**The Meaning of Integer Exponents**

To better understand exponential functions let’s review the meaning of exponents and how to simplify exponential expressions.

**Meaning of whole number exponents:**

The exponent tells you how many times to repeat multiplication of the base by itself.

EX: 32  = 3∙3 = 9 EX: 74 = 7∙7∙7∙7 = 2401

EX: (–5)3  = (–5)∙(–5)∙(–5) = – 125 EX: –23 = – (2∙2∙2) = – 8

**Be careful!**

The negative sign is not part of the base!

Why? Because of order of operations.

EX: $\left(\frac{5}{7}\right)^{2}= \frac{5}{7}∙\frac{5}{7} = \frac{25}{49}$

1. Simplify each expression by first writing out what it means. You can leave your answer in exponential form (show the base(s) and exponent(s) – you do not need to evaluate.)

EX:$\left(5\right)^{2}∙\left(5\right)^{4}=\left(5∙5\right)∙\left(5∙5∙5∙5\right)=5∙5∙5∙5∙5∙5= 5^{6}$

EX: $\left(4^{3}\right)^{2} = \left(4∙4∙4\right)^{2} = \left(4∙4∙4\right)∙\left(4∙4∙4\right)= 4∙4∙4∙4∙4∙4= 4^{6}$

EX: $\frac{2^{3}∙5^{5}}{2^{6}∙5^{2}} = \frac{(2∙2∙2)∙(5∙5∙5∙5∙5)}{(2∙2∙2∙2∙2∙2)∙(5∙5)} = \frac{5∙5∙5}{2∙2∙2} =\frac{5^{3}}{2^{3}} $

1. $8^{3}∙8^{5}$
2. $\left(2^{3}\right)^{5}$
3. $\left(\frac{1}{7}\right)^{3}$
4. $\frac{3^{4}∙4^{6}}{3^{7}∙4^{5}}$
5. Simplify each expression by first writing out what it means. You can leave your answer in exponential form (show the base(s) and exponent(s) – you do not need to evaluate for now).
6. $5^{4}∙5^{6}$
7. $\left(7^{5}\right)^{2}$
8. $\left(2^{3}\right)^{3}$
9. $\frac{6^{4}∙8^{6}}{6^{7}∙8^{5}}$
10. $\left(\frac{2}{9}\right)^{4}$
11. $\left(\frac{3}{8}\right)^{5}$
12. $\frac{2^{3}∙3^{7}}{3^{3}∙2^{2}}$
13. $\frac{5^{3}∙3^{2}}{3^{4}∙5^{6}}$
14. $3^{2}∙3^{6}$
15. $6^{4}∙6$
16. $5^{4}∙5^{6}$
17. $\left(9^{4}\right)^{3}$
18. $\left(4^{4}\right)^{2}$

You wrote out the exponential expressions on the last page to show their meaning and to simplify them. What patterns do you notice?

1. Write down any patterns that you observe.
2. Now make up some examples of your own to show each pattern you found.
3. Discuss the patterns and examples with your class.

Now let’s look at an exponential pattern in a table. We have examined exponential patterns like this before. We can get from one output to the next output by using the constant multiplier 10. The arrows and the box with the constant multiplier show this.

1. What is the pattern when we go backwards? Fill in the blank box below to describe how we move from an output to the previous output. Then use the pattern to figure out the meaning of zero and negative exponents. Write the values in decimal and in fraction form.

|  |  |  |  |
| --- | --- | --- | --- |
| ***x*** | **10*x*** | **Meaning** | **Value** |
| –3 | 10–3 |  |  |
| –2  | 10–2 |  |  |
| –1  | 10–1 |  |  |
| 0 | 100 |  |  |
| 1 | 101 | 10 | 10 |
| 2 | 102 | 10∙10 | 100 |
| 3 | 103 | 10∙10∙10 | 1,000 |
| 4 | 104 | 10∙10∙10∙10 | 10,000 |
| 5 | 105 | 10∙10∙10∙10∙10 | 100,000 |
| 6 | 106 | 10∙10∙10∙10∙10∙10 | 1,000,000 |
| 7 | 107 | 10∙10∙10∙10∙10∙10∙10 | 10,000,000 |
| 8 | 108 | 10∙10∙10∙10∙10∙10∙10∙10 | 100,000,000 |

× 10

1. Explain in your own words what you think a zero exponent means. Then test your theory by trying a zero exponent with a variety of bases on your calculator.
2. Explain in your own words what you think a negative exponent means. Then test your theory by trying some negative exponents with a variety of bases on your calculator. (Try integer and fractional bases. Convert all results to fractions to ‘see’ what happens.)
3. Discuss these ideas with the class.
4. Use the meaning of exponents to write out each exponential expression to show what it means and then simplify it.

EX: $2^{-3}= \left(\frac{1}{2}\right)^{3}= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)= \frac{1∙1∙1}{2∙2∙2}=\frac{1^{3}}{2^{3} }= \frac{1}{8}$

EX: $\left(\frac{3}{5}\right)^{-2}= \left(\frac{5}{3}\right)^{2}= \left(\frac{5}{3}\right)\left(\frac{5}{3}\right)= \frac{5∙5}{3∙3}= \frac{5^{2}}{3^{2}}= \frac{25}{9}$

1. $10^{-4}$
2. $5^{-6}$
3. $\left(\frac{2}{7}\right)^{-3}$
4. $\left(\frac{1}{4}\right)^{-2}$
5. $9^{-2}$
6. $8^{-5}$
7. $\left(\frac{8}{3}\right)^{-4}$