**Activity 2.2.1 SAS Congruence**

In this activity you will discover and prove our first theorem about congruent triangles.



1. Included angles. For each pair of sides in ∆*XYZ*, name the included angle.

Sides: and Included Angle:

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2. Experiment. Work with one other student. You will each draw two triangles using a ruler and protractor.

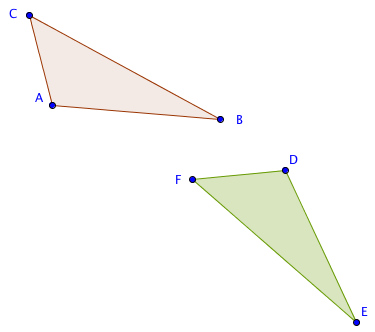
a. Agree upon the measure of two sides of the triangle and the included angle.

Our two sides measure \_\_\_\_\_\_\_\_\_\_\_\_\_\_ and \_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Our included angle measures \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

b. Now draw your triangles. Cut one triangle out and place it on the other. What do you notice?

c. Formulate a conjecture: If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle then \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

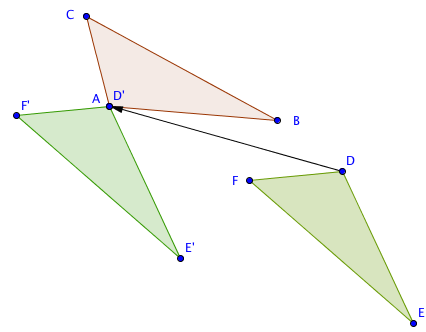
3. Proving the SAS Congruence Theorem. Study this proof and fill in the blanks.

Given ∆*ABC* and ∆*DEF* with

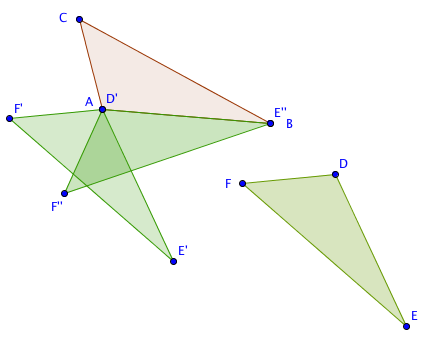
*AB = DE*,  
*AC = DF*, and

m*BAC* = m*EDF*

Prove ∆*ABC*∆*DEF*.

*Step 1*. If *A* and *D* do not coincide then translate ∆*DEF* by the vector from *D* to *A*.

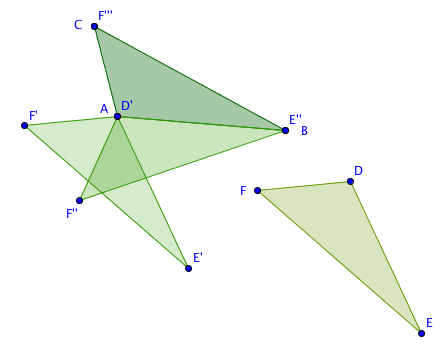
Now *D*’ is the same point as \_\_\_\_\_\_\_.



*Step 2*. If and do not coincide then rotate ∆*D’E’F’* about point *A* through *E’AB*

*Step 3.* will now coincide with since we were given that *AB* = \_\_\_\_\_\_\_.

*Step 4.* If *F’’* is on the opposite side of from *C,* reflect *∆D’’E’’F’’* over .



*D’’* is the same as which point? \_\_\_\_\_

*Step 5*. *E’’’D’’’F’’’* will now coincide with *BAC* since we were given m*BAC* = m*EDF*, and

will coincide with since we were given *AC* = *DF*.

*Step 6*. Therefore *F’’’* coincides with *C* and *E’’’* coincides with *B*.

How many lines can be drawn from point *B* to point *C*? \_\_\_\_\_\_\_\_\_\_\_

Explain why must coincide with :

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

*Step 7*. Since ∆*ABC* is the image of ∆*DEF* under an isometry, ∆*ABC*∆*DEF*.

4. Now state the theorem you have just proved. We will call this the SAS Congruence Theorem.

If \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_,

then \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

5. What properties of transformations were used in the proof?

6. Do you think this proof would work if the angle were not included between the two sides? Explain your thinking.