

Mathematics Instructional Cycle Guide

Geometry (6.G.A.1)

Created Cheryl R. Kerison, 2014 Connecticut Dream Team teacher

CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

Insert the cluster heading and Content Standard(s) here.

This Instructional Cycle Guide also relates to the following *Standards for Mathematical Practice* in the *CT Core Standards for Mathematics*:

Insert the relevant Standard(s) for Mathematical Practice here.

WHAT IS INCLUDED IN THIS DOCUMENT?

- A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings (page 2)
- > A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint (pages 3-6)
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed (pages 7-10)
- Supporting lesson materials (pages 11-18)
- Precursory research and review of standard 6.G.A.1 and assessment items that illustrate the standard (pages 19-21)

HOW TO USE THIS DOCUMENT

- 1) Before the lesson, administer the **Dividing the Playroom** *Mathematical Checkpoint* individually to students to elicit evidence of student understanding.
- 2) Analyze and interpret the student work using the Student Response Guide

3) Use the next steps or *follow-up lesson plan* to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint

4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

MATERIALS REQUIRED

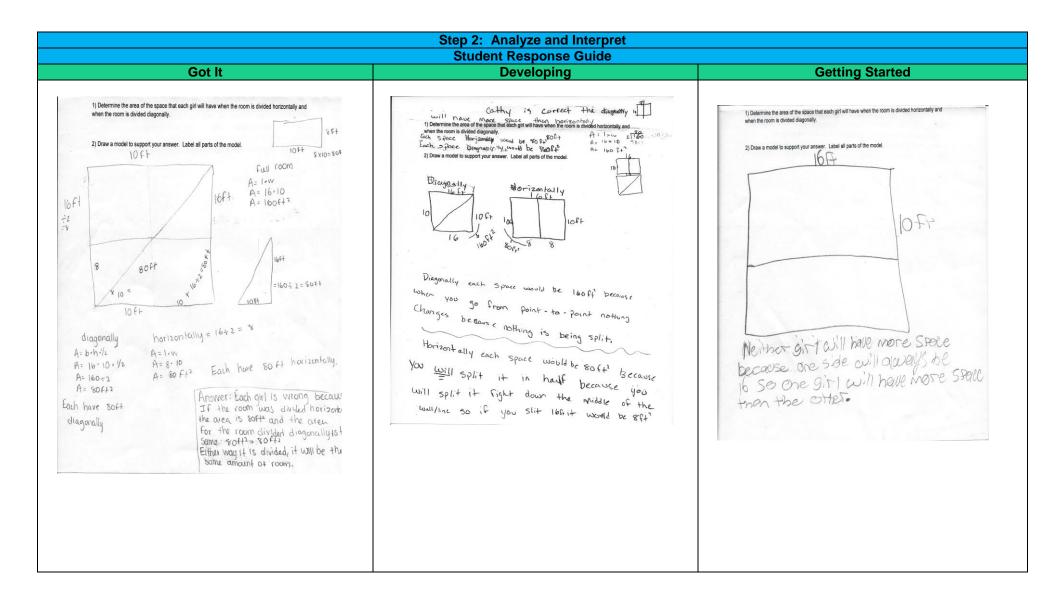
- Dry-erase board or chalkboard
- Chart paper to facilitate the sharing of student work and responses
- Projector
- Graph Paper (1 centimeter squares)
- Color Pencils
- Rulers
- Student Response sheets (included in this document)

TIME NEEDED

Dividing a Playroom administration: 15 minutes Follow-Up Lesson Plan: 1 to 2 instructional blocks

Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

	of student understanding	
Question(s)	Mathematical Checkpoint Purpose	
Kari and Cathy share a playroom that is 16 ft. long and 10 ft wide. They want to divide the playroom in half and rearrange their furniture. They thought of two ways that they could divide the room: horizontally or diagonally. Kari thinks that each girl will have more space if the playroom is divided horizontally, but Cathy thinks that each girl will have more space if the playroom is divided diagonally. Who is correct?	CT Core Standard:	Standard: <u>6.G.A.1</u> Find the area of right triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
 Determine the area of the space that each girl will have when the room is divided horizontally and when the room is divided diagonally. Draw a model to support your answer. Label all parts of the model. Explain your answer. 	Target question addressed by this checkpoint:	Does the student understand that the area of a decomposed composite figure is equal to the area of a composed composite figure?



Getting Started			
Student Response Example	Indicators		
1) Determine the sea of the space that each girl will have when the noon is divided horizontally and when the room is divided diagonally. 2) Drev a model to support your answer. Label all parts of the model. IDEA DOEA Neither girl outling have more space because one side will alwearys the to so one side will alwearys the to so one side will have more space than the othest.	 Student shows understanding of length and width of a rectangle. Students' response shows an understanding that the length of the figure does not change after being divided horizontally. Model does not show an understanding of how a rectangle can be decomposed into two triangles. Model does not show that a formula can be applied to calculate the area of a rectangle. Model does not show an understanding that the area of a rectangle or triangle is labeled in square units. Model does not show that the area of a composite figure is equal to the sum of the decomposed figures 		
In the Moment Questions/Prompts	Closing the Loop (Interventions/Extensions)		
 Q: How would you find the area of the entire space before you divided the space in half horizontally? Q: After dividing the space in half horizontally, what do you notice about the lengths of each of the smaller rectangles? What do you notice about the widths of each of the smaller rectangles? Q: How could you find the area of each of the two smaller rectangles that were formed from the larger figure? Q: Tell me about the two ways the girls thought of dividing the space. Q: After dividing the space in half diagonally, how many triangles to you see in the space? 	LZ video lesson links that may help develop conceptual understanding and procedural skill needed If no LZ video lessons address the error or misunderstanding, provide strategies or notes that could be useful in planning follow up action Use area models to find the area of rectangles <u>http://learnzillion.com/lessons/2374</u> Find the area of a rectangle using the standard formula <u>http://learnzillion.com/lessons/2535</u> Decompose a rectangle by using benchmark numbers <u>http://learnzillion.com/lessons/3720</u> Find the area of a figure by decomposing it <u>http://learnzillion.com/lessons/3746</u>		

D	eveloping
Student Response Example	Indicators
Cathy is correct the dragady if will never more affecting the thorizon balls and 1) Determine the area of the space will be stadt the space the power will be stadt 2) Draw a model to support your answer. Label all parts of the model. The space the power wave is the stadt 2) Draw a model to support your answer. Label all parts of the model. The space to be power in the stadt of the model. The space to be power in the stadt of the model. The space to be power in the stadt of the model. The space to be power in the stadt of the model. The space to be power in the stadt of the model. The space to be space would be is off because there you go from point to point nothing the space to be space would be is off because there you go from point to point nothing the space to be space would be soft because the space to be space would be soft because the space to be the space would be soft because the space to be space would be soft because you will split it in hadf because good will split it fight down the middle of the would be so if you slit iber it would be soft be sails	 Student model shows an understanding of decomposing a figure into more than one shape. Student response indicates an understanding of how to apply a formula calculate the area of a rectangle. Model does not show an understanding of how a formula can be applied to calculate the area of a triangle. Student response does not indicate that the area of a rectangle equals the sum of the area of two right triangles.
In the Moment Questions/Prompts	Closing the Loop (Interventions/Extensions)
 How could you find the area of each of the two triangles that were formed on the rectangle /Q: Find the sum of the areas of the two triangles. Compare that number to be area of the large rectangle before you divided it. What do you notice? /Q: Compare the area of one triangle to the area of one of the smaller ectangles. What do you notice? /Q: What conclusion can you make about the area of the large rectangle hen it is divided either horizontally or diagonally? 	Find the area of a triangle by composing into a rectangle <u>http://learnzillion.com/lessons/1883</u> Guided Practice for 'Find the area of a right triangle' <u>https://learnzillion.com/lessons/1883#video-preview-modal-z0pr329k1p</u>

	Got it
Student Response Example	Indicators
1) Determine the area of the space that each girl will have when the room is divided horizontally and when the room is divided diagonally. 2) Drew a model to support your answer. Label all parts of the model. 10 Ft 10 Ft	 Student model shows an understanding of decomposing a figure into rectangles and triangles. Student response indicates an understanding of how to apply a formula to calculate the area of a rectangle. Student model indicates an understanding of how to apply a formula to calculate the area of a triangle. Student model indicates an understanding that the area of a rectangle or a triangle is labeled in square units. Student model indicates an understanding that the sum of the area of the two triangles equals the area of the rectangle. Student model indicates an understanding that the sum of the area of the two smaller rectangles equals the area of the larger rectangle.
In the Moment Questions/Prompts	Closing the Loop (Interventions/Extensions)
 Tell me about your model. What other ways could you divide the large rectangle and show that its rea does not change? 	Find the area of polygons by decomposing into triangles, rectangles, parallelograms, and trapezoids. https://learnzillion.com/lessons/1061-find-the-area-of-polygons-by-decomposing-into-triangles-rectangles-parallelograms-and-trapezoids

Lesson Objective:		Student Work and Adjust Instruction
	• •	nposing into rectangles or decomposing into triangles and that this does not change the area of the shape.
Content Standard(s):	6.G.A.1 Find the area of right triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	
Targeted Practice Standard :	Do students apply form	els and symbols to represent the problem? nulas to solve the problem? y explain the models and symbolic representations to
Mathematical Goals		Success Criteria
be decomposed intoOnderstand that the account of the second se	area of a composite figure can triangles and other shapes. area of a composite figure is he decomposed figures.	 Draw a model to represent composing or decomposing a figure. Apply the formulas for finding the area of a rectangle and a triangle. Accurately explain the models and symbolic representations to describe solutions to problem solving tasks.
Launch (Probe and Build	Background Knowledge)	
shapes.		e that is divided into 4 smaller squares and another 2 x 2
,	· · ·	e screen and outline the outside borders in red.
2	2 units	2 units
		its

- How does the area of one square in the figure on the left compare to the area of one triangle in the figure on the right?
- How can you prove your answers to those questions?

Instructional Task

Purpose: Students will complete a problem-solving task to experience decomposing a composite shape into triangles and other shapes and to discover that the area of a composite figure is equal to the sum of the decomposed shapes.

Engage (Setting Up the Task)

1) Introduce the task (found on page 12) by projecting the following word problem:

Sandy wants to make a quilt with the dimensions of 6 feet by 6 feet. In addition, the quilt must include the following:

- The area of $\frac{1}{2}$ of the quilt must be blue and the area of $\frac{1}{2}$ of the quilt must be yellow.
 - The shapes in the quilt must be squares and triangles.

Create 2 different models for Sandy's quilt to show that the area of $\frac{1}{2}$ of it will be blue and the area of $\frac{1}{2}$ of it will be yellow.

Calculate the area of the entire quilt.

Calculate the area of each of the colors used in the quilt.

Color and label all parts of each model.

Explain your model and calculations.

2) Instruct students to tell a classmate what the task is asking them to do.

3) Discuss the questions students identified as having to answer in order to solve the quilt task.

4) Review the formulas for the area of a rectangle and the area of a triangle.

5) Distribute Student Response Sheets. Project an example of a sheet on a screen.

5) Explain to students that they will work on the quilt task with their group.

7) Explain to students that they may use any tools that are available in the classroom to complete the task. For example:

- Graph paper (one-inch, one-half inch, or one centimeter squares)
- Color pencils
- Rulers
- Color Tiles
- Tangram pieces

Explore (Solving the Task)

1) Provide time for the students to complete the following:

- Create 2 models of the quilt.
- Calculate the area of the entire quilt and each of the colors used in the quilt.
- Use the Student Response Sheet to organize their area calculations.

2) Possible questions/prompts to use as students engage in the task are:

Focusing Questions	Probing Questions	Advancing Questions
What is this problem asking?	What do the numbers used in the	Explain what you did to solve the
How could you start this problem?	problem represent?	problem.
What tools/manipulatives might help	What patterns do you see?	Compare your answer to another
you?	What connections do you see?	classmate's answer.
Which tool/manipulative would be	How do you know that your answer is	What do you notice about the sum of
best for this problem?	accurate?	the areas of each of the colors used
	What formula might apply to this	in the quilt compared to the area of
	problem?	the entire quilt?

Elaborate (Discuss Task and Related Mathematical Concepts)

1) After all of the teams have completed the task, ask students to post their responses in the room for all their classmates to see.

2) Pose the following questions for discussion:

- What do you notice about the models?
- What do you notice about the calculations of the area of the quilt?
- What do you notice about the calculations of the area of each of the colors of the quilt? When you add up the areas of each of the colors of the quilt, what do you notice?
- What conclusion(s) can you make about the area of a figure when it is decomposed into other shapes?

Checking for Understanding

Purpose: Pose the following questions to elicit evidence of students' understanding that composite figures can be composed into rectangles or decomposed into triangles and other shapes, and that the overall area of the composite figure does not change.

- Explain what this problem asking you to do?
- What do the numbers used in the problem represent?
- How can you visually represent the problem?
- Why did you decide to use this method of solving the problem?
- How can you organize the information needed to complete the task?
- What formula might apply in this situation to help you solve the problem?

Common Misunderstanding

Purpose: Address a common misunderstanding students often have about the multiple ways that figures can be composed and decomposed into rectangles, triangles and other shapes and that this does not change the area of the shape.

- What did you notice about the way the two squares were divided?
- What did you notice about the area of each square?
- Does the way the figures are divided change the total area for each figure?

Checking for Underst	anding			
•	ollowing questions to elig sum of the areas of the o		nderstanding th	at the area of a composite
P: Ask students to res	pond "True" or "False" to t	he following questions:		
 The sum of the (True) 	area of the different shap	ecompose it into different sha ses that compose a figure is t are are all the same size and	the same as the	original area of the figure.
P: For each statement	that is False, ask student	s to edit the wording to make	the statement T	rue.
P: Ask students to prov	vide an example from the l	esson activities to support th	eir responses to	all of the statements.
Closure				
Purpose: Provide stu		ty to self-assess their own g the students with a copy		
1) I can draw a model	to represent composing o	r decomposing a figure.		
Not at all 1	2	Sometimes 3	4	Absolutely 5
2) I can apply formulas	s for finding the area of a r	ectangle and a square		
Not at all 1	2	Sometimes 3	4	Absolutely 5
3) I can explain the mo	dels and how I calculated	the area composed and dec	omposed figures	
Not at all 1	2	Sometimes 3	4	Absolutely 5
Extension Task				
		-solving task by decomposir composite figure is equal to the		
Extend the Quilt proble	m-solving task by solving	the following problem:		
Madison wants to make	e a quilt with the dimensio	ns of 6 feet by 6 feet. Howev	ver, she decided	that her quilt must
include the following ch	naracteristics:			
		sed of a combination of any t ed shapes must equal to the		
 Calculate the a Calculate the s Color and labe Explain your m 	um of the areas of the pol I all parts of the model. Iodel and calculations.	the area of each of the polygy ygons used in the quilt. the area of a figure when it is		

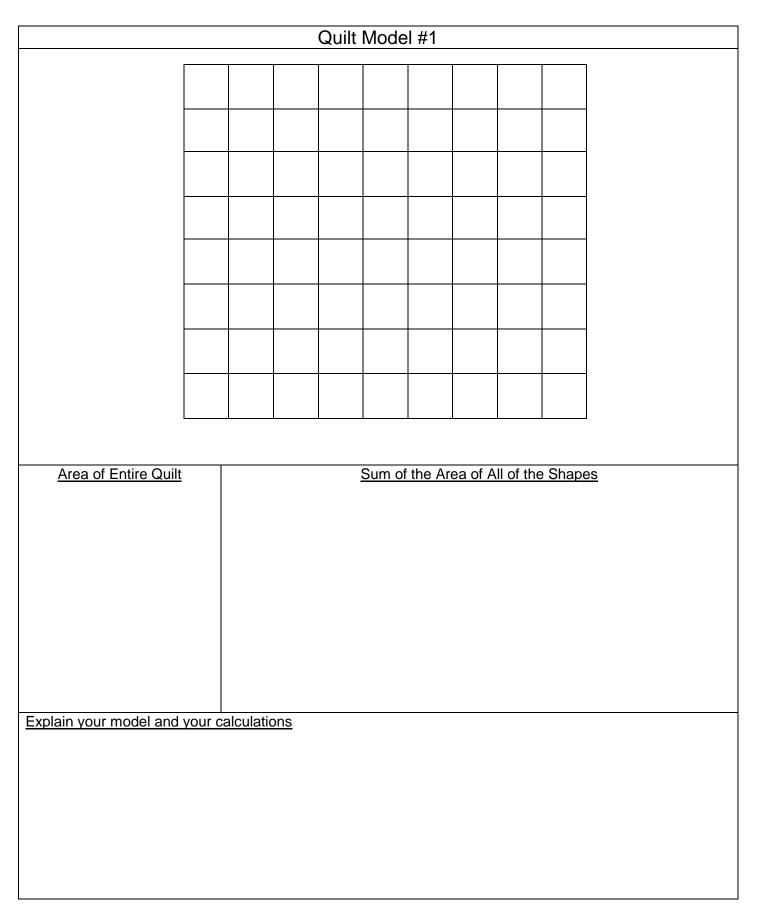
Quilt Task

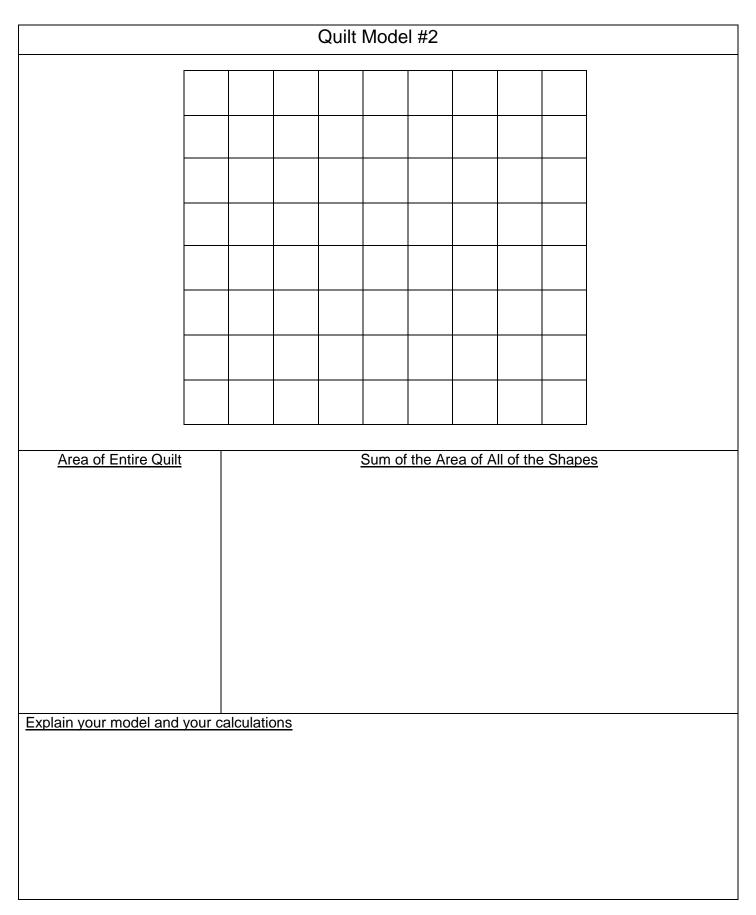
Sandy wants to make a quilt with the dimensions of 6 feet by 6 feet. In addition, the quilt must include the following:

- The area of ½ of the quilt must be blue and the area of ½ of the quilt must be yellow.
- The shapes in the quilt must be squares and triangles.

Your Task:

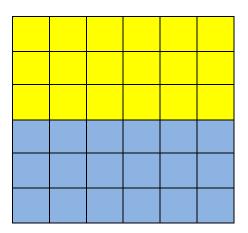
- Create 2 different models for Sandy's quilt to show that the area of ½ of it will be blue and the area of ½ of it will be yellow.
- Calculate the area of the entire quilt.
- Calculate the sum of the area of all of the shapes used in the quilt.
- Color and label all parts of each model.
- Explain your model and calculations.

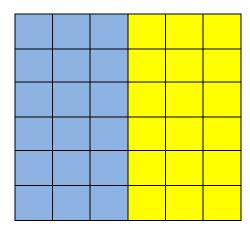


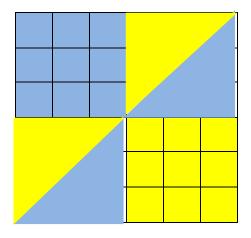


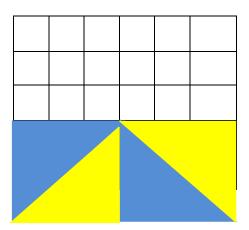
Quilt Task:

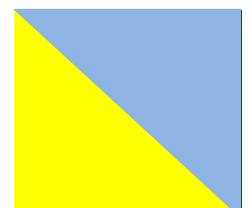
Possible Solution Paths for Creating the Models



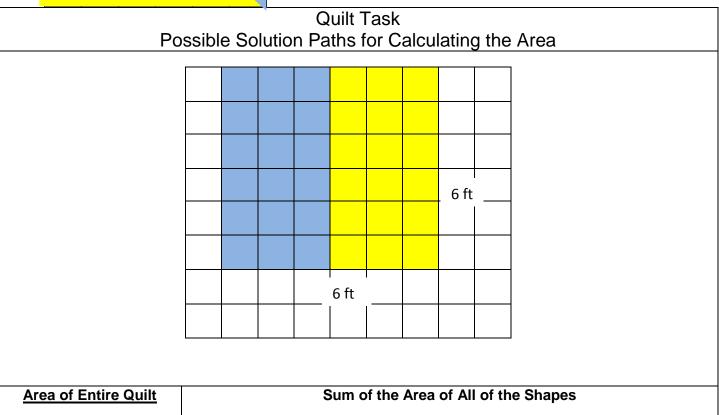








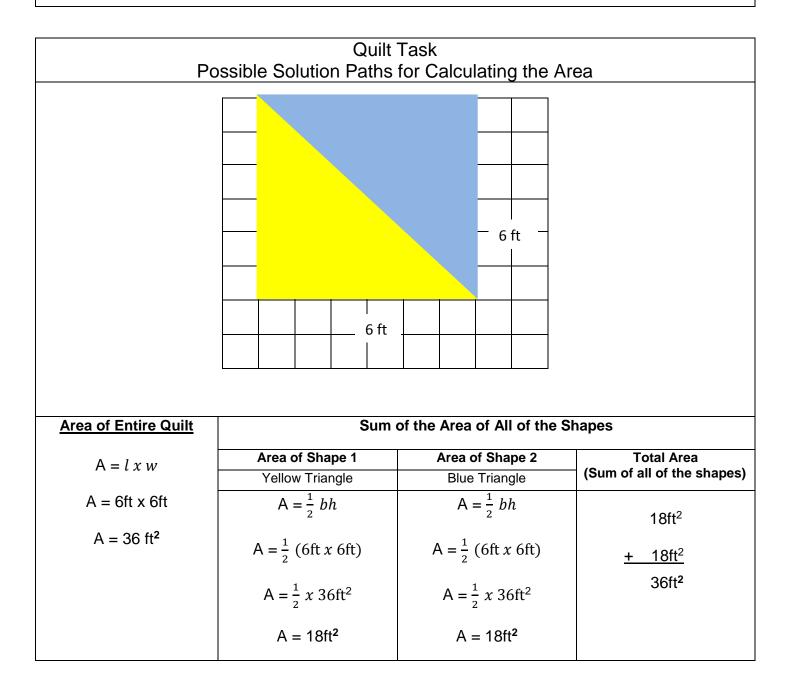




A = l x w	Area of Shape 1 Blue Rectangle	Area of Shape 2 Yellow Rectangle	Total Area (Sum of all of the shapes)
$A = 6ft \times 6ft$	$A = l \ x \ w$	A = l x w	18ft ²
$A = 36ft^2$	$A = 3ft \times 6ft$	A = 3ft x 6ft	<u>+ 18ft²</u>
	$A = 18ft^2$	$A = 18ft^2$	36ft ²

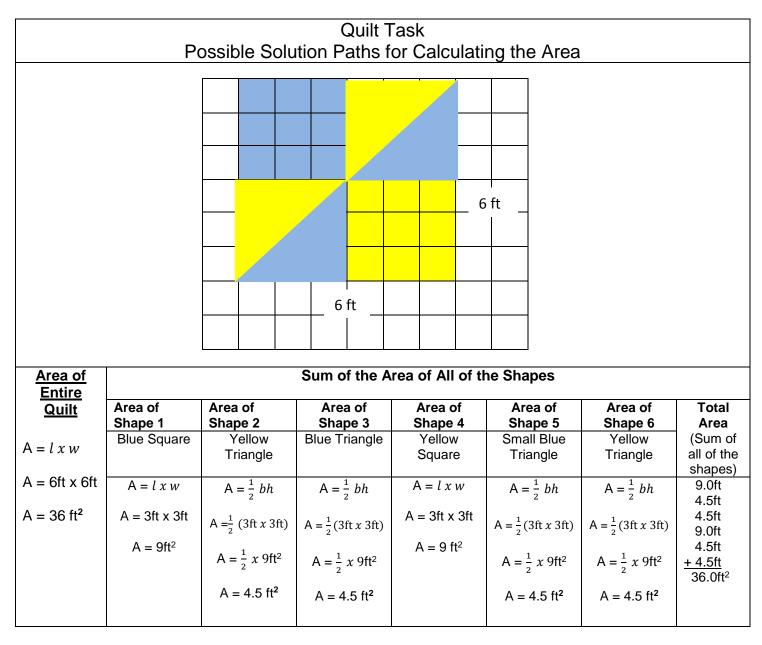
Explain your model and your calculations.

- Students should explain that ½ of the model is blue and ½ is yellow. They should explain that 18 units on the grid are blue and 18 are yellow for a total of 36 units.
- Students should explain that their calculations for finding the area of the entire quilt support that ½ of the quilt is blue and ½ of the quilt is yellow.
- Students should explain that the sum of the blue and yellow areas on the quilt is equal to the area of the entire quilt. Students should explain that this was demonstrated by their model and calculations.



Explain your model and your calculations.

- Students should explain that ½ of the model is blue and ½ is yellow. They should explain that the model shows that each triangle has an area of 18 units for a total of 36 units.
- Students should explain that their calculations for finding the area of the entire quilt support that ½ of the quilt is blue and ½ of the quilt is yellow.
- Students should explain that the sum of the blue and yellow areas on the quilt is equal to the area of the entire quilt. Students should explain that this was demonstrated by their model and calculations.



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Explain your model and your calculations

- Students should explain that ½ of the model is blue and ½ is yellow. For example, the model shows that the area of each of the two yellow triangles is 4.5units². Students should explain that both triangles will make a square that has an area of 9 square units. In addition, the yellow square has an area of 9 square units. The two yellow triangles and the yellow square equal to ½ of the quilt. The students should explain the blue areas of the quilt using similar reasoning.
- Students should explain that their calculations for finding the area of the entire quilt support that ½ of the quilt is blue and ½ of the quilt is yellow. Students should explain that all of the shapes compose a quilt of 36 square feet.
- Students should explain that the sum of the blue and yellow areas on the quilt is equal to the area of the entire quilt. Students should support their responses with specific examples from their models and calculations.

Student Self-Assessment

1) I can draw a model to represent composing or decomposing a figure.

Not at all		Sometimes		Absolutely
1	2	3	4	5
For example, ir	the lesson I			
2) I can apply f	formulas for finding the	e area of a rectangle and a squa	res.	
Not at all 1	2	Sometimes 3	4	Absolutely 5
For example, ir	the lesson I			

3) I can explain the models and how I calculated the area composed and decomposed figures.

Not at all		Sometimes		Absolutely	
1	2	3	4	5	
For example, in	the lesson I				

Research	n and review of standard
Content Standard(s):	Standard(s) for Mathematical Practice:
Standard: <u>6.G.A.1</u> Find the area of right triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	MP. 4 Model with mathematics. Mathematically proficient students: apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
Smarter Balanced Claim	Smarter Balanced Item
	631
Claim #1 – Concepts & Procedures "Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency."	The trapezoid shown is divided into a right triangle and a rectangle.
Claim #4 – Modeling & Data Analysis "Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems."	Use the Equation Tool to create an expression that could be used to determine the area of the trapezoid. $\begin{array}{c} \bullet \bullet$
CPR Pre-Requisites (Conceptual Understanding, Procedural	Conceptual Understanding and Knowledge
Skills, and Representations)	Understand base, height and area
	 Understand that two right triangles can also be formed from a rectangle by dividing the rectangle into 2 with a diagonal cut
	 Understand that the area of a right triangle with base b and height h must be half that of a rectangle or ½ * b * h.
	Understand the area formula for a rectangle
	Relate area to multiplication and addition
	Procedural Skills

Apply the formula for the area of a rectangle		
• Apply the formula for the area of a right triangle		
 Find the area of composite figures by decomposing into right triangles, rectangles and other shapes. 		
Representational		
 Write, read and evaluate expressions in which letters stand for numbers (Math.6.EE.2) 		
 Evaluate expressions that arise from formulas used in real-world problems. (Math 6.EE.A.2b) 		
Social knowledge		
Area		
 Composite figures – Composite figures are made up of two or more geometric shapes such as rectangles and triangles. 		
 Compose; Composing – Compose means to make something by putting things together 		
 Decompose; Decomposing – Decompose means to take things apart or to look for shapes within a shape. 		
Polygon		

Standards Progression			
Grade(s) below	Target grade	Grade(s) above	
4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.	6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and	7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle	
	mathematical problems. 6.G.4 Represent three- dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of	7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.	

solving real-world and mathematical problems.	
6.EE.A.2 Write, read and evaluate expressions in which letters stand for numbers.	
6.EE.2.C Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional	
order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area or a cube with sides of length s = $\frac{1}{2}$.	

Common Misconceptions/Roadblocks

What characteristics of this problem may confuse students?

- Students confuse the concepts of area with perimeter.
- Students may be confused that the labels for the sides of a rectangle are length and width, whereas the labels for a triangle are base and height.
- Students are unable to identify the appropriate formulas to use for calculating the area of a rectangle and the area of a right triangle.
- Students do not add the areas of the separate components of the polygon to find the area of the entire polygon.
- Students may not understand that, after decomposing a polygon, they may be required to use different formulas to find the area of each figure within the polygon.

What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?

- Students do not understand how to apply number values to a formula for the area of a rectangle.
- Students do not understand how to apply number values to a formula for the area of a right triangle.
- Students do not understand the meaning of the word compose as it applies to geometry.
- Students do not understand the meaning of the word decompose as it applies to geometry.

- Students experience difficulty with dividing by 1/2.
- Students do not understand that area is measured in square units.
- Students do not understand how to apply number values to the formula for the area of a trapezoid.

What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?

• Students may believe that all composite shapes must be divided the same way.