**Unit 4: Investigation 4 (4 Days)**

**EFFECTS OF CHANGING PARAMETERS OF AN EQUATION IN SLOPE-INTERCEPT FORM: *y* = *mx* + *b***

***CCSS:*** F-LE2, F-LE5, F-IF7, F-IF7a, G-GPE 5

**Overview**

Students examine how changes in the parameters *m* and *b* in slope-intercept form affect the function’s graph.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Describe the changes in a line that occur when the *y*-intercept increases or decreases.
* Describe the changes in a line that occur when the slope increases or decreases.
* Graph a line given the slope-intercept form of a line by first plotting the *y*-intercept then using slope to find a second point on the line.
* Explain the meaning of a change in slope or a change in *y*-intercept in the context of a real world problem.
* Identify the slope and *y*-intercept of the line from the graph of a linear function.
* Find the slope-intercept form of the equation of a line given its graph with the *y*-intercept and an indicated point.
* Identify parallel lines as having the same slope, but distinct *y*-intercepts.
* Identify perpendicular lines as having slopes that are opposite reciprocals, or equivalently, slopes whose product is -1.

**Assessment Strategies: How Will They Show What They Know?**

**Exit Slip 4.4.1** assesses student understanding of the parameters *m* and *b*.

**Exit Slip 4.4.2** has students apply the slope-intercept form of the line to a real world context.

**Journal Entry** asks students to think about the conditions that determine a line.

**Launch Notes**

This may be the first intensive use of the graphing feature of the calculator, so you may want to have a brief discussion about the calculator as a laboratory tool that will help us discover mathematics. It is important to know when it is appropriate, even necessary, to use the calculator and when the calculator my lead you astray. A brief history of computing tools and computing machines in mathematics may be in order. You could have students look up one fact about technology in mathematics and bring that fact to class to share with their classmates. You could assign the following topics to different students and ask them to do a quick search: Pascal’s calculating machine, the slide ruler, the first computers of various types (main frame, desktop), the first calculators of various types (adding machines, scientific, graphing), costs and capabilities of calculators since the 1960’s, computer algebra systems. Inform students how advances in technology have changed mathematics and changed which mathematical disciplines are important.

**Closure Notes**

Point out to students that since Unit 1, they have been working with explicit rules for arithmetic sequences and real life scenarios such as the Hydrocarbon activity that involve rates of change (*m*) and fixed starting amounts (*b*). We have moved from an intuitive understanding of the slope-intercept form of a linear function to a more formal mathematical view of *y* = *mx* + *b* which includes mathematical tools and skills students will need to work effectively to understand this ubiquitous pattern.

**Teaching Strategies**

1. Students start **Activity** **4.4.1 Effects of Changing Parameters** by graphing a set of linear functions having the same *y*-intercept but different slopes. Students will discover that changing the magnitude of the parameter *m,* the slope of the line, causes changes in the steepness of the graph, and changing the sign of *m* changes the direction of the graph.

Students will then graph a set of linear functions having the same slopes but different *y*-intercepts on the same set of axes. Students will discover that changing the *y*-intercept of a linear function causes vertical shifts in the function’s graph. Hence *b* plays two roles – that of the vertical shift and the *y*-intercept. In this activity students may discover that parallel lines have the same slope. This will be discussed later in this investigation.

Help students to arrive at a definition for “parameter,” and help them distinguish parameters from variables. In the case of linear functions, the parameters are *m* and *b*, and the variables are *x* and *y.* For any particular linear function the values of *m* and *b* are constant. Often they are given or there is enough information to enable us to find them. We can understand the effect of these parameters by letting them vary and comparing one linear function with another. The concept of parameter will be revisited in Unit 7 with exponential functions and in Unit 8 with quadratic functions.

Students should practice drawing accurate graphs and labeling the graphs clearly. Graphing calculators or graphing software can provide a dynamic picture of the changes in the graph as the parameters increase or decrease.

**Exit Slip 4.4.1** assesses what students have learned so far about the parameters *m* and *b*. It is designed for a group activity in which each pair of students is given a different linear function, but it can be modified for use as an individual assessment.

1. In **Activity 4.4.2 Slope-Intercept Form**, students identify the slope as *m* and the *y*-intercept as *b* from multiple representations of linear functions. From the algebraic equation, have students create a table of values and graph the function by plotting points. From the table or the graph, have them identify the slope and the *y*-intercept as they did in Investigation 3. Lead students to discover that the slope is the coefficient of *x* and the *y*-intercept is the constant term. Encourage students to graph a function in slope-intercept form by plotting the *y*-intercept and then using slope to find a second point on the line. Reverse the process by presenting a graph and asking students to identify the slope and *y-*intercept and then write the equation. **Activity 4.4.3** **Practice with Slope-Intercept Form** provides additional practice and is suitable for homework. Students should realize that the slope and the *y*-intercept are sufficient characteristics for determining a linear function.

In **Activity 4.4.4 Making a Profit**,students explore a real world application and analyze the effect changes in the slope and the *y*-intercept have on a linear function in context.

**Group Activity**

Several of the activities in this investigation, particularly **Activity 4.4.4,** are well suited for group work. Consider doing a jigsaw puzzle style of group work whereby students from an original group are reorganized into ability groups unbeknownst to the students. Assign a problem to each group according to their ability. Then the original groups reconvene for each student to share their work.

1. Provide additional practice with problems concerning distance versus time or dollars per item so students may interpret the *y*-intercept as the “start” and the slope as how things change. Knowing where to start and how to move is sufficient for finding a solution. In previous activities, students used the slope-intercept form of the line without calling it by name. **Activity 4.4.5 Applications of Slope-Intercept Form** contains questions which students may find familiar as well as questions related to slope-intercept form and interpreting the slope and *y-*intercept. This activity could be used as a jigsaw group work activity or may be assigned for homework.

**Exit Slip 4.4.2** prompts students to solve a contextual problem with an equation in slope-intercept form.

1. To complete this investigation of slope-intercept form, students should discover properties of slopes of parallel and perpendicular lines. To place the idea of parallel lines in context, you might describe two walkers traveling at the same rate of speed but starting from different places. Ask the students if the two would ever meet. Have students graph several pairs of equations in slope-intercept form that are either parallel or perpendicular. You may give them tables of data that will result in parallel and perpendicular lines and have students plot the points and compare the equations to the graphs. See if students can then use the comparisons to explain how to algebraically determine whether lines are parallel or perpendicular given the equation or the slope of each line. For additional reinforcement, you may give students, working with a partner, a pair of points on each of two lines and observe how they decide to determine whether the lines are parallel or perpendicular. Have students share their results.

**Activity 4.4.6 Parallel and Perpendicular Lines** allows students to explore relationships between parallel and perpendicular lines and contains questions similar to the ones outlined above.

If you have students graph perpendicular lines on the calculator caution them that the lines will look perpendicular only if they use a square grid. Any window on the calculator may be made square by using the Zoom Square command (Zoom 5 ZSquare) in the Zoom menu.

Additional practice with parallel and perpendicular lines may be found in **Activity 4.4.7 More Parallel and Perpendicular Lines**.

Here are two additional activities which reinforce the main ideas from this investigation:

1. Students may complete the *Movement with Functions: Lesson 2, How Did I Move* activity sheet from *NCTM Illuminations* (or an equivalent activity). This kinetic activity requires groups of three students to go to a football field (or indoor space containing position markers) to implement specific movement scenarios. Each group of students has a stop watch and a set of index cards that specifies certain movement tasks. As one student performs a movement task, another student tracks their position, and a third student records the time. Following the activity, students use the data to graph functions describing each student’s movement.
2. Students may complete the *Equations of Attack* activity sheet from *NCTM Illuminations* (or an equivalent activity). The activity consists of students working in pairs to take turns finding lines (representing cannon shots) to connect from one point (their cannon) to another point (their opponent’s battleship). Once one of the students finds all the lines that properly connect their cannons to their opponents’ battleships, that student wins the competition. Each student places five battleships on lattice points within the first quadrant, and students are assigned five alternating positions on the positive *y*-axis. Since the cannon positions are on the *y*-axis, students must determine the necessary slope to connect a line from their cannon to their opponent’s battleship. To differentiate instruction, the cannon and battleship positions may be placed throughout all four quadrants within the coordinate plane, making the task of finding the line of attack more difficult.

**Differentiated Instruction (For Learners Needing More Help)**

Maintain students’ note cards and the class bulletin board. Provide directions on how to graph a function in slope-intercept form.

When doing a jigsaw puzzle style of group work, carefully assign problems that suit students’ capabilities.

**Differentiated Instruction (Enrichment)**

Challenge students to graph an equation such as $y=-^{5}/\_{3}x-8$ on a 10-by-10 coordinate plane centered at the origin. Students will likely interpret the slope as “down 5, right 3”, requiring them to “go off” the bottom of the graph unless they interpret the slope as “up 5, left 3”.

Give students equations with decimal numbers and non-integer intercepts to graph.

**Journal Prompt**

You may have heard that “two points determine a line”. What does this mean?

Based on your work with linear equations, what other two pieces of information determine a line? Explain.

**Resources and Materials**

* **Activity** **4.4.1** Effects of Changing Parameters
* **Activity 4.4.2** Slope-Intercept Form
* **Activity 4.4.3** Practice with Slope-Intercept Form
* **Activity 4.4.4** Making a Profit
* **Activity 4.4.5** Applications of Slope-Intercept Form
* **Activity 4.4.6** Parallel and Perpendicular Lines
* **Activity 4.4.7** More Parallel and Perpendicular Lines
* **Exit Slips 4.4.1** and **4.4.2**
* Straight Edges for drawing linear graphs
* Bulletin Board for key concepts
* Graphing Calculators