**Unit 6: Investigation 2 (4 Days)**

**SOLVING SYSTEMS OF LINEAR EQUATIONS BY SUBSTITUTION**

**CCSS:** A-REI #5, A-REI #6

**Overview**

In this investigation, students will use the substitution method to solve systems of linear equations. Students will learn that the underlying mathematical justification for the substitution technique is the substitution property of equality – that one can substitute equivalent expressions for each other. To date, they have experienced substituting a single value for a variable when evaluating algebraic expressions. In this investigation, students substitute algebraic expressions for a variable.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Students will solve a system of linear equations using the substitution method.
* Students will explain what the solution to a system of linear equations means in the context of a real world problem.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 6.2.1** requires students to solve a system of equations by substitution.
* **Exit Slip 6.2.2** requires students to use a system of equations to find the break-even point.
* **Journal Entry 1** asks students to explain the meaning of the word “substitution.”
* **Journal Entry 2** asks students to explain how to use a table, a graph, or the substitution method to solve a system of equations.

**Launch Notes**

Heifer International is a charitable organization that sends farm animals and seeds, and provides agricultural and business instruction, to people who do not have enough to eat. Two major tenets of the Heifer Project are “Pass on the Gift” whereby the receiver becomes a giver when new animals are born, and “Give a man a fish, he eats for a day. Teach a man to fish, he eats for a lifetime.”

You may write to Heifer International or go to [www.heifereducation.org](http://www.heifereducation.org) in advance in order to have a classroom set of informational brochures, stickers and other materials on hand for the first day of the lesson. Start the lesson by inviting students to read about the Heifer International projects. Students can summarize and share information from the brochures and discuss the philosophy of the Heifer Project, what they do, how they do it, who donates, and who receives Heifer gifts. You may choose among other options for launching this investigation depending upon students’ interests and what they are studying in other courses. Some suggestions include:

1. Have students write an essay or discuss in groups either “Pass on the Gift” (which is also *Pay it Forward* from the book and movie of that name) or the saying “Give a man a fish….”
2. Show a video clip from Heifer International at [www.Heifer.org](http://www.Heifer.org) or [www.HeiferEducation.org](http://www.HeiferEducation.org).
3. Ask students to research Heifer International before class and write down two facts about the Heifer program to share with the class.

**Closure Notes**

By the end of the lesson, students should know two techniques for solving a system of equations—graphing and substitution, and be able to describe the advantages of each. You may check their understanding with the two exit slips described below.

**Teaching Strategies**

1. Begin the lesson by having the students learn about Heifer International, described above, and connect the scenario, as appropriate, to world hunger and nutrition, community service projects, world cultures, geography, or agriculture.

Arrange students into pairs or small groups and have them answer Question 1 on **Activity 6.2.1** **Passing on the Gift**. Do not tell the students how to work the problem. Allow them time to use the mathematical ideas they already know to solve the problem. The goal is to have students discover the substitution method for solving systems of equations.

Students may use ideas from the previous investigation to solve the problem such as making numerical tables or writing algebraic equations. Ask probing questions and encourage students to develop a variety of methods for solving the problem. Some students may graph the information, some may write equations, and others may make tables of the following type:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Number of Goats, *g* | 10 | 9 | 8 | 7 |
| Number of flocks of chickens, *c* | 0 | 1 | 2 | 3 |
| Total Cost | $120(10)+$20(0)  = $1200 | $120(9)+$20(1)  =$1100 | $120(8)+$20(2)  =$1000 | $120(7)+$20(3)  =$900 |

A table is a useful step in the transition from verbal statements to algebraic equations, so encourage its use, and, later, ask students if they can write the system of two equations.

Since students will have just solved systems by graphing, some will use that method. Some students will be able to write two equations, and may figure out the substitution method on their own.

You may walk around to each group and encourage different groups to solve the problem using different methods according to the group’s inclinations. If a group finishes early, you may ask that group to solve the problem in a different way – numerically, graphically, or algebraically depending on which method is not yet developed by the group. For example, if a group has solved the problem by graphing two lines, the teacher can ask the students to find algebraic equations for the lines and use the equations to find the solution. One group may show signs of using the substitution method for solving a system of equations. Ask probing questions to confirm students’ thinking. “What are the variables and how are they defined? What are the two equations you can write with the two variables? Could you solve one equation if it used only one variable? Can you change one equation so that one variable is written in terms of another variable?”

Be sure to have students present to each other a variety of ways to attack the problem. If a group is close to solving the system using substitution, you may ask leading questions to help them discover the substitution method. During the whole class discussion, be sure the students articulate the idea that the point of intersection of the two lines is the point that makes both equations true.

Elicit ideas about why graphing and numerical solutions might be cumbersome. Have a whole class discussion on the substitution method. Begin by asking them for examples of substitution in their experience. They may think of substitutes on the soccer field, substitute teachers, and cooking substitutions. A substitute is one thing that stands in for another. In math class students have substituted values for variables when evaluating expressions for given values. The substitution method for solving a system of equations involves finding an equivalent expression to replace one variable in order to transform an equation in two variables to an equation in one variable.

Ask students to explain what the solution means in the context of the problem. Confirm that the various methods for solving the problem lead to the same result. Explain that the problem that they just solved is called *a system of linear equations*. Have students help define what a system of equations represents. If one group was able to solve by substitution, be sure to reference the work of that group, and say that we will now formalize that method. Otherwise, ask the class probing questions to lead them to the method.

Discuss the essential requirement for substituting in mathematics: the substitute expression must be equivalent to the original expression. Also ask, “When have we seen substitution in math class?” Point the discussion in the direction of evaluating expressions with defined replacement variables (evaluate the expression 3*x*+2*y* for *x*=1 and *y*=–2) or the *f(x)* notation (Given *f(x),* evaluate *f*(2)). Develop the idea that expressions, not just numbers, can be substituted in for variables. On the board, you can write a system that uses geometric shapes or a nonsense word instead of the variables *x* and *y*. Show that we could substitute these shapes or nonsense words into the equation – though we have not defined operations on shapes or words leaving the operations undefined, we could write the expression. If you used a geometric shape, you could fill in the shape with a mathematical expression to show how an expression such as 5*x* + 7 can be substituted in for *y*. Thus, you can help the students become visually comfortable with substituting multiple part expressions in for a single variable. Emphasize the mathematical concept underlying this method -- “the substitution property of equality” that states “equivalent expressions can be substituted for each other”, or “If *a* = *b*, then “*a*” can be substituted for “*b*” in a mathematical expression.” Ask students to identify different manifestations of the substitution property of equality that have already occurred in their math experience.

In a whole class guided discussion, work through the steps involved in solving a system of equations by substitution. Have students write process cards to place in their math toolkit or the “Formula Reference” section of their notebooks.

Next students may work in pairs to tackle the other two problems in **Activity 6.2.1 Passing on the Gift** using substitution. As you circulate, note which pairs solve using one variable or another or one equation or another. Be sure that the pairs which show different approaches present their solution to the class, so that the class can see that it does not matter which variable you choose to solve for, nor which equation you use to solve for the variable. It is likely that some students will try to substitute the variable they solve for back into the same equation. Explain that this leads to an identity, does not utilize the other equation in the system, and does not lead to a solution of the system. If any of these variations do not present themselves, direct one of the faster pairs to try out what would happen if, for example, you solved for a variable in the first equation, and then substituted the found expression back into the first equation. Ask another pair to try solving for the other variable first.

Some students may use only one variable to identify the two pieces of information being asked for. For example, in the first problem, “Let *g* = the number of goats, and let 10 – *g* be the number of flocks of chickens”. Note that this method of solving problems would give the equation 120*g* + 20(10–*g*) = 1000 and is mathematically equivalent to the substitution method for solving a system of equations if we let *f* = number of flocks, solve for *f* in *f* + *g* = 10 and substitute for *f* in the equation 120*g* + 20*f* = 1000. Be sure the students see that the mathematics is the same regardless of the approach. Comparing the two approaches shows that by judicious naming of variables, one can bypass the step where one solves for a variable in one equation to substitute into the other equation. Using two variables, however, has the advantage of being more transparent in setting up the equations. Insist that the students learn to write a system of equations in this unit.

Wrap up by reviewing the steps in solving a system using the substitution method, noting that it is easier to solve for a variable when the coefficient is one. Discuss the pros and cons of solving a system algebraically compared with graphically.

You may use **Exit Slip 6.2.1** requiring the students to solve a system of equations by substitution. **Activity 6.2.2 Solving Systems by the Substitution Method** may be used as homework. Additional practice is provided in **Activity 6.2.3 More Practice with the Substitution Method**, which may be used as needed.

**Differentiated Instruction (For Learners Needing More Help)**

Students may need some guidance in identifying the two variables and writing the two equations. Underline the words that describe the variables and translate the given information into equations. Have the students write the solution as an ordered pair, and write in words what the solution means in the context of the problem.

You may allow students to use the “Formula Reference Section” from their notebooks, which could include a procedure card on how to solve a system by substitution.

**Group Work**

Have students work in pairs to solve problems 2 and 3 on **Activity 6.2.1**. Hopefully student pairs will raise questions such as whether it matters which equation is used to solve for a variable or which variable is solved for. Have them try the problem one way then the other way and compare results.

**Journal Entry 1**

Look up the word “substitution” in the dictionary and explain why this method is called the “substitution method.”

1. **Activity 6.2.4 Drag Racing** yields a system of equations where both equations are in slope-intercept form. Students will receive practice using variables other than *x* and *y*. Note that the car starts at the 0-meter mark on the track and the truck starts at the 120- meter mark. Help students to see that the slope is the rate of change or speed of the vehicle in meters per second and the starting point on the track is the *y*-intercept. The equations will be *d* = 30*t* + 120 and *d* = 45*t*. In solving a system of equations with both equations in the form *y = mx + b*, one approach is to substitute the second expression for *y* in for the *y* in the first equation, thereby “setting the *y*’s equal to each other” and then solving the one resulting equation in *x*. In fact, in any system, students may prefer to solve both equations for *y*, set the equations equal to each other and solve. An advantage to this approach is the ease in checking the answer with the graphing method on a calculator.

You can set the stage for The Drag Racing Activity by asking students if they have ever gone to a car race. Drag racing videos are available at <http://www.dragtimes.com/drag-racing-videos.php> or by searching “drag race videos”. Emphasize that drag racing on public streets is illegal and deadly for not only the racers, but for innocent bystanders, too.

“Would it be fair for a pickup truck to race a Mustang? How can you make the race fair if one vehicle is much faster than the other?” “To be Fair” could be defined as “each person has an equal opportunity to win”. Discuss giving a head start to the slower vehicle. In this activity the head start will take the form of the slower vehicle starting 120 meters closer to the finish line. (This is mathematically equivalent to giving the truck a 4-second head start).

There is an applet simulating the drag race that you can show to the class. Go to <http://uhaweb.hartford.edu/rdecker/> and click on the link to *Algebra Curriculum Project*. Directions for using the applet are under the Applet Help menu. Click on and move the slider in the box below the graphs to see the race on the left and the corresponding time-distance plot on the right. Point out to the class how the applet helps them visualize various events along the race such as “Where does the Mustang overtake the pickup?” and “How far ahead is the Mustang at the end of the race?” Students may finish the worksheet for homework.

**Differentiated Instruction (For Learners Needing More Help)**

Students could walk a simulation of a race. If you haven’t already done so in earlier units, use a motion detector to show that the time distance graph of a person walking at a constant rate is a straight line. Contrast this with a person speeding up (concave up graph) or slowing down (concave down graph).

As an extension, students might use the race applet to change the parameters of the problem by using different starting points or different speeds. Ask students to predict the results and then check using the applet or the graphing calculator.

**Group Work**

Have students begin work on **Activity 6.2.4** in groups. Review student work as you circulate among the groups. Be sure everyone has the equations *d = 45t* and

*d = 30t + 120* before they leave the class. Note that solving a system of equations by setting the *y*’s equal to each other is an example of the transitive property of equality: if 45*x* = *y* and *y* = 35*x* + 120, then 45*x* = 35*x* + 120. (If *a* = *b* and *b* = *c*, then *a* = *c.)*

**Journal Entry 2**

If you have a system of equations, explain how you would solve the system using:

1. a table
2. a graph
3. substitution method
4. In **Activity 6.2.5 Break-Even Analysis**, students create a system of equations in which both equations are in slope-intercept form. In this example, however, the dependent variables are not the same: one is total cost and the other is revenue. The break-even point is where revenue equals cost so we set the *y*’s (i.e. the dependent variables) equal to each other.” To launch this activity, you can ask students if they have ever participated in a fundraiser or business venture. Have the students identify the fixed costs, variable costs and revenue for the situations the students describe.

Students should learn the vocabulary of revenue, cost, variable cost, fixed cost and break-even point. Have the students summarize the information by writing semi-algebraic expressions such as:

The break-even point occurs is when .

Call the class to attention after about 10 minutes to be sure everyone has the correct total cost function *C(x) = 0.15x + 450* and revenue function *R(x) = 1x*. Highlight the different ways students arrived at the equations. Did they make tables? Did they just fill in for slope and *y*-intercept in the slope intercept form of an equation? Why is the intercept for the revenue equation zero? Once the students understand how to develop the cost and revenue equations, have them continue working on the Popcorn problem in their groups.

When everyone has completed **Activity 6.2.5**, have one group present their solution. Discuss the advantages or disadvantages of the algebraic substitution method over the graphical method. In the solution, students should realize that the break-even point is when *Revenue = Cost*, and write *R(x) = C(x)* on the board. Then replace *R(x)* with an equivalent expression *1x*. Similarly, replace *C(x)* with *0.15x + 450*. You now have *1x = 0.15x + 450*. The solution, *x* ≈ 529.41 is found with greater precision using the substitution method than with a graph. In the real world context, however, students should realize that a fractional bag of popcorn does not make sense. They will have to sell 530 bags in order to break even. Note that Question 10 requires representing an inequality on a number line as introduced in Unit 2.

**Differentiated Instruction (Enrichment)**

Some students may want to talk about profit (see **Activity 6.2.5**). Observe that *Profit = Revenue –Cost* or *P = R – C*. Since the break-even point is where Profit is 0, substitute 0 in for *P*: *0 = R – C* which becomes: *R = C*. If profit is 0 when *R = C*, ask when profits will be positive (when *R>C*) and when one operates at a loss (*R<C*). One could find the break-even point by finding a profit function *P = 1x – (0.15x + 450)* and solving for *P = 0*. Graph all three equations on the graphing calculator: profit, revenue and cost. Note that the *x*-intercept of profit function is the same *x* value as the intersection of the revenue and cost functions. Notice that the interval on which the cost function is greater (higher) than the revenue function is where the profit is negative (the business is losing money) and the interval where revenue is greater than cost is where the profit function is positive.

**Group Work**

**Activity 6.2.5** is appropriate for heterogeneous groups. Although there is a lot of new vocabulary (cost, revenue, break-even point, etc.), the more able students should be able to help their partners figure out what to do.

Students can finish the break-even analysis problem on **Activity Sheet 6.2.5** for homework. You may also create a few homework problems that use topics of student interest, and/or have the students create systems of equations to solve an interesting problem. This is a good opportunity to personalize the problems for the students.

**Exit Slip 6.2.2** presents another situation in which students must find the break-even point.

1. **Activity 6.2.6 Systems of Equations in Slope-Intercept Form** includes several systems of equations that have equations in slope-intercept form. The systems involve negative integers. Two systems have a single solution, one has no solution, and one has infinitely many solutions. This provides a review of the concepts from the first investigation. You might have the class come together as a whole. Ask the students as a class to describe what happens algebraically when you try to solve a system with no solution (a patently false statement results) and when you try to solve a system with infinitely many solutions (a patently true statement results). Ask the students to describe how they can tell whether there is no solution before they even begin to graph or solve the system algebraically. (The slopes of the two lines are equal, but the *y*-intercepts are different.) Have the students check their work by graphing the system on a graphing calculator. Give students a few minutes to write down any questions or concerns they have about solving a system of equations by the substitution method. Ask students to share what they wrote.

**Differentiated Instruction (For Learners Needing More Help)**

If the students need more practice transforming verbal descriptions to algebraic equations, you might have them work on three different scenarios that result in identical equations. Point out that the math is independent of the real-world contexts, which is part of the power of mathematics as a problem-solving tool. **Activity 6.2.7 One (Equation) for All and All for One** is an example of a set of problems with different contexts, but identical equations.

**Resources and Materials**

* **Activity 6.2.1** Passing on the Gift
* **Activity 6.2.2** Solving Systems Using Substitution
* **Activity 6.2.3** More Practice with the Substitution Method
* **Activity 6.2.4** Drag Racing
* **Activity 6.2.5** Break-Even Analysis
* **Activity 6.2.6** Solutions to Systems of Equations
* **Activity 6.2.7** One for All
* **Exit Slip 6.2.1** Substitution Method
* **Exit Slip 6.2.2** Breaking Even
* Bulletin board for key concepts
* Graphing Calculators
* Student Journals