Activity 2.2.4 Teacher Notes

This activity gives students some practice and background theory about how to work with the absolute value operator, its relationship with the principal square root of an expression squared, and why we use a ± symbol when we take the square root of both sides of an equation.

Be sure that students understand that a solution $\pm 3 $means 3 or -3. For at least a few equations, have students write out the two solutions separately, not using the ± symbol. Do likewise when students write the solutions obtained from the quadratic formula or completing the square. Have them write $\frac{-b+\sqrt{b^{2}-4ac}}{2a }$ and $\frac{-b-\sqrt{b^{2}-4ac}}{2a }$ as two separate solutions.

Shortly before or after students do the last page of Activity Sheet 2.2.4 on solving equations of the type x2 =c , ask them,

“Why do we write $\pm \sqrt{c}$ when solving an equation of the type x2 =c ?”

Recall that$\sqrt{x^{2}}=\left|x\right|$ . The example shows why this fact yields the positive and the negative of the principal square root when you take the square root of both sides of an equation.

Example: Solve for x: $\left(x-5\right)^{2}=49$

 Since the inverse of squaring is square rooting, take the square root of both sides.

 $\sqrt{\left(x-5\right)^{2}}=\sqrt{49}$

 The square root of a squared expression is the absolute value of the expression.

 $\left|x-5\right|=\sqrt{49}$

There are two square roots of 49: +7 and -7, because 72=49 and (-7)2=49 . The radical symbol indicates the principal square root of 49. The principal (positive) square root of 49 is 7

 $\left|x-5\right|=7$

The expression inside the absolute value bars is either equal to 7 or to -7

For a short cut, we can write that x-5 equal positive or negative 7 this way:

 $x-5=\pm 7$

This is a compound sentence using the conjunction ‘or’ that can be split into its two parts:

$$x-5 equals positive 7 or x-5 equal negative 7$$

 $x-5=7 or x-5= -7$

Solve each of the two equations for x by adding 5 to each side of the equal sign:
 $x=7+5 or x= -7+5$

$ x=12 or x=-2 $

The two solutions to the equation $\left(x-5\right)^{2}=49$ are 12 and -2

If we want to think of this equation and its solution in terms of distance from a point ‘5’, we can say: One solution for x is 12, because the distance between 12 and 5 squared is 49 . The other solution for x is -2, because the distance between -2 and 5 all squared is 49. We found two numbers for x that are 7 units from 5 on the number line. One number was 7 units to the right of 5, the other is 7 units to the left of 5.