**Unit 4: Investigation 2 (4 Days)**

# RECOGNIZING LINEAR FUNCTIONS FROM WORDS, TABLES AND GRAPHS

***CCSS:*** F-IF6, F-LE1, F-LE1A

**Overview**

Students recognize linear functions in tabular and graphical forms and represent functions with verbal descriptions, equations, graphs, and tables. Students will develop methods for identifying the characteristics of linear functions and an understanding of rate of change and initial value in a real word context.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Distinguish between a linear and non-linear function from a table of values and from a graph.
* Transform a function from one representation to another.
* Identify a linear function’s constant average rate of change and *y*-intercept and interpret them in a non-contextual setting.
* Use an equation or a graph of a function that models a real world situation to produce a particular ordered pair and give an appropriate interpretation of its meaning in context.
* Choose appropriate increments and scales to construct tables and four-quadrant graphs and select the appropriate table set up and windows when using technology and use the trace feature to demonstrate the relationship between an ordered pair and a point on the graph.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit Slip 4.2** asks students to identify a function as linear or non-linear from a table and to explain their reasoning.
* **Journal Entry** asks students to compare arithmetic and geometric sequences with linear and non-linear functions.
* **Quiz on Investigations 4.1 and 4.2** assesses key ideas from the first two investigations.

**Launch Notes**

Begin the lesson by showing a variety of pizza menus and leading a short class discussion about what size pizza is the best buy. Now distribute **Activity 4.2.1 Pizza Problems**. This activity asks students to complete a table of values for the area of a pizza as a function of radius (non-linear), and for the circumference of a pizza as a function of radius (linear).  Ask students to specify the domains of the relations and decide if these are functions and if the functions are linear or non-linear.

**Closure Notes**

Be sure that students can use multiple representations of a function to distinguish linear from non-linear functions. They should be able to transform functions between multiple representations and use each representation to confirm whether or not a function is linear or not. Develop a deep understanding of the idea of constant average rate of change, the *y*-intercept, increasing and decreasing functions, and the meaning of a coordinate pair in context. Avoid providing students a formula until the ideas makes sense to them. The slope formula is introduced in Investigation 3 and the slope-intercept form of the line is introduced in Investigation 4.

**Teaching Strategies**

1. Following the launch, display two data tables of your choice, one a linear function and the other a non-linear function. Ask students to consider how they might identify linearity solely from a function’s data table. At first, students will probably note that a function is linear if the *x*-values and the *y*-values in a table each form an arithmetic sequence.

Next, present more data tables including at least one in which the increment between *x*-values is constant and not one (Δ*x* not necessarily equal to 1). You can use **Activity 4.2.2 Recognizing Linear Functions**. Allow students time to identify whether or not each table represents a linear relationship and discuss the rationale used.

Then ask students to describe how they could identify linearity if the data table has *x*-values with increments that are NOT constant. Allow students time to develop the method for finding the average rate of change between two data points. Ask probing questions so that you guide students to compare various changes in the *y*-values with the corresponding changes in the *x-*values, but save the definition and formula for slope for the next investigation. Students might calculate and compare average rates of changes among data tables that represent the special functions examined in Unit 3. This may require a review of equivalent fractions. Students will be able to articulate that data is linear if and only if the rate of change between any two points is the same.

Students should continue to investigate a variety of linear models (positive slope and positive *y*-intercept, negative slope and positive *y*-intercept, positive slope and negative *y*-intercept, negative slope and negative *y*-intercept). For each case, students construct a table of ordered pairs given a verbal description of the function, analyze how changes in one variable impact the other variable, plot the corresponding ordered pairs to form a line, and then analyze the line in the context of the real world situation. Students will observe that the graph of a line is increasing when the slope is positive and decreasing when the slope is negative.

Next, students will examine the relationship between a function’s equation, table and graph. Given equations, students will first construct tables by hand. Negative values of *x* will be included as inputs. The teacher will then demonstrate how to use the TABLE feature of the calculator, including the difference between “Ask” and “Auto”, and how to set up the table with the appropriate initial value (TblStart) and increment (ΔTbl). Students will use their calculators to check tables made by hand. Stress that when we compute average rates of change for linear functions (from a table or the graph of the function), the average rate of change never changes regardless of the two points used on the line. Mathematicians refer to this constant value as the slope of the line.

Then, students will use the tables to draw four-quadrant graphs by hand, using the standard window (-10 ≤ X ≤ 10; -10 ≤ Y ≤ 10) introduced in Unit 3. They will then use the calculator to display the same graphs in the same window (Zoom 6). Demonstrate how to adjust the window and ask students to experiment displaying the same function in different windows. Students will describe how the visual representation changes and discuss what makes a “good” window. Students will also learn to use Zoom Decimal and Zoom Integer to create “square” windows. They will notice that in a square window the line *y = x* makes a 45 degree angle with the positive *x*-axis. Students will also explore connections between tables and graphs using the vertical split screen (Mode G-T) and the TRACE feature of the calculator. The teacher may guide students to use windows that give “friendly” outputs, e.g. (-4.7 ≤ X ≤ 4.7; -10 ≤ Y ≤ 10) for Y1 = 2*x* – 5. Students will trace along the graph while reading corresponding values from the table.

For homework you may assign **Activity 4.2.3 Using Tables to Determine If a Function Is Linear**.

1. Students should continue engaging in activities that highlight the capability of linear functions to model a wide range of real world situations. They can do **Activity 4.2.4 Draining a Swimming Pool** and **Activity 4.2.5 Ordering DVD’s**. At least one of these might be assigned for homework. Students will match verbal descriptions, graphs and tables. Students will be able to identify (a) the constant average rate of change, (b) increasing/ decreasing functions, (c) positive and negative *y*-intercepts, (d) domain and range and (e) how these features of a linear function may be interpreted in the real world context of the given problem.

Note: When identifying *x*-intercepts often the table of values may not include the *x*-intercept or the *y*-intercept despite the fact that one is visible on the graph. Or a table of values might include the *x*-intercept or *y*-intercept but the portion of the line graphed may not. Students need to discuss the fact that just because a graph or table does not explicitly show the *x*- or *y*-intercept DOES NOT necessarily mean the function does not have one. However, there are some applications where the intercepts do not make sense, for example the relationship between the number of sides of a polygon and the sum of the interior angles in **Activity 4.2.6** below.

**Exit Slip 4.2** asks students to identify linear functions from tables.

1. In **Activity 4.2.6 Linear Functions in Geometry**, students explore patterns concerning the relationships of angles in polygons that are modeled with linear functions. The sum of the interior angles of a convex polygon is a function of the number of sides: *f(x) =* 180*x* – 360. To discover this, students can triangulate polygons on a separate worksheet, fill in a table, and make a graph. The fact that non-integral and non-positive values of *x* do not make sense in this context should be emphasized. With the screen (0 ≤ X ≤ 9.4; –500 ≤ Y ≤ 1500) students can use TRACE to find pairs where *x* (representing the number of sides) is an integer in the range 3 ≤ *x* ≤ 9. Students will be asked to make statements relating the graph to context such as, “The point (5, 540) on the graph indicates that if the polygon has 5 sides, then the sum of its interior angles is 540 degrees.” *Note:* Requisite prior knowledge: the sum of the interior angles of a triangle is 180 degrees.

In a right triangle the measure of one acute angle is a function of the measure of the other acute angle: *f(x) =* 90 – *x*. Here the variable *x* is continuous with domain: 0 < *x* < 90. An appropriate window is (0 ≤ X ≤ 94; 0 ≤ Y ≤ 100). Students will be asked to make statements such as “The point (42, 48) on the graph indicates that if one of the acute angles measures 42°, then the other one measures 48°.

For more practice, give students **Activity 4.2.7 Teddy Bear Sales**

At this point students should be ready for **Unit 4** **Investigations 1 and 2 Quiz**. You may want to select only some items from those provided depending upon how much time you want to give for this assessment.

**Differentiated Instruction (For Learners Needing More Help)**

Have students make note cards that they can use as reference while doing the exercises. For example one card will picture the graph of an increasing function with the word “increasing” and a bicycle driving up the hill of the graph. Another card entitled “*y*-intercept” can display the *y*-intercept on a graph, the coordinate pair (0,?) and a table with the *y*-intercept circled. A third card entitled “average rate of change” can show a graph, a table and the words . Have students identify which card to use as they answer each question in an exercise. Also put these key ideas on a bulletin board.

**Differentiated Instruction (Enrichment)**

Students can begin to invent their own real world situations using events they are involved in. They may invent their own linear scenarios based on fundraisers they are involved in, sports they are playing, or academic content they are studying in their other classes. Challenge students to create an exercise for their classmates to work on, or have them put a power point together analyzing a timely local event that they can show the class.

**Group Activity**

Several of the activities in this investigation, particularly **Activities 4.2.3** and **4.2.4** are well suited for group work. Use the jigsaw model of group work and have students complete problems in newly formed groups. Then have the students report back to their original group on the activity that they did or report to the class about their activity.

**Journal Prompt**

Compare the patterns of arithmetic and geometric sequences that you learned in Unit1 with linear and non-linear functions that you are learning in Unit 4.

**Resources and Materials**

* **Activity 4.2.1** Pizza Problems
* **Activity 4.2.2** Recognizing Linear Functions
* **Activity 4.2.3** Using Tables to Determine if a Function is Linear
* **Activity 4.2.4** Draining a Swimming Pool
* **Activity 4.2.5** Ordering DVD’s
* **Activity 4.2.6** Linear Functions in Geometry
* **Activity 4.2.7** Teddy Bear Sales
* **Exit Slip 4.2** Is it Linear?
* **Unit 4 Investigations 1 and 2 Quiz**
* Straight Edges for drawing linear graphs
* Bulletin Board for key concepts
* Student Journals
* Graphing Calculators