**Unit 8: Investigation 5 (4 Days)**

**FACTORING QUADRATIC TRINOMIALS**

***CCSS: A-SSE 3a***

**Overview**

Students will factor quadratic trinomials of various forms and convert quadratic functions in standard form to quadratic functions in factored form. They will learn that factoring polynomials is the inverse operation of multiplying polynomials. Students will then be able to take a quadratic equation of the form $ax^{2}+bx+c=0$ and, if possible, solve it by factoring the left side and using the Zero Product Property.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Factor quadratic trinomials in various forms
* Check factorizations using multiplication
* Convert quadratic functions in standard form to factored form
* Solve a quadratic equation by factoring or determine that a quadratic equation cannot be solved in this way

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slips 8.5a and 8.5b** ask students to show steps in converting a quadratic function from standard form to factored form.
* **Journal Entry 1** asks students how many factor pairs need to be tested in order to determine that a particular quadratic trinomial cannot be factored.
* **Journal Entry 2** asks students to create two quadratic equations, one that can be solved by factoring and one that cannot. They should solve the first one and explain why factoring does not work on the second one.

**Launch Notes**

Show students the Futures Channel video on Windsail design (1 minute, 42 seconds) at <http://thefutureschannel.com/videogallery/windsails/>. Students should note that the shape of the masts for these high speed surf boards is parabolic. Designing these masts involves working with quadratic functions in factored form. Here is the basic design of the parabolic mast:



The mast can be represented by the portion of a parabola that lies above the *x*-axis as shown.

The length of the mast (which is also called the “height” when the mast is held upright) is the distance between the *x-*intercepts. The designers are also interested in the maximum amount of bend, shown by *m* in the diagram. Suppose the equation for the parabola is
 $y=f(x)=-.002x^{2}+.2x$, with all dimensions given in inches. Pose the question: can we find the length of the mast and the value of *m*?

**Closure Notes**

We have learned to solve some quadratic equations by factoring. However, we have also seen that not all quadratic trinomials can be factored. In the next investigation we will learn more general methods that can be used to solve all quadratic equations.

**Teaching Strategies**

I. Lead students in a discussion of how to solve the windsail problem posed above. The *x*-intercepts are found by setting *y* = 0 and solving for *x*. Help students see that multiplying both sides by 1000 will give a simpler equation with integer coefficients:
 $-2x^{2}+200x=0$.

Some students may want to solve the equation by adding $2x^{2}$ on both sides and then dividing by 2*x*. This yields a solution *x* = 100. However, division by 0 is undefined so we can only perform the operation if *x* ≠ 0. To get around this difficulty a preferred approach is to factor the left side of the equation and apply the Zero Product Property introduced in Investigation 4.

Ask students if they can find a way to express the left side as a product. You may want to use an area model to help students answer the question. The two terms in the product must have a common factor, which represents the height of the rectangle. In this case the greatest common factor is –2*x*. They can then work backwards to find the two terms that lie along the upper side of the rectangle.





Thus$ -2x^{2}+200x=-2x(x-100)$ and so the equation becomes $-2x\left(x-100\right)=0$ which leads to two solutions, *x* = 0 or *x =* 100. They can conclude that the length of the mast is 100 inches. They should recognize that *m* is the *y*-coordinate of the vertex, and that it can be found by evaluating *f* (50) to find that *m* = 5 inches.

You may now use **Activity 4.5.1 Finding Common Monomial Factors** to give students practice with expressions that have a common monomial factor and applying the zero product property.

 II. The next step is to have students learn to factor quadratic trinomials as the product of two binomial factors. Begin by having the students multiply to find the binomial expansions of the following:

 (*x* + 3)(*x* + 4) and (*x* + 5)(*x* – 2)

This will serve both as a review of Investigation 4 and a lead-in to developing the algorithm for factoring.

After students have had time to work on the multiplications, present this scenario to students: Your Aunt Ramona has decided to bake some cookies for you to bring into class. She wants every student to have the same number of cookies, but she doesn’t know how many students there are in your class. She decides to make 48 cookies. What are all the possible class sizes to ensure that everyone gets the same number of cookies?

Hopefully, students realized that they needed to find the factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48. They should notice that these factors come in pairs:

If there are 48 students in the class each students gets 1 cookie.

If there are 24 students in the class, each student gets 2 cookies.

If there are 16 students in the class, each student gets 3 cookies, etc.

In the context of the problem some cases may not be valid solutions (e.g. 48 students may be too many, 4 students too few.)

Now go back to the two binomial expansions from the launch. Make sure that students all got the trinomials *x*2 + 7*x* + 12 and *x*2 + 3*x* –10.

Review also from Investigation 4 that the trinomials are now in standard form. In the first case, *a*=1, *b*=7, and *c*=12. Ask if they see any relationships between the binomials they multiplied and the terms in standard form.

Students should recognize that the sum of the constants (in the binomials) is the linear term coefficient in the trinomial, and the product of the constants (in the binomials) is the constant term in the trinomial. This suggests a strategy for factoring a trinomial such as $x^{2}+19x+48$:

“Look for factor pairs for 48 that give a sum of 19.” The factor pair is 16, 3.

The factors are (*x* + 16)(*x* + 3)

Explain that the factorizations can be checked through multiplication.

Continue with examples where *b* and/or *c* are negative to help students figure out strategies for determining which signs to use.

Here is a video showing an example of factoring when the leading coefficient is 1: <http://www.wtamu.edu/academic/anns/mps/math/mathlab/video.htm?video=algebra/college/07/05>. Note that this video refers to the “FOIL” method.

Here is a video showing an example of factoring when the leading coefficient is not 1: <http://www.wtamu.edu/academic/anns/mps/math/mathlab/video.htm?video=algebra/college/07/07>. This video also refers to “FOIL” and promotes a trial and error approach to finding the factors.

As an alternative you may show students the following algorithm for factoring trinomials whose leading coefficient is not equal to 1 (*a* ≠ 1).

Step 1: Identify *a*, *b*, and *c.*

Step 2: Multiply *ac.*

Step 3: Identify pairs of factors of *ac*, and find the pair of factors whose sum equals *b.* If there is no pair of factors whose sum equals *b*, the original quadratic expression cannot be factored.

Step 4: Split the middle linear term into two terms whose coefficients are the factors of *ac.*

Step 5: Factor by grouping.

For example, given the quadratic expression 2*x*2 + 11*x* + 12, *a* = 2, *b* = 11, and *c* = 12. Therefore, *ac* = 24, and factors of 24 include (1 x 24), (2 x 12), (3 x 8), and (4 x 6). (3 x 8) is the unique factor pair whose sum equals 11 (*b* = 11). Using these factors, we can split the middle term into two terms, and then factor by grouping. Factoring by grouping requires that we find a common factor in the first two terms, and find a common factor in the last two terms, and factor these terms out. Once a common binomial factor appears, we factor it out as well.

2*x*2 + 11*x* + 12 = 2*x*2 + 3*x +* 8*x* + 12 = *x*(2*x* +3) + 4(2*x*+3) = (2*x*+3)(*x* + 4)

A more concrete approach is have students use area models or algebra tiles to examine the relationships between the quadratic expression and the binomial factors. This essentially involves undoing the multiplication of polynomials with area models demonstrated in Investigation 4.

For example, to factor $x^{2}+8x+12$, start by assigning $x^{2}$ and 12 to two regions as shown.

The next step is to find two numbers whose product is 12 and whose sum is 8. 6 and 2 fit the bill. So split the middle term 8*x* into 6*x* and 2*x* and assign these quantities to the two remaining rectangles.



The final step is to find the common factor for each row and column as shown. The result is that the trinomial $x^{2}+8x+12$ can be factored as (*x* + 6)(*x* + 2).

Virtual algebra tiles may be found at <http://illuminations.nctm.org/ActivityDetail.aspx?ID=216>

In addition, students can check their factorizations using the Wolfram Alpha ([www.wolframalpha.com](http://www.wolframalpha.com)).

**Differentiated Instruction (For Learners Needing More Help)**

For students who have difficulty finding factors of integers, the web site [www.**calculatorsoup.com**/calculators/math/factors.php](http://www.calculatorsoup.com/calculators/math/factors.php) can be used as an aid. This will enable them to focus on the process of factoring polynomials rather than getting stuck on the first step.

Have students work on **Activity 8.5.2** **Factoring Trinomials** to practice factoring trinomials both when the leading coefficient is 1 and when it is not 1.

**Differentiated Instructions (For Learners Needing More Help)**

The first five problems in the activity are routine. The sixth and seventh, however are not. You may choose not to give these two problems to students who are struggling with the concept.

The sixth problem in the activity is an example of a difference of squares. Students should be able to solve this if they realize that the *x* term simply has a coefficient of zero. If students do get it, you may want to give them *x*2 – 16 to try and see if they see the pattern.

The seventh problem is an example of a trinomial that can’t be factored. This will lead into the importance of the quadratic formula in upcoming investigations.

III. This group activity will give students additional practice in factoring.

**Group Activity**

Cut up the cards in **Activity 8.5.3 Find Your Match.** Cut out one card for each student. If you have an odd number of students, use the alternative card listed at the end. The goal is for each student to find his or her match. For example, if my slip was (2*x* + 4)(*x* – 1), I would need to find the person whose slip had 2*x*2 + 2*x* – 4 on it. On the original activity form—which is only for the teacher—each correct factored expression is next to its matching trinomial.

At this point you may want to assign for homework part of Activity 8.4.5 **Solving Quadratic Equations by Factoring.**

Use **Exit Slip 8.5a or 8.5b** to assess students’ ability to factor a quadratic trinomial.

**Differentiated Instruction (For Learners Needing More Help)**

Use Exit Slip 8.5b with leading coefficient equal to 1 rather than Exit Slip 8.5a.

**Journal Entry**

Suppose you are trying to factor the quadratic trinomial $x^{2}+12x-36$. How many pairs of factors will you need to test in order to decide that it cannot be factored?

IV. Another real world example to motivate need to solve quadratic equations is projectile motion. First show students the following video on the physics of circus juggling: <http://video.pbs.org/video/1602463762/>**.** An alternative video discusses the physics of football punting: <http://science360.gov/obj/tkn-video/fc729ef0-22ee-4f61-bb2a-b6c07685fb02>

Now offer the following situation: A football is being launched upwards at a rate of 44 feet/second. The equation *y* = –16*t*2 +44*t* + 12 can be used to model the path of the ball. *t* represents seconds and *y* represents the height of the ball. Now, we need to know how long it will take until the ball falls to the ground.

Ask students what the value of *y* would be when the ball hits the ground. The answer is 0. Substituting 0 for *y* gives the quadratic equation 0= –16*t*2 +44*t* + 12. Discuss the need to transform the right side of this equation from standard form to factored form.

So, have students work in pairs to try to find the solution. Once they get the trinomial into factored form, they may apply the Zero Product Property.

Note that there are two ways to proceed with factoring. Many students may come up with
(–4*t* + 12)(4*t* + 1). Some students may realize that you can factor a –4 out first leaving you with –4(–4*t*2 – 11*t* – 3). This now factors into 4(*t* – 3)(4*t* + 1).

Either way, the solutions to the quadratic are t = - ¼ and t = 3. Since we cannot have negative time, the solution to this problem is that the ball will reach the ground in 3 seconds.

After attempting this projectile motion Students can examine the simulation at: <http://phet.colorado.edu/sims/projectile-motion/projectile-motion_en.html>.

Note that length is measured in meters and not feet.

Another site allows you to play a game where you determine coefficients that will allow you to sink a pirate ship: <http://library.thinkquest.org/27585/lab/sim_pirates.html>

**Activity 8.5.4 Solving Quadratic Equations by Factoring** gives students the opportunity to practice the skill of factoring in the context of solving equations. It also introduces the concept that if all possible factor pairs have been tried without success, the quadratic expression cannot be factored and this method will not produce a solution to the equation. In fact, only a small proportion of all quadratic equations with parameters within a certain range (say between –10 and 10) can actually be solved by factoring.

In **Activity 8.5.5 Building Fences** students apply the skill of solving equations by factoring to another real world context. In this activity there are a few problems in which students will first have to get the equation in the form $ax^{2}+bx+c=0$ before they can factor.

You may tell students that they will learn more general methods for solving quadratics in the next investigation.

**Resources and Materials**

* Activity 8.5.1 Finding a Common Monomial Factor
* Activity 8.5.2 Factoring Trinomials
* Activity 8.5.3 Find Your Match
* Exit Slips 8.5a and 8.5b
* Activity 8.5.4 Solving Quadratic Equations by Factoring
* Activity 8.5.5 Building Fences
* Graphing Calculators
* Projector for video clips