

Module 3 – EL  
Participant Guide

Accessing the Connecticut  
Core Standards in  
Mathematics

# Meeting the Challenge: CT Core Standards Success for English Learners and Students with Disabilities

Grades K–12

*A Professional Learning Series for  
School Teams Dedicated to the Success  
of ALL Students*



### Connecticut Core Standards Systems of Professional Learning

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The *Meeting the Challenge* project includes a series of professional learning experiences for school teams on Connecticut Core Standards Success for English Learners and Students with Disabilities.

Participants will have continued support for the implementation of the new standards through virtual networking opportunities and online resources to support the training of educators throughout the state of Connecticut.

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## Warm-Up Activities

## Warm-Up Activities

### Think-Pair-Share: Your Learning and Applications from Modules 1 and 2

#### DESCRIPTION

Participants reflect on previous learning from Modules 1 and 2 and then share their reflections with a partner.

#### DIRECTIONS

**Think:** Reflecting on previous learning from Modules 1 and 2, think about:

- an “aha” moment
- a time you “designed to the edges” (term from Module 1 video)
- something that affirmed your current practices
- an application with your school or with your students

**Pair-Share:** Share your reflection with a table partner.

## Benefits for ELs

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How does **Think-Pair-Share** benefit English learner students?

- Provides think time.
    - Suggestion for young learners: “Put on your thinking cap.”
  - Promotes home language retention and development through homogeneous pairings.
  - Promotes English language development when a partner shares in English what his/her non-English speaking partner said: i.e., now that person is also hearing his/her idea expressed in English.
  - Opportunity to rehearse language with a partner in a lower-stress situation prior to sharing; such rehearsal opportunities can have a profound effect on language production.
  - Hearing an idea more than once promotes receptive language development.
  - Every student participates:
    - Students who most need language growth are not typically the ones with their hands up to volunteer.
    - It may also be culturally inappropriate to do so for students with collective cultural backgrounds; however, it is generally appropriate across cultures to share a group or partner idea.
-

- Use differentiated sentence frames to support ELs and provide models for language growth of all learners.
- Easier for students with less confidence/language to share if they have a starting point.
- Structure younger learners’ partner talk further; e.g., ask “birds” to share, then when done, tell “butterflies” to share.
- Tell younger students who their partner is before the sharing begins; e.g., students come to rug in pairs; there would be no more than one triad; no one is left out; no time is wasted.

## Sentence Frames

### DESCRIPTION

Participants share their most important learning from Module 1 using sentence frames.

### DIRECTIONS

1. Think of how you might share your most important learning using one of the frames below:
  - I learned that \_\_\_\_\_.
  - I was surprised to learn that \_\_\_\_\_.
  - \_\_\_\_\_ was the most interesting to me.
  - Another important big idea was \_\_\_\_\_.
  - I felt that \_\_\_\_\_ would have the greatest impact on my teaching.
  - The emphasis on \_\_\_\_\_ seems particularly relevant for teaching students in multicultural, multi-level classes.
2. When prompted, stand up while one person volunteers to share his/her sentence frame (sit down if you had the same/similar idea as the volunteer).
3. The person who shared calls on the next person to share.

## Benefits for ELs

How do **Sentence Frames** benefit English learner students?

- Gives a starting point as well as provides modeling for a range of learners.
- Teacher hears all statements.
- Recommend writing the sentence frames in order of complexity; simplest on top.
- Sentence frames and oral language practice provides models that can later be applied to writing.
- Promote additional sentence variety:

- Use more than sentence starters.
- Provide omissions at the beginning of a sentence.
- Model varied syntax to promote development of complex language; transferable to writing.
- Consider the audience when constructing sentence frames; i.e., with younger ELs use only 2 or 3.
- Use color:
  - Separates frames.
  - Can indicate level of difficulty.

**Note:** When asking students to share learning, use “I learned that” rather than “I learned” so that students are forced to express a complete idea, either an important fact, or better yet, a statement of understanding. Otherwise students will say I learned “about” a topic or “how.” Neither requires statement of important learning.

### Adapted Stand and Share (Kagan)

- Opportunity for rehearsal, repetition, building comprehension, and hearing other ideas.
- Used at the beginning of class provides review of previous learning as well as an orientation for students who were absent the day before.
- Used at the end of class, promotes summary statements, encapsulation of essential learning.
- Productive use of class time:
  - Beginning of class: “bell ringer” activity; student not present do not miss essential new learning.
  - During class: encourages engagement and physical activity; reinforces new learning.
  - End of class period: for older students eliminates “packing up/checking out” before the bell; for younger students utilizes every moment including just before lunch, recess, or dismissal.

### TIPS

- Use when students need to move.
- Use at the beginning of class to review previous learning.
- Use at the end of class to reflect on essential learning.
- Use to support language development but do not use them all the time.
- Write to specifically support the content.
- Observe to make sure most reserved students are ready.
- Observe to seek out students who regularly sit before sharing and appoint them to share first.
- Know that once students are comfortable with this form of sharing, many will try to think of another idea to remain standing.

### RESOURCES

- The Positive Engagement Project. Retrieved from [http://pepnonprofit.org/uploads/2/7/7/2/2772238/language\\_lines\\_sentence\\_frames.pdf](http://pepnonprofit.org/uploads/2/7/7/2/2772238/language_lines_sentence_frames.pdf)

## Module 3 Session Agenda

### Accessing the Connecticut Core Standards in Mathematics

- Warm-Up Activities
- The Language of Mathematics
- Digging Deeper – Language Scaffolds
- Collaboration is Key
- Reflection, Next Steps, and Session Evaluation

### Goal of Today’s Professional Development

Deepen participants’ understanding of the Connecticut Core Standards (CCS) for Mathematics and their impact on instruction in meeting the needs of ALL learners, with an emphasis on English learners.

### Pre-Assessment of Today’s Activities

**Instructions:** Check the box on the scale that best represents your knowledge or feelings about the Connecticut Core Standards in your classroom.

Self-Assessment Questions	No	Somewhat	Yes	Absolutely, and I could teach it to someone else
	1	2	3	4
I know the difference between CCS Mathematical Standards <u>and</u> the Standards for Mathematical Practice.				
I understand the shifts of Focus, Coherence, and Rigor.				
I am fully aware of the practices and the associated language functions that English learner students need in order to engage with mathematics content.				
I have a practical understanding of the full descriptions of the Standards for Mathematical Practice and am able to apply them to math tasks for my English learner students.				
I am able to identify the changing roles of the TESOL professional and can make educated recommendations that benefit English learner students.				

## Part 1: The Language of Mathematics

## Part 1: The Language of Mathematics

### Activity 1: Viewing a Video – *Beyond Right Answers*

#### DESCRIPTION

Participants will engage in a video overview of the CCS utilizing the Cornell Note-Taking strategy.

#### DIRECTIONS

1. View the video, *Beyond Right Answers*. Using the note-taking organizer on the next page, take graphic or text notes in the right-hand column.
  - Key words and ideas
  - Important information
  - Repeated/stressed information
2. After the video, note the most important information in the left-hand column.
  - Main idea(s)
  - Key questions
3. With your team, write a summary of the video in the space at the bottom.

### Benefits for ELs

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How does **Cornell Note-Taking** benefit English learner students?

- Can be partially completed prior to instruction so that new learning is focused.
- Can record information in symbols, graphics, or similar representations.
- Serves to activate higher-order thinking when synthesizing and applying information.
- Working with another student(s) can provide opportunities to process information in the home language and/or hear the group's thoughts in English.

#### RESOURCES

- Video: Teaching Channel. *Beyond Right Answers: Math and the CCSS*. Retrieved from <https://www.teachingchannel.org/videos/teaching-math-ccss>
- Note-Taking Organizer (page 9)

### Note-Taking Organizer

Key points after the video	Information recorded during the video
Summary of the video in your own words	

## Kindergarten - 8th Grade Math Standards

### Early Elementary

- **Counting & Cardinality: K**
- **Operations & Algebraic Thinking: K-5**
- **Number & Operations in Base 10: K-5**
- **Measurement & Data: K-5**
- **Number & Operations - Fractions: 3-5**

### Upper Elementary

- **Ratios & Proportional Relationships: 6-7**
- **The Number System: 6-8**
- **Expressions & Equations: 6-8**
- **Statistics & Probability: 6-8**
- **Functions: 8**

### Geometry: K-8

Is the only domain addressed at all 9 levels

## High School Math Standards

Number and Quantity	Algebra	Functions
<ul style="list-style-type: none"> <li>• The Real Number System</li> <li>• Quantities</li> <li>• The Complex Number System</li> <li>• Vector &amp; Matrix Quantities</li> </ul>	<ul style="list-style-type: none"> <li>• Seeing Structure in Expressions</li> <li>• Arithmetic with Polynomials &amp; Rational Expressions</li> <li>• Creating Equations</li> <li>• Reasoning with Equations &amp; Inequalities</li> </ul>	<ul style="list-style-type: none"> <li>• Interpreting Functions</li> <li>• Building Functions</li> <li>• Linear, Quadratic, &amp; Exponential Models</li> <li>• Trigonometric Functions</li> </ul>
Modeling	Geometry	Statistics & Probability
<p><i>Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).</i></p>	<ul style="list-style-type: none"> <li>• Congruence</li> <li>• Similarity, Right Triangles &amp; Trigonometry</li> <li>• Circles</li> <li>• Expressing Geometric Properties with Equations</li> <li>• Geometric Measurement &amp; Dimension</li> <li>• Modeling with Geometry</li> </ul>	<ul style="list-style-type: none"> <li>• Interpreting Categorical &amp; Quantitative</li> <li>• Making Inferences &amp; Justifying Conclusions</li> <li>• Conditional Probability &amp; the Rules of Probability</li> <li>• Using Probability to Make Decisions</li> </ul>

See additional resources: [http://ctcorestandards.org/?page\\_id=2](http://ctcorestandards.org/?page_id=2)

## 8 Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

### RESOURCES

- Bee Sharp. Posters from presentation retrieved from <http://howtobeagreatteacher.com/wp-content/uploads/2014/01/MP-Grades-K-1-All.pdf>

After viewing and discussing the posters for each of the 8 SMPs, note how they collectively offer Comprehensible Input for ELs:

## Activity 2: The Core Shifts in Mathematics

### DESCRIPTION

Participants will engage in a video to reflect on Focus, Coherence, and Rigor as it pertains to English learner students.

### DIRECTIONS

1. View the video from Achieve the Core on the Shifts in Mathematics.
2. Pay particular attention to the discussion of Focus, Coherence, and Rigor.
3. Consider these questions relative to English learner students and discuss with a table partner:

*What do Focus, Coherence, and Rigor look like for ELs?*

*What challenges do you see to teaching the CCS-Math for ELs?*

### RESOURCES

- Video: Achieve the Core. *The Common Core State Standards Shifts in Mathematics*. Retrieved from <http://achievethecore.org/shifts-mathematics>

## **Activity 3: Illustrated Practices Charts from the Connecticut State Department of Education**

### **DESCRIPTION**

Participants will compare two charts with emphasis on Focus, Coherence, and Rigor.

### **DIRECTIONS**

1. With your team, compare the Grade 3 Illustrated Practices Chart with the Grade 7 Illustrated Practices Chart (pages 14-16).
2. Compare and contrast the elements of Focus, Coherence, and Rigor.
3. Further, consider the two questions that were asked earlier with the video:
  - What do Focus, Coherence and Rigor look like for ELs?
  - What challenges do you see to teaching the Connecticut Core Standards for Mathematics for ELs?

### **RESOURCES**

- Connecticut State Department of Education. Grade 3 and Grade 7: Illustrated Practices Charts. Retrieved from Math Units of Study <http://www.sde.ct.gov/sde/cwp/view.asp?a=2618&q=322592>
  - Scroll to the bottom and find the heading “Mathematics”
  - Click on “K-12 Math Units of Study”
  - Open each grade level
  - Then open “Illustrated Practices”

**Grade Three Standards for Mathematical Practice:** The K-12 Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. This page gives examples of what the practice standards look like at the specified grade level.

<i>Standards</i>	<i>Explanations and Examples</i>
Students are expected to: <b>1. Make sense of problems and persevere in solving them.</b>	In third grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
Students are expected to: <b>2. Reason abstractly and quantitatively.</b>	Third graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.
Students are expected to: <b>3. Construct viable arguments and critique the reasoning of others.</b>	In third grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking.
Students are expected to: <b>4. Model with mathematics.</b>	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Third graders should evaluate their results in the context of the situation and reflect on whether the results make sense.
Students are expected to: <b>5. Use appropriate tools strategically.</b>	Third graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.
Students are expected to: <b>6. Attend to precision.</b>	As third graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.
Students are expected to: <b>7. Look for and make use of structure.</b>	In third grade, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to multiply and divide (commutative and distributive properties).
Students are expected to: <b>8. Look for and express regularity in repeated reasoning.</b>	Students in third grade should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don’t know. For example, if students are asked to find the product of $7 \times 8$ , they might decompose 7 into 5 and 2 and then multiply $5 \times 8$ and $2 \times 8$ to arrive at $40 + 16$ or 56. In addition, third graders continually evaluate their work by asking themselves, “Does this make sense?”

CSDE Retrieved from <http://www.sde.ct.gov/sde/cwp/view.asp?a=2618&q=322592> Adopted from The Arizona Academic Content Standards

**Grade Seven Standards for Mathematical Practice:** The K-12 Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. This page gives examples of what the practice standards look like at the specified grade level.

<i>Standards</i>	<i>Explanations and Examples</i>
Students are expected to: <b>1. Make sense of problems and persevere in solving them.</b>	In grade 7, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”
Students are expected to: <b>2. Reason abstractly and quantitatively.</b>	In grade 7, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
Students are expected to: <b>3. Construct viable arguments and critique the reasoning of others.</b>	In grade 7, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?” “Does that always work?” They explain their thinking to others and respond to others’ thinking.
Students are expected to: <b>4. Model with mathematics.</b>	In grade 7, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students explore covariance and represent two quantities simultaneously. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences, make comparisons and formulate predictions. Students use experiments or simulations to generate data sets and create probability models. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
Students are expected to: <b>5. Use appropriate tools strategically.</b>	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 7 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Students might use physical objects or applets to generate probability data and use graphing calculators or spreadsheets to manage and represent data in different forms.

CSDE Retrieved from <http://www.sde.ct.gov/sde/cwp/view.asp?a=2618&q=322592> Adopted from The Arizona Academic Content Standards

<b>Standards</b>	<b>Explanations and Examples</b>
Students are expected to: <b>6. Attend to precision.</b>	In grade 7, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students define variables, specify units of measure, and label axes accurately. Students use appropriate terminology when referring to rates, ratios, probability models, geometric figures, data displays, and components of expressions, equations or inequalities.
Students are expected to: <b>7. Look for and make use of structure.</b>	Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables making connections between the constant of proportionality in a table with the slope of a graph. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 3(2 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$ , $2c = 12$ by subtraction property of equality), $c=6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving scale drawings, surface area, and volume. Students examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities.
Students are expected to: <b>8. Look for and express regularity in repeated reasoning.</b>	In grade 7, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. They extend their thinking to include complex fractions and rational numbers. Students formally begin to make connections between covariance, rates, and representations showing the relationships between quantities. They create, explain, evaluate, and modify probability models to describe simple and compound events.

CSDE Retrieved from <http://www.sde.ct.gov/sde/cwp/view.asp?a=2618&q=322592> Adopted from The Arizona Academic Content Standards

## Activity 4: Closed Sort – Analytical Tasks and Receptive and Productive Language Functions

### DESCRIPTION

Provided with Closed Sort cards for either SMP 1 or SMP 6, participants will sort practices into the language functions that ELs need to engage with mathematics content.

### DIRECTIONS

1. With your team, locate the following headings within the set of cards you were given:
  - Analytical Tasks
  - Receptive Language Functions
  - Productive Language Functions
2. Now, discuss and sort the practices into the appropriate headings.
3. Pair up with a team who did the other Closed Sort (e.g., one of you did SMP 1 and one of you did SMP 6).
4. Share your Closed Sorts and discuss implications for teaching and learning.

## Benefits for ELs

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How do **Closed Sorts** benefit English learner students?

- Oral English language is modeled by peers.
- Students are provided the opportunity for structured talk.
- Students use prior knowledge to organize and make sense of words and concepts.
- Students may be required to defend their sorting by talking about the common features of the categories and how each specific term/phrase/indicator meets these criteria.
- Students at all levels use higher order thinking skills.
- Incorporates these best practices for ELs:
  - Activating prior knowledge.
  - Explicit vocabulary introduction/practice.
  - Preview of key concepts.

### RESOURCES

- Handout: Council of Chief State School Officers (CCSSO). Framework for English Language Proficiency Development Standards corresponding to the Common Core State Standards and the Next Generation Science Standards. Retrieved from <http://www.ccsso.org/Documents/2012/ELPD%20Framework%20Booklet-Final%20for%20web.pdf>

## Part 2: Digging Deeper – Language Scaffolds

## Part 2: Digging Deeper – Language Scaffolds

### Activity 5a: Standards for Mathematical Practice #6 – Attend to Precision

#### DESCRIPTION

Participants will view a video that explains SMP 6.

#### DIRECTIONS

1. View the video, *Standards for Mathematical Practice #6*.
2. Note the following:
  - How SMP 6 is defined.
  - How SMP 6 is broken down into various components.
  - How the video is organized.
  - Elements of repetition, color, sound, etc.

#### RESOURCES

- Video: YouTube. *Standards for Mathematical Practice #6*. Retrieved from <https://www.youtube.com/watch?v=4whGbDxCora>

## Activity 5b: Your Turn – SMP 1

### DESCRIPTION

Working in teams, participants will create a “performance” to demonstrate SMP 1.

### DIRECTIONS

1. Read and discuss the complete version of SMP 1:

Mathematically proficient students start by explaining to themselves the meaning of the problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, table, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Together, determine a format for explaining to others what SMP 1 is. Some suggestions are a chant, a rap, a role play, a simulation, or a skit.
3. Consider the things that made the video successful and what you might incorporate (e.g., sequencing, repetition, etc.).
4. Be ready to perform for the large group.

## Benefits for ELs

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How do drama and other forms of **Performances** benefit English learner students?

- Authentic, contextualized acquisition of new vocabulary and structure.
  - Practice and use of English: speaking, listening, and understanding.
  - Acquisition of meaningful, fluent interactions in English.
  - Opportunities to practice communicating in different social contexts and in different social roles.
  - Improved sense of confidence in learning a new language.
-

## Activity 6a: Your Turn – Deconstructing SMP 1 and SMP 6

### DESCRIPTION

Participants will break down the lengthy descriptions of SMP 1 and SMP 6 into manageable parts and create a poster to teach others about each SMPs’ components.

### DIRECTIONS

1. Working with your team, break down the statement(s) of your assigned SMP (1 or 6) into simplified and student-centered language; i.e., What does this language mean? What will students know and be able to do?
2. Write your simplified statements in the right-hand side of the tables provided below and on the next page.
3. Now, use your simplified statements to create a poster that makes your SMP comprehensible to others.

### RESOURCES

- Bee Sharp. (A worksheet similar to the one used in this activity is available for each of the 8 SMPs.) The 8 Standards of Mathematical Practice. Retrieved from <http://howtobeagreatteacher.com/the-8-standards-of-mathematical-practice/>
- Connecticut English Language Proficiency Standards (CELP): [http://www.sde.ct.gov/sde/lib/sde/pdf/curriculum/bilingual/celp\\_standards.pdf](http://www.sde.ct.gov/sde/lib/sde/pdf/curriculum/bilingual/celp_standards.pdf).

SMP 6 Statements	SMP 6 Simplified Statements
<b>SMP 6:</b> Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.	
(continuing) They are careful about specifying units of measure, labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.	
(continuing) In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions.	

SMP 1 Statements	SMP 1 Simplified Statements
<p><b>SMP1:</b> Mathematically proficient students start by explaining to themselves the meaning of the problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals.</p>	
<p>(continuing) They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt.</p>	
<p>(continuing) They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need.</p>	
<p>(continuing) Mathematically proficient students can explain correspondences between equations, verbal descriptions, table, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem.</p>	
<p>(continuing) Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?”</p>	
<p>(continuing) They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.</p>	

## Activity 6b: Stay and Stray

### DESCRIPTION

Participants will present the content of the posters to others so that all will be fully acquainted with SMP 1 and SMP 6.

### DIRECTIONS

1. With your team, gather around your poster.
2. Discuss and make sure that each member can explain the poster and its contents to others.
3. When prompted, one person will stay with the poster and the others will move clockwise to the next poster.
4. The person remaining with the poster will explain it to the arriving group.
5. Listen to the presenter for the next set of instructions.

## Benefits for ELs

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How does **Stay and Stray** benefit English learner students?

- Group thinking, discussion, collaboration, and product reduce stress and build confidence.
- Everyone contributes.
- English is modeled and practiced.
- Repetition of concepts and language throughout the process.
- Promotes accountability since any group member could be selected to stay and teach the content to the next group; all are responsible for listening and understanding.
- Since participants act as a teacher, they are able to demonstrate that they understand the concepts.

### TIP

- Have everyone in each group number off. The presenter can call a number and have that person from each group stay with their poster. For the next round, call a different number, and so on until everyone has a chance to “stay” and teach a small group the content of a poster.

## Activity 7: What Fraction of this Shape is Red?

### DESCRIPTION

Participants will engage with a video seeking evidence of SMP 1 and SMP 6 using a graphic organizer.

### DIRECTIONS

1. View the video, *What Fraction of this Shape is Red?*
2. Complete the 3-Column Graphic Organizer on the next page.

## Benefits for ELs

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How do **Graphic Organizers** benefit English learner students?

- Organizers help students to understand text structure and promote organized note-taking.
- Students don't have room to just copy the text; they need to paraphrase and summarize.
- Students need to use some of their own words to retell.
- All students can have assignments to complete a common graphic organizer but could use different readings or media to do so.
- Whenever students share their graphic organizers with others, the information is organized and easier to process.
- Organizers prepared by more advanced students one year can provide a "preview read" to an EL in subsequent years.
- Ways to organize information may differ across cultures.

### RESOURCES

- Video: Teaching Channel. *What Fraction of this Shape is Red?* Retrieved from <https://www.teachingchannel.org/videos/teaching-fractions>
- 3-Column Graphic Organizer (page 25)
- While not used in today's workshop, this is a quality list of questions to ask when observing children and the SMPs: Mathematics Practices – Questions for Observation. Retrieved from <http://commoncore.fcoe.org/sites/commoncore.fcoe.org/files/resources/8%20practices%20questions.pdf>

### 3-Column Graphic Organizer

**SMP 1:** Make sense of problems and persevere in solving them

**SMP 2:** Reason abstractly & quantitatively

**SMP 3:** Construct viable arguments & critique the reasoning of others

**SMP 4:** Model with mathematics

**SMP 5:** Use appropriate tools strategically

**SMP 6:** Attend to precision

**SMP 7:** Look for & make use of structure

**SMP 8:** Look for & express regularity in repeated reasoning

Evidence of SMP 1 & SMP 6	Evidence of other SMPs	Missed Opportunities

## Teaching Mathematics to English Language Learners

*“Mathematics teachers must attend to all students, including those who speak a first language other than English or have related cultural differences, and ensure that all have access and opportunities to learn mathematics and to reveal what they know. Every student’s cultural and linguistic heritage should be respected and celebrated for the diversity that it contributes to the learning environment. Expanded learning opportunities and instructional accommodations should be available to English language learners (ELLs) who need them to develop mathematical understanding and proficiency.”*

National Council of Teachers of Mathematics. Position Statement, 2013. Retrieved from <http://www.nctm.org/Standards-and-Positions/Position-Statements/Teaching-Mathematics-to-English-Language-Learners/>

### NCTM Four Dimensions of Instruction for English Language Learners

1. Establish **learning environments and classroom norms** that support the active engagement of all students, including ELLs. Such classrooms honor the diverse ways in which students approach mathematics, communicate their mathematical thinking (e.g., code-switching) and record their strategies and solutions to exercises and problems. For example, in some countries, one billion is written as  $10^{12}$  rather than  $10^9$ .
2. Identify and use **instructional strategies** that make content more accessible, and consider how to implement culturally relevant pedagogy in mathematics classrooms.
3. Orchestrate **classroom discussions** in ways that support acquisition of mathematics concepts and language development. It is important for all students, but especially critical for ELLs, to have opportunities to speak, write, read and listen in mathematics classes, with teachers providing appropriate linguistic support and encouragement.
4. **Assess** ELLs in ways that permit them to show what they know and are able to do. This requires providing test accommodations that lessen the language complexity without reducing the rigor of mathematics under investigation.

National Council of Teachers of Mathematics. Retrieved from <http://www.nctm.org/ELLMathematics/>

## Activity 8: Dimensions of Instruction Foldable

### DESCRIPTION

In teams, participants will construct a “foldable” that explains how either SMP 1 or SMP 6 “looks” in each of four dimensions of instruction.

### DIRECTIONS

1. Work with the same group from the poster activity (SMP 1 or SMP 6 / Stay and Stray).
2. Construct a “foldable” to explain how that SMP looks in each of the four Dimensions of Instruction as defined by the National Council of Teachers of Mathematics:
  - Learning Environments and Class Norms
  - Instructional Strategies
  - Classroom Discussions
  - Assessment
3. Place your foldable on display as directed by the presenter.

## Benefits for ELs

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How do **Foldables** benefit English learner students?

- A foldable is an interactive graphic organizer; all the benefits of graphic organizers apply here.
- Teacher can determine initial understandings through the product.
- ELs build factual knowledge through schema.
- Encourages metacognition through active learning.

### RESOURCES

- Diagrams retrieved from [www.averyschools.net](http://www.averyschools.net)
- One of many resources: [www.dinah.com](http://www.dinah.com)

## Activity 9: Annotated Math Tasks

### DESCRIPTION

Participants will review the math task “Making Matchsticks,” which is a model for students to learn to read and understand word problems, communicate about mathematics, build disciplinary and academic vocabulary, and develop practices in math.

### DIRECTIONS

1. Review the math task, “Making Matchsticks,” looking for evidence of:
  - Learning to read and understand word problems.
  - Communicating about mathematics.
  - Building disciplinary and academic vocabulary.
  - Developing practices in math.
2. Look for evidence of and opportunities for ELA scaffolds:
  - Teaching academic vocabulary.
  - Integrating oral and written language instruction into content area teaching.
  - Providing regular, structured opportunities to write.
  - Capitalizing on students’ home language and knowledge.
3. Look for evidence of and opportunities for math scaffolds:
  - Providing concrete and visual models.
  - Providing graphic organizers and foldables.
  - Using multimedia to enhance ELs’ comprehension of mathematical concepts.

### RESOURCES

- Handout: Understanding Language Project, Stanford Graduate School of Education. Making Matchsticks. Retrieved from: [http://ell.stanford.edu/sites/default/files/math\\_pdf/8.MS-Matchsticks%207-2-13%20copy.pdf](http://ell.stanford.edu/sites/default/files/math_pdf/8.MS-Matchsticks%207-2-13%20copy.pdf)

ELA Scaffolds	Math Scaffolds

## Activity 10: Your Turn – Annotating a Math Task

### DESCRIPTION

Participants will annotate “Digging Dinosaurs,” a grade 5 math task.

### DIRECTIONS

1. Work with your team to annotate the math task “Digging Dinosaurs.”
2. Employ any of these ELA scaffolds:
  - Teach academic vocabulary.
  - Integrate oral and written language instruction into content area teaching.
  - Provide regular, structured opportunities to write.
  - Capitalize on students’ home language and knowledge.
3. Employ any of these math scaffolds:
  - Provide concrete and visual models.
  - Provide graphic organizers and foldables.
  - Use multimedia to enhance ELs’ comprehension of mathematical concepts.
4. Share as directed by the presenter.

### RESOURCES

- Handout: “Digging Dinosaurs.” Inside Mathematics. Problems of the Month:  
<http://www.insidemathematics.org/assets/problems-of-the-month/digging%20dinosaurs.pdf>

**Part 3: Collaboration is Key**

## Part 3: Collaboration is Key

### Activity 11: Collaborating to Plan a Lesson

#### DESCRIPTION

Participants will view and respond to a video that depicts teachers modeling collaboration.

#### DIRECTIONS

1. View the video from the Teaching Channel on collaborating to plan a math lesson.
2. Consider and be ready to respond to these questions:

What interactions do you see between these teachers?	
How do these teachers engage in reflection?	

3. Brainstorm how these interactions/reflections would be different if it was a math and TESOL teacher collaborating?

#### RESOURCES

- Video: Teaching Channel. *Collaborating to Plan a Common Core Math Lesson*. Retrieved from <https://www.teachingchannel.org/videos/teacher-collaboration-on-common-core-nea#>

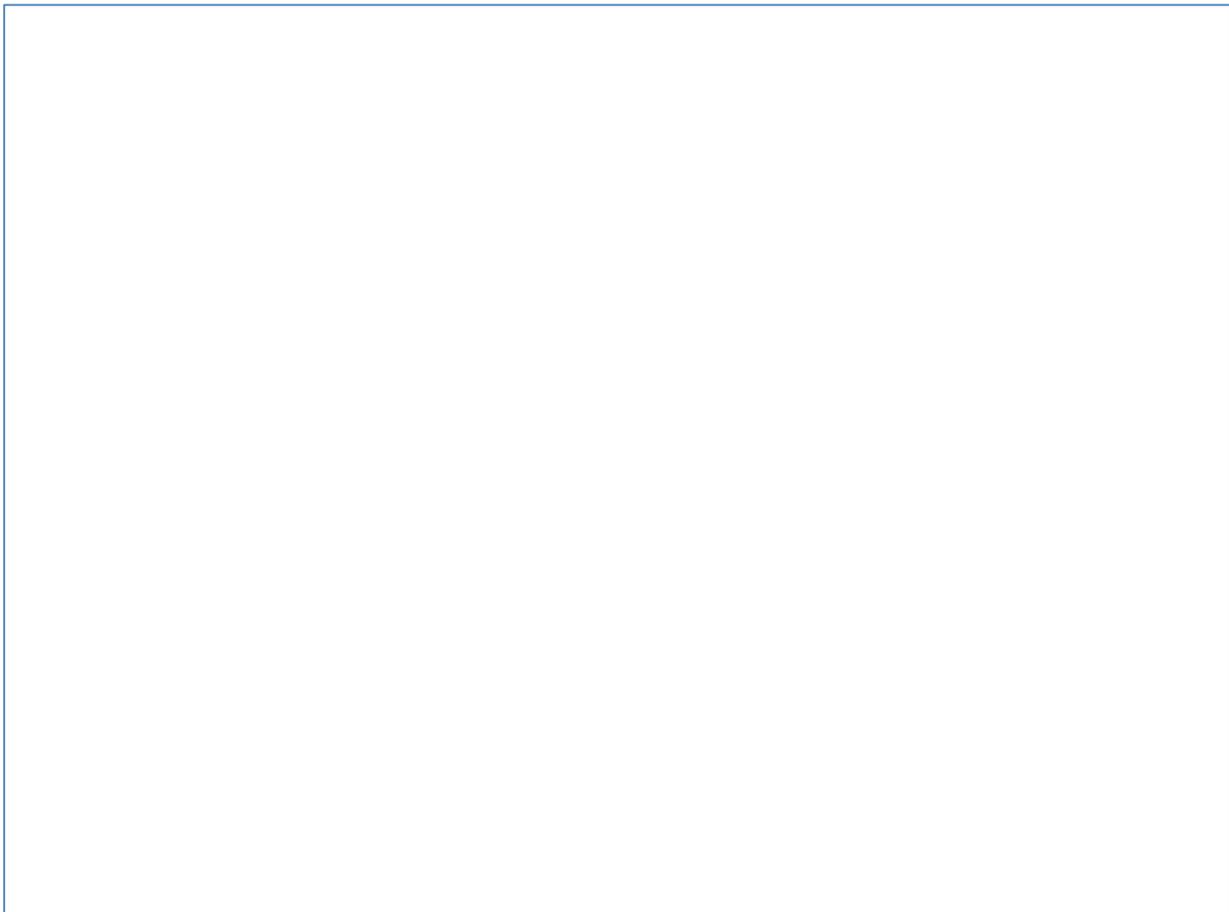
## Activity 12: Multiple Representations

### DESCRIPTION

Participants discuss the idea that mathematics calls for multiple representations and create at least two representations to illustrate the idea.

### DIRECTIONS

1. Collaborate with your team to come up with at least two representations to illustrate the ideal collaboration between math and TESOL teachers (drawing, number sentence, words, symbols, equation, etc.).
2. Use the space below to brainstorm ideas, then create your multiple representations on chart paper.
3. As teams, share your poster with the whole group.

A large, empty rectangular box with a thin blue border, intended for students to create their multiple representations as described in the directions.

## Closing Activities

## Closing Activities

### Our Team’s Next Steps

<p>What is the role of the TESOL professionals in your school and has it changed in recent years?</p> <ul style="list-style-type: none"> <li>•</li> <li>•</li> <li>•</li> </ul>	<p>Next Steps:</p> <ul style="list-style-type: none"> <li>•</li> <li>•</li> <li>•</li> </ul>
<p>How can TESOL professionals continue to serve the general education teachers in your school by acting as experts, advocates, or consultants on language development in addition to serving students directly?</p> <ul style="list-style-type: none"> <li>•</li> <li>•</li> <li>•</li> </ul>	<p>Next Steps:</p> <ul style="list-style-type: none"> <li>•</li> <li>•</li> <li>•</li> </ul>
<p>What school structures would make it easier for TESOL teachers to fulfill this role?</p> <ul style="list-style-type: none"> <li>•</li> <li>•</li> <li>•</li> </ul>	<p>Next Steps:</p> <ul style="list-style-type: none"> <li>•</li> <li>•</li> <li>•</li> </ul>

### Post-Assessment of Today’s Activities

**Instructions:** Check the box on the scale that best represents your knowledge or feelings about the Connecticut Core Standards in your classroom.

Self-Assessment Questions	No	Somewhat	Yes	Absolutely, and I could teach it to someone else
	1	2	3	4
I know the difference between CCS Mathematical Standards <u>and</u> the Standards for Mathematical Practice.				
I understand the shifts of Focus, Coherence, and Rigor.				
I am fully aware of the practices and the associated language functions that English learner students need in order to engage with mathematics content.				
I have a practical understanding of the full descriptions of the Standards for Mathematical Practice and am able to apply them to math tasks for my English learner students.				
I am able to identify the changing roles of the TESOL professional and can make educated recommendations that benefit English learner students.				

### Session Evaluation

Thank you for attending Module 3. Your feedback is very important to us! Please fill out a short survey about this afternoon’s session.

The survey is located here: <http://surveys.pcgus.com/s3/CT-Module-3-EL>

## Resources

- Connecticut State Department of Education. Grade 3 and Grade 7 Illustrated Practices Charts. Retrieved from Math Units of Study <http://www.sde.ct.gov/sde/cwp/view.asp?a=2618&q=322592>
- Council of Chief State School Officers (CCSSO). Framework for English Language Proficiency Development Standards corresponding to the Common Core State Standards and the Next Generation Science Standards. Retrieved from <http://www.ccsso.org/Documents/2012/ELPD%20Framework%20Booklet-Final%20for%20web.pdf>
- Bee Sharp. How to be a great teacher: <http://howtobeagreatteacher.com/wp-content/uploads/2014/01/MP-Grades-K-1-All.pdf>. Posters retrieved from <http://howtobeagreatteacher.com/wp-content/uploads/2014/01/MP-Grades-K-1-All.pdf>
- Bee Sharp. The 8 Standards of Mathematical Practice. Retrieved from <http://howtobeagreatteacher.com/the-8-standards-of-mathematical-practice/> (A worksheet similar to the one used in this activity is available for each of the 8 SMPs.)
- Inside Mathematics. Problems of the Month: <http://www.insidemathematics.org/problems-of-the-month/download-problems-of-the-month>
- Mathematics Practices – Questions for Observation. Retrieved from <http://commoncore.fcoe.org/sites/commoncore.fcoe.org/files/resources/8%20practices%20questions.pdf>
- The Positive Engagement Project: [http://pepnonprofit.org/uploads/2/7/7/2/2772238/language\\_lines\\_sentence\\_frames.pdf](http://pepnonprofit.org/uploads/2/7/7/2/2772238/language_lines_sentence_frames.pdf)
- Understanding Language Project, Stanford Graduate School of Education. Retrieved from: [http://ell.stanford.edu/teaching\\_resources/math](http://ell.stanford.edu/teaching_resources/math)
- Understanding Language Project, Stanford Graduate School of Education. Making Matchsticks. Retrieved from: [http://ell.stanford.edu/sites/default/files/math\\_pdf/8.MS-Matchsticks%207-2-13%20copy.pdf](http://ell.stanford.edu/sites/default/files/math_pdf/8.MS-Matchsticks%207-2-13%20copy.pdf)

## Videos

- Achieve the Core. *The Common Core State Standards Shifts in Mathematics*. Retrieved from <http://achievethecore.org/shifts-mathematics>
- Teaching Channel. *Beyond Right Answers: Math and the CCSS*. Retrieved from <https://www.teachingchannel.org/videos/teaching-math-ccss>
- Teaching Channel. *Collaborating to Plan a Common Core Math Lesson*. Retrieved from <https://www.teachingchannel.org/videos/teacher-collaboration-on-common-core-nea#>
- Teaching Channel. *What Fraction of this Shape is Red?* Retrieved from <https://www.teachingchannel.org/videos/teaching-fractions>
- YouTube. *Standards for Mathematical Practice #6*. Retrieved from <https://www.youtube.com/watch?v=4whGbDxCora>