. That is vy0 ≈ 4.3m/sec . The initial velocity in the horizontal direction is approximately 1.2 m/s , and can be found by solving cos(75°) m/sec= vx0/4.5)

Have students graph the trajectory of the projectile. Then have them to observe just the vertical motion of the projectile. What is the highest point that they estimated the ball reaches? (It should be less than 3 meters) . Did it hit the ground in more or less time than compared to the ball that was tossed straight up? (less time)

You want to stress the independence of the vertical and horizontal motions of the projectile. The horizontal distance is given by “distance = rate (time)” or “ sx(t) = vx0(t)” where sx is the distance traveled in the horizontal direction and vx0 is the velocity in the horizontal direction.

Have student #4 toss the crumpled paper with the same initial velocity of 4.5 m/ s in a horizontal direction, parallel to the floor. The initial velocity in the vertical direction is 0.

Regardless of the horizontal motion, the vertical position is given by

h(t) = -.5 g(t2) +vy0 (t) +h0

Therefore, the vertical position function for a ball that is dropped from 2 meters high, h(t)= -4.9t2+2, is the same as if you gently roll a ball off a table 2 meters high, or if you hit the ball hard with a pool cue stick off the table.

A nice animation for understanding that the independence of vertical motion and horizontal motion for projectiles motion is available at The Physics Classroom, Parabolic Motion of Projectiles <http://www.physicsclassroom.com/mmedia/vectors/bds.cfm> ,and the “The Monkey and the Zookeeper” at <http://www.physicsclassroom.com/mmedia/vectors/mzi.cfm> .

If you want more background information about the horizontal and vertical components of projectile motion see <http://www.physicsclassroom.com/class/vectors> , [the Physics Classroom](http://www.physicsclassroom.com/) » [Physics Tutorial](http://www.physicsclassroom.com/class) » Vectors - Motion and Forces in Two Dimensions