**Galileo in Dubai**

Galileo was a great Italian scientist in the 17th Century. Before Galileo conducted some experiments, most people thought that heavy bodies fall faster than light ones do. In fact, if you have not before thought about this you might be thinking so too. Galileo studied the motion of falling bodies by climbing up the Tower of Pisa and dropping objects of different weights. He measured the distance an object falls in a given amount of time. The Tower of Pisa, is now called the “Leaning Tower” since it is no longer completely vertical. It is 183 feet high on one side and 186 feet high on the other.

Let us pretend to repeat Galileo’s experiment and drop a heavy object from the top of the 2723-foot Khalefa Tower in Dubai, the world’s tallest building in 2012. The table of values is given below. One variable is the time *x* the object has been falling, measured in seconds. The other variable is the total distance *y* the object has fallen in *x* seconds, and it is measured in feet.

|  |  |
| --- | --- |
| **Time object has fallen, *x*** | **Distance object has travelled, *y*** |
| 0 | 0 |
| 1 | 16 |
| 2 | 64 |
| 3 | 144 |
| 4 | 256 |
| 5 | 400 |
| 6 | ? |

1. Graph the data above using time in seconds as the independent variable. Label your axes and title your graph. Plan your scale so that your graph uses at least half of the graph paper. Do not connect the points with straight line segments. Your teacher will give you graph paper. Note: Your graph for this data should look very much like your TI-made graphs in class and from the experiment in **Activity 8.1.2**.
2. Earlier in this course you studied *direct variation*. Galileo, before he performed his experiments thought that the distance an object would fall would be directly proportional to the time it took to fall that distance. That means he thought he would come up with an equation of the form *y = kx*, where *k* is a constant. Does your graph support this idea of Galileo’s? \_\_\_\_\_\_\_ Explain why or why not.
3. The same table appears below. Another column has been added so that you can find the change in *y*. After you compute Δ*y* it should help explain why your graph looks the way it does. Should your points appear to lie on a line? \_\_\_\_\_\_\_ Explain.

|  |  |  |
| --- | --- | --- |
| ***x*** | ***y*** | **First Difference****Δ*y*** |
| 0 | 0 | ----- |
| 1 | 16 |  |
| 2 | 64 |  |
| 3 | 144 |  |
| 4 | 256 |  |
| 5 | 400 |  |
| 6 | ? |  |

1. Could your table be describing an exponential function? \_\_\_\_\_\_\_ Why or why not?
2. We can examine the table more carefully to see if the data is quadratic by looking at the second differences. Find Δ(Δ*y*).

|  |  |  |  |
| --- | --- | --- | --- |
| ***x*** | ***y*** | **First Difference****Δ*y*** | **Second difference** **Δ (Δ*y*)** |
| 0 | 0 | ----- |  |
| 1 | 16 |  |  |
| 2 | 64 |  |  |
| 3 | 144 |  |  |
| 4 | 256 |  |  |
| 5 | 400 |  |  |
| 6 | ? |  |  |

Does the table describe a quadratic function? \_\_\_\_\_\_\_

1. Look at the table and the graph to model your function with an equation of the form
 $f(x)=ax^{2}+bx+c.$ You should find *b* and *c* based on the table and graph:

*b = \_\_\_\_\_\_\_\_\_\_ c* = \_\_\_\_\_\_\_\_\_\_\_\_. Explain how you found them.

Now experiment with different values of *a* to find one that fits the data.

*a* = \_\_\_\_\_\_\_\_\_\_\_. Write your equation here: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

1. Do you notice a connection between the value of *a* and the second differences for *y*? Explain.
2. Using your equation, fill in the output in the table when time is 6 seconds.
3. Could we let the time be 1.5 seconds? \_\_\_\_\_\_\_ 2.6 seconds? \_\_\_\_\_\_\_ Why or why not?
4. Would we let the time be –3 seconds? \_\_\_\_\_\_\_ –2.6 seconds? \_\_\_\_\_\_\_Why or why not?
5. Draw a smooth curve through the points on your graph. Let your answer to number 8 above help you. Use your smooth curve to estimate the distance travelled *y* when *t* = 6.
6. Is your estimate in number 11 close to your answer in number 8? \_\_\_\_\_\_\_ If it isn’t try to repair your equation and redo number 8.
7. Use the equation to show how far the object will fall:
8. After 10 seconds?
9. After 15 seconds?
10. When do you think the object will hit the ground? Explain your reasoning.
11. For this real world problem:
12. What is a reasonable domain?
13. What is a reasonable range?
14. If the Khalefa Tower in Dubai had been built in Galileo’s time, do you think he would have used it or the Tower of Pisa for his experiment? Why?