**Doubling Time and Half-Life**

1. Suppose you have $1000 and you are given an opportunity to invest it at an annual interest rate of 6% compounded annually. You want to know how long it will take for your money to double to $2000.
2. Write a function that will give the amount (*y*) you will have after (*x*) years.
3. Use your function to make a table of values:
4. Graph the function. Label and scale the axes.

|  |  |
| --- | --- |
| ***x*** | ***y*** |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |
| 9 |  |
| 10 |  |
| 11 |  |
| 12 |  |
| 13 |  |
| 14 |  |
| 15 |  |
| 16 |  |

1. Use the table or the graph to find the first year when the investment is worth at least $2000.
2. The amount of time it takes an investment to double its value is called the “doubling time.” What is the doubling time (to the nearest year) for an investment earning 6% interest?
3. If you had started with $2000 how long would it take your money to double to $4000? Explain how you can quickly arrive at the answer without much work.
4. Working in a group, find the doubling time for each of these investments. (Suggestion: divide up the interest rates among members of the group. You may use tables or graphs, or estimate solutions to equations using a guess and check method.)

In every case you start with $1000 and the interest in compounded annually.

1. Interest rate = 2%
2. Interest rate = 3%
3. Interest rate = 4%
4. Interest rate = 6%
5. Interest rate = 8%
6. Interest rate = 9%
7. Interest rate = 10%
8. Interest rate = 12%
9. Interest rate = 18%
10. Record your results from questions 1 and 2 in this table.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Interest rate** | 2% | 3% | 4% | 6% | 8% | 9% | 10% | 12% | 18% |
| **Doubling time (years)** |  |  |  |  |  |  |  |  |  |

1. Discuss in your group any patterns you see in the table.
2. Multiply any one of the interest rates by its doubling time? What do you notice?
3. For many years (before the age of calculators and computers) investors used a “rule of 72” to find doubling time. In your own words, what is the rule of 72?
4. A new automobile is purchased for $20,000. It loses its value at the rate of 20% per year. This is called depreciation.
5. What is the decay factor?
6. Find an exponential function that models this situation?
7. Make a table of values for the function.

|  |  |
| --- | --- |
| **Time since purchase**  **(in years)** | **Value of automobile**  **(in dollars)** |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |

1. Estimate (to the nearest 0.1 years) how long it will take the automobile to be worth half its original value. This is called the “half-life.”
2. Estimate (to the nearest 0.1 years) how long it will take the automobile to worth $5000, which is one-fourth the original value.
3. How many half-life periods does it take for the value of the automobile to drop from $20,000 to $5000?
4. When will the automobile be worth $2500? Explain how you can use half-life to arrive at the answer.
5. When radioactive substances decay, they become transformed into different isotopes of the same element or even different elements. They do this by emitting an alpha particle or a beta particle. You may learn more about this in a chemistry or physics class.

Let’s explore situations where the concept of half-life can be applied.

1. Polonium 218 decays to become lead 214 at the rate of 17% per day. What is the decay factor?
2. Write an exponential model for the decay of Polonium 218. Assume you start with 100 grams of Polonium 218.
3. Use a table or a graph to estimate the half-life of Polonium 218 to the nearest 0.1 day.
4. Uranium 238 decays to become Thorium 234 at the rate of 14.3% every billion years. Use a table or a graph to estimate the half-life of Uranium 238.
5. Bismuth 210 decays to become Polonium 210 in 5 days. Assume that you start with 100 grams of Bismuth 210. Experiment with different values of *b* in the exponential model to estimate the decay factor and the rate of decrease.
6. Does the half-life of a radioactive substance depend upon the amount of material you start with? Explain.