**Unit 6: Investigation 1 (3 Days)**

**SOLVING SYSTEMS OF LINEAR EQUATIONS**

**CCSS:** A-REI #6, A-REI #11

**Overview**

In this investigation, students will solve systems of linear equations by making tables, solving linear equations in one variable, and graphing lines (both by hand and with the graphing calculator). They will find and interpret solutions of systems of linear equations and use systems of linear equations to solve real world problems.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Students will write equations to model a situation, graph equations (both by hand and using the graphing calculator), find the point of intersection and interpret the solution in the context of the problem.
* Students will solve a system of linear equations that represents a real world situation graphically and numerically.
* Students will explain what the solution to a system of linear equations means in the context of the problem.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 6.1** asks students to solve a system of linear equations graphically and to explain what the solution of that system represents.
* **Journal Entry** asks students to explain in their own words how to find the solution of a system of linear equations graphically.

**Launch Notes**

The gender wage-gap is a real-life context which can be used to engage students in the first activity. Historically, women in the labor force have earned less than men. Salaries in the range $50,000 to $74,999 have traditionally been considered “middle” to “upper middle” class. Not surprisingly, the percent of men in this group is greater than the percent of women. However, this may change in the future. Display the data table on an overhead projector or digital board and begin the lesson by asking the question: “Will the percentage of women earning $50,000 - $74,999 per year ever be equal to or surpass the percentage of men earning $50,000 - $74,999 per year?”

Inform students that they will use a system of linear equations to solve this problem and that they will discover throughout this unit that systems of linear equations can be used to solve problems, make decisions, and make predictions.

**Closure Notes**

The investigation culminates with students successfully demonstrating their ability to solve a system of linear equations graphically. This may be done with **Exit Slip 6.1**. Immediately before or after administering the exit slip, have students discuss the meaning of the solution of a system of equations. They should recognize that the point of intersection on the graph represents an ordered pair that satisfies both equations.

**Teaching Strategies**

**Differentiated Instruction (For Learners Needing More Help)**

You may start by giving students two equations that are easy to plot. Ask them to graph the system by hand and locate the point of intersection. Then move into the example that arises from the real world situation.

1. Begin the class by displaying the data table below and asking the class, “Will the percentage of women earning $50,000 - $74,999 per year ever be equal to or surpass the percentage of men earning $50,000 - $74,999 per year?” (See **Activity 6.1.1a Will Women Catch the Men?**). Let the students think about the information for a few minutes and offer any comments.

|  |  |  |
| --- | --- | --- |
| **Number of years since 2000** | **% of men**  **earning**  **$50,000 – $74,999** | **% of women**  **earning**  **$50,000 – $74,999** |
| 2 | 20.1 | 13.0 |
| 3 | 20.2 | 13.3 |
| 4 | 20.5 | 14.2 |
| 5 | 20.7 | 15.1 |

The students may be able to predict that the women within the income bracket specified will indeed ‘catch’ the men since the percentage of women earning $50,000 - $74,999 is increasing at a more rapid rate then the percentage of men. If so, then ask them, “If these trends continue, when will the women catch the men and, at that moment, what percentage of women will be earning $50,000 - $74,999?”

If students do not think about using scatter plots and trend lines to represent the data, then probe them by asking, “What strategies have you learned this year that may be useful in this situation?” and/or “What do you notice about the data?”

Students may choose to make a scatter plot and fit trend lines by hand or they may choose to use calculators to find the least squares regression lines. Students should notice that the slope of the linear model for the women is greater than the slope of the linear model for the men, indicating that, if the rates stay constant, eventually the women will catch up. Ask them “How will you use these equations to answer the question: If these trends continue, when will the women catch the men and what percentage of women will be earning $50,000–$74,999?”

Students should realize that by extending the trend lines, they can find the point of intersection, and answer the question. If they are using a calculator they may trace to the point of intersection or use the intersect feature. To do so they select 2nd Trace (Calculate), 5: intersect, and choose their two lines as the 1st curve and 2nd curve. The point of intersection is (16.20, 23.04). You should then help them interpret the meaning of this point. The women should catch up to the men in the year 2016 at which time approximately 23% of each gender will be earning between $50,000 and $74,999.

In the course of the discussion you should revisit important ideas from Unit 5, including the meaning of the correlation coefficient—in both cases *r* ≈.98—and the difference between interpolation and extrapolation. Students may suggest that the recession of 2008 may affect the trends and lessen the accuracy of the prediction.

You may ask students to help develop the definition of a system of linear equations and what is meant by a solution of a system of equations. As they solve more problems in this unit, students may refine their definitions.

Note that **Activity 6.1.1a Will the Women Catch the Men** is designed for an open-ended approach to this problem. You may choose to use **Activity 6.1.1b** **Will the Women Catch the Men** which is more structured version of the activity if you wish to have students work in groups after the problem has been presented to the entire class.

**Differentiated Instruction (For Learners Needing More Help)**

This is an opportunity to build on the skills learned in Unit 5 by calculating two linear regression equations and finding the point of intersection.

Allow students to use the formula reference section of their notebook, which could include a procedure card that shows how to create a scatter plot on the graphing calculator, calculate a linear regression, and calculate a point of intersection.

**Technology Tips**: If students use technology to find regression lines, guide students to see that solving this problem using technology utilizes skills that they learned in Unit 5. Note that only three lists are needed instead of four – the independent variable is the same for both dependent variables, thus it only needs to be entered once. Therefore, the students could input the data in the table into Lists 1, 2, and 3. Then, they need to use Lists 1 and 2 (number of years since 2000 and men’s data) to graph scatter plot 1. Next, they should use the data in Lists 1 and 3 (number of years since 2000 and women’s data) to graph scatter plot 2.

Note that when calculating the linear regression for the men, they can use the command *LinReg(ax+b) L1, L2.* The equation they should get is

. When calculating the linear regression equation for women they can use the command *LinReg(ax+b) L1, L3.* The equation they should get is . They may then enter the equations in *Y1* and *Y2* in the “*Y =”* menu. Some newer versions of the TI-83 family of calculators allow you to do this in one step via the commands *LinReg(ax+b) L1, L2, Y1* and *LinReg(ax+b) L1, L3, Y2.*

Be sure to ask students to explain the meaning of the slopes of the two lines. Also, when they find the point of intersection, have them substitute the coordinates into *both* equations to show that both equations are satisfied at the point of intersection.

Students may get an error message on their calculator depending on what they chose for their window settings. Remind them that they must be able to see the point of intersection in order to calculate it. Thus, they must modify their window.

**Differentiated Instruction (Enrichment)**

If students are interested in exploring data for earnings between $75,000 and $99,999 a year, they may use the table below. You may choose to use these data as an extension to the lesson as well.



Linear regression equation for men: *y* = 0.2*x* + 7.4667

Linear regression equation for women:  *y* = 0.45*x* + 2.3667

Intersection Point: (20.4, 11.55)

Another possible extension would be for students to go the U.S. Census Bureau website and research the earnings for other years and/or income levels. Source:

<http://usgovinfo.about.com/od/censusandstatistics/Census_and_Statistics.htm>

1. **Activity 6.1.2 Choosing a Gym** provides students an opportunity to solve real world problems using multiple approaches. As students work on problem 1, circulate to see the approaches that students are using to solve the problem. Students may solve the problem by making a table, using a guess and check approach, solving the equation , graphing by hand, or graphing on the graphing calculator. If they have answered the question by using one of these methods, then challenge them to see if they can represent the situation in a different manner or solve the problem using a different method.

You may have different pairs of students present their solutions to the class so that solving the problem graphically, algebraically, and via a table are all presented. Discuss the pros and cons of each method. Discuss the answer to part (d) and ask the class what it means to be a solution of a system of linear equations.

Problems 2 and 3 in **Activity 6.1.2** present similar scenarios. They introduce students to two special situations. In problem 2 the two equations are represented by the same line and there are an infinite number of solutions. In problem 3 the two lines are parallel so there is no point of intersection. Ellie’s gym is less expensive no matter how many months Natasha uses the gym.

**Group Activity**

Structure the assignment of **Activity 6.1.2** to be given to pairs of students. For example, one member may do question 1a while the other is working on 1b. They can then compare answers and work together on 1c and 1d. For question 2b, each student can be responsible for writing one of the equations, then checking the other’s work. Have students take joint responsibility for the final question in each set (1d, 2e, and 3e) in which they must explain the rationale for their choices and ask them to be prepared to share their reasoning with the class.

1. **Activity 6.1.3 Solving Systems of Equations by Graphing** provides students additional practice in using the graphing method to solve systems of equations. Students may need to be reminded of the different techniques they learned in Unit 4 for graphing the equation of a line. They may pick two or more points, or start with the *y-*intercept and use the slope to find a second point. This selection of systems includes points of intersection in all four quadrants and equations with fractional coefficients.

Re-emphasize that a solution to a system of equations consists of an ordered pair (*x, y*) and that they should check to be sure the solution satisfies both equations.

In problems 6 and 7 students again encounter situations where there is an infinite number of solutions and where there is no solution. Dependent and inconsistent systems will be studied in more detail in the next investigation. Finally, in Problem 8, the intersection point lies outside of the graph given. Students will need to reason that when the slopes are different the lines will eventually intersect. They may use another sheet of graph paper or a calculator to find the solution, which is (6, 23).

Up until this point, all the equations have been given in slope-intercept form. **Activity 6.1.4 Systems with Equations in Different Forms** extends the graphing method to systems that contain lines in standard form or slope-intercept form. Have students graph these equations in two ways: (1) by finding two or more points to make a graph by hand and (2) by solving for *y* in terms of *x* so that the equation is in slope-intercept form and can be entered into the Y= menu on the calculator.

Have students complete **Exit Slip 6.1**, which requires them to graph a system of linear equations graphically and explain what the solution represents.

**Journal Entry**

Explain, in your own words, how to find the solution of a system of linear equations graphically.

**Resources and Materials**

* **Activity 6.1.1a** Will the Women Catch the Men
* **Activity 6.1.1b** Will the Women Catch the Men
* **Activity 6.1.2** Choosing a Gym
* **Activity 6.1.3** Solving Systems of Equations by Graphing
* **Activity 6.1.4** Systems with Equations in Different Forms
* **Exit Slip 6.1** Solving a System by Graphing
* Graph paper
* Calculators
* Bulletin board for key concepts