**Unit 8: Investigation 2 (4 Days)**

**QUADRATIC FUNCTIONS IN VERTEX FORM**

***CCSS: F-IF4, F-IF7a, F-BF3***

**Overview**

Students will discover the line of symmetry of a parabola whose equation is given in standard form. Given a function in vertex form, students will investigate the effects of the parameters *a*, *h* and *k* on the graph of a parabola, identify the vertex and determine whether the parabola opens up or down. Students will reverse the process and write an equation in vertex form for a parabola given its vertex and one other point. Students will learn to transform a quadratic function from standard form to vertex form.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Find the vertex of a quadratic function from its equation given an equation in vertex form or standard form.
* Model a real world situation by writing the equation of quadratic function given the vertex and one other point.
* Transform a quadratic function in standard form to a function in vertex form by finding $h=\frac{-b}{2a}$ and $k=f\left(\frac{-b}{2a}\right)$.
* Graph a quadratic function in vertex form.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 8.2** asks students to describe how they investigated the role of the parameters *a, h* and *k* on the graph of a quadratic function.
* **Journal Entry 1** prompts students to apply their understanding of the vertex form of a quadratic function to the design of a park fountain.
* **Journal Entry 2** prompts students to compare and contrast the parameters for the standard form and the vertex form of a quadratic function.

**Launch Notes**

Begin this investigation by activating prior knowledge and continuing with the bowl theme from Investigation 1. Present the following National Geographic video clip about solar cookers: <http://video.nationalgeographic.com/video/environment/energy-environment/solar-cooking/>

Following the video, challenge students to think about effective designs for solar cookers. Small groups should consider things such as the shape of the cooker, capacity, and focus of the sun’s rays. Following the group activity, use pictures of solar cookers and the main tank ripple applet to generate a consensus among the entire class on what makes a good solar cooker.

**Closure Notes**

The investigation should close with an opportunity for students to explain or demonstrate how quadratic functions in vertex form model real world situations.

**Teaching Strategies**

**Group Activity**

Following the video about solar cookers, challenge students to think of effective designs for solar cookers. In **Activity 8.2.1a Design a Solar Cooker**, students work in small groups to design the shape and capacity of a solar cooker and predict the focus of the sun’s rays. After the group activity, show pictures of various solar cookers and show the main tank ripple applet so students understand what makes a good solar cooker.

1. **Activity 8.2.1a Design a Solar Cooker** builds upon the bowl theme from Investigation 1 and explores parabolic solar cookers. Following the group activity, students are presented a quadratic equation in standard form that models the largest cross section of a solar cooker. Students are challenged to discover a way to algebraically determine the line of symmetry of the parabola. The line of symmetry is useful for this problem as it helps to determine the location of the cooking pot within a parabolic solar cooker.

**Differentiated Instruction (For Learners Needing More Help)**

The fractional coefficients in the above function can be challenging. As an alternative use **Activity 8.2.1b**, where a similar function with measurements in feet is

$f\left(x\right)=0.4x^{2}-1.2x$.

Students will make the link between real world phenomena like parabolic solar cookers and quadratic functions. Students are asked to explain how the incoming rays of heat correspond to the line of symmetry of the parabola and are asked to sketch how the incoming rays reflect on a parabolic surface. After students share their ideas, discuss with students what the line of symmetry of a parabola represents in terms of reflected light, sound or heat waves coming in or out of the parabola. Students should realize that the hottest point in a solar cooker is some point along the line of symmetry.

Present a diagram similar to the one on the right that shows how heat or light waves will reflect off a parabolic reflector. Students should see that a parabolic mirror reflects the light or heat waves in lines parallel to the line of symmetry.

For homework, you can give the students a photo of a parabolic automobile headlight and ask them to draw a parabolic cross section of the headlight showing light source and rays to represent the light waves. Have them sketch and label the line of symmetry.

You can also show students the parabola applet <http://www.falstad.com/ripple/ex-parabola.html>. Click on “Main Ripple Tank Applet”, open the “Zip archive of this applet” then “Run”, and use the pull down “Setup” menu at the top to scroll down to “Parabolic Mirror 2”. Students do not need to be able to find the focus of a parabola, but they should understand the role of the line of symmetry. The “Parabolic Mirror 1” applet shows waves emanating from a single source such as in an automobile headlight.

1. In **Activity 8.2.2** **Graphing Quadratic Functions in Vertex Form** students will look at several quadratic functions in vertex form $y=a(x-h)^{2}+k$. Challenge students to find the line of symmetry and the vertex from the equation. Have students draw a few graphs by hand. The teacher will need to guide students in choosing appropriate independent variables so that a complete graph can be obtained – the complete graph will show the concavity, the vertex and the increasing and decreasing portions of the parabola. One approach is to identify that the lowest (or highest) value for the function will occur when the base of the quadratic term is 0. That is, when *x – h* = 0 the quadratic will have achieved its extreme value. Using *x = h* for the middle value of *x*, students can then choose a few other values for *x* on either side of the *x =h,* create a table of values and plot points. Ask students about the rates of change between various points on the parabola to reinforce the concept that the quadratic function does not have a constant rate of change. Thus, a quadratic function is non-linear.

**Activity 8.2.3 Exploring Parameters with Geometer’s Sketchpad** offers students a dynamic way to vary the parameters. It may be used in a computer lab or as a demonstration to the class.

After graphing two or three quadratics in vertex form by hand, let students use a graphing calculator to experiment with various values for each of the parameters *a*, *h* and *k* so they can discover the effect of each parameter on the graph. Have students write their discovery in their own words. Use this lesson as an opportunity to assess how well students learned to design and carry out their own mathematical experiments. Have the students complete **Exit Slip 8.2**, which requires them to list a summary of the steps they took to explore the effects of each parameter.

Next, ask students how we can find the vertex of a parabola when the quadratic equation is given in standard form. You may have students take another look at pages 7 and 8 of **Activity 8.1.6** and ask if they can find a relationship between *b* and the axis of symmetry. They may arrive at the conjecture that the *x*-coordinate of the vertex is $-\frac{b}{2}$. This seems to work in cases where *a* = 1. Have them test some functions where *a* ≠ 1 to arrive at a more general formula.

If they have Internet access, students can explore the relationship between the parameters of a function in standard form and the axis of symmetry at <http://www.mathwarehouse.com/geometry/parabola/axis-of-symmetry.php>. They should discover that the *x*-coordinate of the vertex is $h= \frac{-b}{2a}$ and the equation of the line of symmetry is $x= \frac{-b}{2a}$.

**Differentiated Instruction (Enrichment)**

Develop the fact that 3 non-collinear points determine a parabola, which is analogous to the fact that 2 points determine a line. In the special case that the student has a vertex and a point, then the third point is automatically pre-determined by symmetry. Sometimes students will need to fit a quadratic to data even if the data represent only a portion of the graph of a quadratic, because the data may not include the complete graph of a quadratic.

**Journal Entry 1**

RAFT Writing Strategy

Role – Park fountain engineer

Audience – City Park and Recreation Department

Format – Blueprints

Topic – Provide several blueprint options for various parabolas. Explain how changing the parameters of a quadratic function will change the shape of the water coming from the fountain.

1. In **Activity 8.2.4 Modeling with Quadratic Functions in Vertex Form**, students are given graphs with the vertex and at least one other point on a parabola, and they must find an equation for the graph or data. If you use integer values for *a* then the students can plot the vertex and the point on the calculator, type in their guess for the equation and change the parameters on the equation until the equation contains the points. Fitting a curve to data this way reinforces logical thinking and knowledge of the effects of each parameter.

It is important that students do not get bogged down in algebraic manipulation; rather, they should learn to understand the interplay among the algebraic, graphical and tabular form of a quadratic function. For students who are ready for more algebraic manipulation, you can point out that they can find *a* by substituting the known values for *h*, *k*, *x* and *y* into the equation and solving for *a*. This process is analogous to finding *b* in the slope-intercept form of a line given the slope and a point. The general theme is to substitute into an equation what one knows in order to find out what one does not know.

Another activity that has students model data with quadratic functions is **Activity 8.2.5 Bouncing Ball.** In this activity students use a motion detector to collect data on the height of a ball as it rebounds off the ground and they use a calculator to fit a quadratic function to data.

**Differentiated Instruction (For Learners Needing More Help)**

Provide an opportunity for struggling learners to use Geometer’s Sketchpad (**Activity 8.2.3 Exploring Parameters with Geometer’s Sketchpad**) to continue their exploration of parameters and quadratic functions. The dynamic nature of the software and the opportunity to manipulate parameters to match the graph of a quadratic equation may help visual learners to make sense of how the function is affected by each parameter.

**Journal Entry 2**

Compare and contrast the parameters *a, b,* and *c* in standard form with the parameters *a, h,* and *k* in vertex form.

1. In **Activity 8.2.6** **Transforming Quadratic Functions in Standard Form to Vertex Form**, students are given opportunities to transform quadratic functions in standard form to vertex form.

Teach students to transform equations from standard form to vertex form by applying what they have already learned: the *x*-coordinate of the vertex is given by $h= \frac{-b}{2a}$. Knowing *h*, one can find *k* by evaluating *f(h)* which is equivalent to$ f\left(\frac{-b}{2a}\right)$. The parameter ‘*a*’ is the same in both the standard and the vertex forms of the equation. Knowing *a*, *h* and *k* students can write a quadratic in vertex form.

In addition to the applications in this lesson, you can have students convert the solar cooker equations they studied at the start of this investigation. Instruct students to convert the equations into vertex form. The identification of the vertex is important to the design of the solar cooker.

**Resources and Materials**

* **Activity 8.2.1a** Design a Solar Cooker
* **Activity 8.2.1b** Design a Solar Cooker
* **Activity 8.2.2** Graphing Quadratic Functions in Vertex Form
* **Activity 8.2.3** Exploring Parameters with Geometer’s Sketchpad
* **Geometer’ Sketchpad (GSP) Files** (needed for Activity 8.2.3)
* **Activity 8.2.4** Modeling with Quadratic Functions in Vertex Form
* **Activity 8.2.5a** Bouncing Balls TI-84+
* **Activity 8.2.5b** Bouncing Balls TI-NSpire
* **Activity 8.2.6** Transforming Functions in Standard form to Vertex Form
* **Exit Slip 8.2** Investigating Parameters
* Computer with Geometer’s Sketchpad (needed for Activity 8.2.3)
* CBR (needed for Activity 8.2.5)
* Balls (needed for Activity 8.2.5)
* Solar Cooker Video: <http://video.nationalgeographic.com/video/environment/energy-environment/solar-cooking/>
* Solar Cooker Images: <http://www.solarcookers.org>
* Parameter Exploration: <http://www.mathwarehouse.com/geometry/parabola/axis-of-symmetry.php>
* Student Journals
* Graphing Calculators