

# Mathematics Instructional Cycle Guide 

Geometry (6.G.A.1)

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## CT CORE STANDARDS

This Instructional Cycle Guide relates to the following Standards for Mathematical Content in the CT Core Standards for Mathematics:

Insert the cluster heading and Content Standard(s) here.
This Instructional Cycle Guide also relates to the following Standards for Mathematical Practice in the CT Core Standards for Mathematics:

## Insert the relevant Standard(s) for Mathematical Practice here.

## WHAT IS INCLUDED IN THIS DOCUMENT?

> A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings (page 2)
> A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint (pages 3-6)
> A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed (pages 7-10)
> Supporting lesson materials (pages 11-18)
> Precursory research and review of standard 6.G.A. 1 and assessment items that illustrate the standard (pages 1921)

## HOW TO USE THIS DOCUMENT

1) Before the lesson, administer the Dividing the Playroom Mathematical Checkpoint individually to students to elicit evidence of student understanding.
2) Analyze and interpret the student work using the Student Response Guide
3) Use the next steps or follow-up lesson plan to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint
4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

## MATERIALS REQUIRED

- Dry-erase board or chalkboard
- Chart paper to facilitate the sharing of student work and responses
- Projector
- Graph Paper ( 1 centimeter squares)
- Color Pencils
- Rulers
- Student Response sheets (included in this document)


## TIME NEEDED

Dividing a Playroom administration: 15 minutes
Follow-Up Lesson Plan: 1 to 2 instructional blocks
Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

| Step 1: Elicit evidence of student understanding |  |  |
| :---: | :---: | :---: |
| Mathematical Checkpoint |  |  |
| Question(s) |  | Purpose |
| Kari and Cathy share a playroom that is 16 ft . long and 10 ft wide. They want to divide the playroom in half and rearrange their furniture. They thought of two ways that they could divide the room: horizontally or diagonally. Kari thinks that each girl will have more space if the playroom is divided horizontally, but Cathy thinks that each girl will have more space if the playroom is divided diagonally. Who is correct? | CT Core Standard: | Standard: 6.G.A. 1 Find the area of right triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. |
| 1) Determine the area of the space that each girl will have when the room is divided horizontally and when the room is divided diagonally. <br> 2) Draw a model to support your answer. Label all parts of the model. <br> 3) Explain your answer. | Target question addressed by this checkpoint: | Does the student understand that the area of a decomposed composite figure is equal to the area of a composed composite figure? |


| Step 2: Analyze and Interpret |  |  |
| :---: | :---: | :---: |
| Student Response Guide |  |  |
| Got It | Developing | Getting Started |
| 1) Determine the area of the space that each girl will have when the room is divided horizontally and when the room is divided diagonally. <br> 2) Draw a model to support your answer. Label all parts of the model. 10 ft <br> full room <br> $A=1 . \mathrm{w}$ <br> $A=16 \cdot 10$ <br> $A=160 \mathrm{ft}^{2}$ <br> diagonally $\text { horizontally }=16 \div 2=8$ <br> $A=b \cdot h \cdot 1 / 2$ <br> $A=1, w$ <br> $A=16 \cdot 10 \cdot \frac{1}{2}$ <br> $A=8 \cdot 10$ <br> Each have so ft horizentally. <br> $A=160 \div 2$ <br> $A=80 \mathrm{ft} 2$ <br> Each have soft diagonally <br> Answer: Each girl is wrong becaus If the room was diviled horizates the area is $80 \mathrm{ft}^{2}$ and the aren For the room divjded diagenallyist same. $80+t^{2}=80 t+2$ <br> Either way it is divided, it will be the same amount of room. | Cathy is correct the diageanyy IIT] <br>  1) Determine the area of the spoom is divided diagonally. <br> Each Space Horizomily woud be ofo $\mathrm{ft}{ }^{80 \mathrm{Ct}}$ 2) Draw a model to support your answer. Label all parts of the model <br>  $A=160 \mathrm{Ct}$ <br> Diagmally each spacee would be 160 ft because When you go from point-to-point nothing Changes berause nothing is being split. <br> Horizontally each space would be soft because You will split it in half because you will split it right down the middle of the walline so if you slit lbfit wowld be 8ft' | 1) Determine the area of the space that each giri will have when the room is divided horizontally and when the room is divided diagonally. <br> Neither girl will have more spece because one side will qwelys be 16 so one girl will halle mose spall than the other. |


| Getting Started |  |
| :---: | :---: |
| Student Response Example | Indicators |
| 1) Determine the area of the space that each girl will have when the room is fivided horizontally and <br> Ne ither girt will have more spece because one side will always he if so one gitl will have mose spalc than the olter. | - Student shows understanding of length and width of a rectangle. <br> - Students' response shows an understanding that the length of the figure does not change after being divided horizontally. <br> - Model does not show an understanding of how a rectangle can be decomposed into two triangles. <br> - Model does not show that a formula can be applied to calculate the area of a rectangle. <br> - Model does not show an understanding that the area of a rectangle or triangle is labeled in square units. <br> - Model does not show that the area of a composite figure is equal to the sum of the decomposed figures |
| In the Moment Questions/Prompts | Closing the Loop (Interventions/Extensions) |
| Q: How would you find the area of the entire space before you divided the space in half horizontally? <br> Q: After dividing the space in half horizontally, what do you notice about the lengths of each of the smaller rectangles? What do you notice about the widths of each of the smaller rectangles? <br> Q: How could you find the area of each of the two smaller rectangles that were formed from the larger figure? <br> Q: Tell me about the two ways the girls thought of dividing the space. <br> Q: After dividing the space in half diagonally, how many triangles to you see in the space? | LZ video lesson links that may help develop conceptual understanding and procedural skill needed <br> If no LZ video lessons address the error or misunderstanding, provide strategies or notes that could be useful in planning follow up action <br> Use area models to find the area of rectangles http://learnzillion.com/lessons/2374 <br> Find the area of a rectangle using the standard formula http://learnzillion.com/lessons/2535 <br> Decompose a rectangle by using benchmark numbers http://learnzillion.com/lessons/3720 <br> Find the area of a figure by decomposing it http://learnzillion.com/lessons/3746 |


| Developing |  |
| :---: | :---: |
| Student Response Example | Indicators |
| Cathy is currect the diaserany uIt <br>  when the room is divided diagonally. fach S Qace Harjzandaly $\qquad$ 2) Draw a model to support your answer. Label all parts of the model. <br> Diagomally each space would be 160 ff because Changes you go from point-to-point notwing Changes becourse nothing is being split. <br> thrizontally each space would be soft because You will split it in half because you will split it right down the midde of the watline so if you slit lofit would be $8 f t^{\prime}$ walline so if you slit 16 fit wound fitt | - Student model shows an understanding of decomposing a figure into more than one shape. <br> - Student response indicates an understanding of how to apply a formula to calculate the area of a rectangle. <br> - Model does not show an understanding of how a formula can be applied to calculate the area of a triangle. <br> - Student response does not indicate that the area of a rectangle equals the sum of the area of two right triangles. |
| In the Moment Questions/Prompts | Closing the Loop (Interventions/Extensions) |
| Q: How could you find the area of each of the two triangles that were formed from the rectangle <br> $\mathbf{P} / \mathbf{Q}$ : Find the sum of the areas of the two triangles. Compare that number to the area of the large rectangle before you divided it. What do you notice? <br> P/Q: Compare the area of one triangle to the area of one of the smaller rectangles. What do you notice? <br> P/Q: What conclusion can you make about the area of the large rectangle when it is divided either horizontally or diagonally? | Find the area of a triangle by composing into a rectangle http://learnzillion.com/lessons/1883 <br> Guided Practice for 'Find the area of a right triangle' https://learnzillion.com/lessons/1883\#video-preview-modal-zOpr329k1p |


| Got it |  |
| :---: | :---: |
| Student Response Example | Indicators |
|  | - Student model shows an understanding of decomposing a figure into rectangles and triangles. <br> - Student response indicates an understanding of how to apply a formula to calculate the area of a rectangle. <br> - Student model indicates an understanding of how to apply a formula to calculate the area of a triangle. <br> - Student model indicates an understanding that the area of a rectangle or a triangle is labeled in square units. <br> - Student model indicates an understanding that the sum of the area of the two triangles equals the area of the rectangle. <br> - Student model indicates an understanding that the sum of the area of the two smaller rectangles equals the area of the larger rectangle. |
| In the Moment Questions/Prompts | Closing the Loop (Interventions/Extensions) |
| P: Tell me about your model. <br> Q: What other ways could you divide the large rectangle and show that its area does not change? | Find the area of polygons by decomposing into triangles, rectangles, parallelograms, and trapezoids. <br> https://learnzillion.com/lessons/1061-find-the-area-of-polygons-by-decomposing-into-triangles-rectangles-parallelograms-and-trapezoids |



## Instructional Task

Purpose: Students will complete a problem-solving task to experience decomposing a composite shape into triangles and other shapes and to discover that the area of a composite figure is equal to the sum of the decomposed shapes.

## Engage (Setting Up the Task)

1) Introduce the task (found on page 12) by projecting the following word problem:

Sandy wants to make a quilt with the dimensions of 6 feet by 6 feet. In addition, the quilt must include the following:

- The area of $1 / 2$ of the quilt must be blue and the area of $1 / 2$ of the quilt must be yellow.
- The shapes in the quilt must be squares and triangles.

Create 2 different models for Sandy's quilt to show that the area of $1 / 2$ of it will be blue and the area of $1 / 2$ of it will be yellow.

Calculate the area of the entire quilt.
Calculate the area of each of the colors used in the quilt.
Color and label all parts of each model.
Explain your model and calculations.
2) Instruct students to tell a classmate what the task is asking them to do.
3) Discuss the questions students identified as having to answer in order to solve the quilt task.
4) Review the formulas for the area of a rectangle and the area of a triangle.
5) Distribute Student Response Sheets. Project an example of a sheet on a screen.
5) Explain to students that they will work on the quilt task with their group.
7) Explain to students that they may use any tools that are available in the classroom to complete the task. For example:

- Graph paper (one-inch, one-half inch, or one centimeter squares)
- Color pencils
- Rulers
- Color Tiles
- Tangram pieces


## Explore (Solving the Task)

1) Provide time for the students to complete the following:

- Create 2 models of the quilt.
- Calculate the area of the entire quilt and each of the colors used in the quilt.
- Use the Student Response Sheet to organize their area calculations.

2) Possible questions/prompts to use as students engage in the task are:

| Focusing Questions | Probing Questions | Advancing Questions |
| :--- | :--- | :--- |
| What is this problem asking? | What do the numbers used in the | Explain what you did to solve the |
| How could you start this problem? | problem represent? | problem. |
| What tools/manipulatives might help | What patterns do you see? | Compare your answer to another |
| you? | What connections do you see? | classmate's answer. |
| Which tool/manipulative would be | How do you know that your answer is | What do you notice about the sum of |
| best for this problem? | accurate? | the areas of each of the colors used |
|  | What formula might apply to this | in the quilt compared to the area of <br> problem? |
|  |  |  |
|  |  |  |

## Elaborate (Discuss Task and Related Mathematical Concepts)

1) After all of the teams have completed the task, ask students to post their responses in the room for all their classmates to see.
2) Pose the following questions for discussion:

- What do you notice about the models?
- What do you notice about the calculations of the area of the quilt?
- What do you notice about the calculations of the area of each of the colors of the quilt? When you add up the areas of each of the colors of the quilt, what do you notice?
- What conclusion(s) can you make about the area of a figure when it is decomposed into other shapes?


## Checking for Understanding

Purpose: Pose the following questions to elicit evidence of students' understanding that composite figures can be composed into rectangles or decomposed into triangles and other shapes, and that the overall area of the composite figure does not change.

- Explain what this problem asking you to do?
- What do the numbers used in the problem represent?
- How can you visually represent the problem?
- Why did you decide to use this method of solving the problem?
- How can you organize the information needed to complete the task?
- What formula might apply in this situation to help you solve the problem?


## Common Misunderstanding

Purpose: Address a common misunderstanding students often have about the multiple ways that figures can be composed and decomposed into rectangles, triangles and other shapes and that this does not change the area of the shape.

- What did you notice about the way the two squares were divided?
- What did you notice about the area of each square?
- Does the way the figures are divided change the total area for each figure?


## Checking for Understanding

Purpose: Pose the following questions to elicit evidence of students' understanding that the area of a composite figure is equal to the sum of the areas of the decomposed figures.

P: Ask students to respond "True" or "False" to the following questions:

- The area of a figure changes when we decompose it into different shapes. (False)
- The sum of the area of the different shapes that compose a figure is the same as the original area of the figure. (True)
- The different shapes that compose a figure are all the same size and shape. (False)

P: For each statement that is False, ask students to edit the wording to make the statement True.
P: Ask students to provide an example from the lesson activities to support their responses to all of the statements.

## Closure

Purpose: Provide students with an opportunity to self-assess their own learning related to the Success Criteria by projecting the questions below or providing the students with a copy of self-assessment sheets to complete.

1) I can draw a model to represent composing or decomposing a figure.
Not at all
12
Sometimes
3
4
Absolutely
5
2) I can apply formulas for finding the area of a rectangle and a square

| Not at all | Sometimes |  | Absolutely |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 |

3) I can explain the models and how I calculated the area composed and decomposed figures.

| Not at all | Sometimes | Absolutely |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 |

## Extension Task

Purpose: Students will extend the Quilt problem-solving task by decomposing a composite shape into two-dimensional geometric shapes to discover that the area of a composite figure is equal to the sum of the decomposed shapes.

Extend the Quilt problem-solving task by solving the following problem:
Madison wants to make a quilt with the dimensions of 6 feet by 6 feet. However, she decided that her quilt must include the following characteristics:

- The shapes in the quilt may be composed of a combination of any two-dimensional regular polygon.
- The sum of the areas of the decomposed shapes must equal to the area of the $6 \mathrm{ft} \times 6 \mathrm{ft}$ quilt.
- Create a model for Madison's quilt.
- Calculate the area of the entire quilt and the area of each of the polygons used in the quilt.
- Calculate the sum of the areas of the polygons used in the quilt.
- Color and label all parts of the model.
- Explain your model and calculations.
- What conclusion(s) can you make about the area of a figure when it is decomposed into other shapes?


## Quilt Task

Sandy wants to make a quilt with the dimensions of 6 feet by 6 feet. In addition, the quilt must include the following:

- The area of $1 / 2$ of the quilt must be blue and the area of $1 / 2$ of the quilt must be yellow.
- The shapes in the quilt must be squares and triangles.


## Your Task:

- Create 2 different models for Sandy's quilt to show that the area of $1 / 2$ of it will be blue and the area of $1 / 2$ of it will be yellow.
- Calculate the area of the entire quilt.
- Calculate the sum of the area of all of the shapes used in the quilt.
- Color and label all parts of each model.
- Explain your model and calculations.




## Quilt Task:

Possible Solution Paths for Creating the Models





## Explain your model and your calculations.

- Students should explain that $1 / 2$ of the model is blue and $1 / 2$ is yellow. They should explain that 18 units on the grid are blue and 18 are yellow for a total of 36 units.
- Students should explain that their calculations for finding the area of the entire quilt support that $1 / 2$ of the quilt is blue and $1 / 2$ of the quilt is yellow.
- Students should explain that the sum of the blue and yellow areas on the quilt is equal to the area of the entire quilt. Students should explain that this was demonstrated by their model and calculations.



## Explain your model and your calculations.

- Students should explain that $1 / 2$ of the model is blue and $1 / 2$ is yellow. They should explain that the model shows that each triangle has an area of 18 units for a total of 36 units.
- Students should explain that their calculations for finding the area of the entire quilt support that $1 / 2$ of the quilt is blue and $1 / 2$ of the quilt is yellow.
- Students should explain that the sum of the blue and yellow areas on the quilt is equal to the area of the entire quilt. Students should explain that this was demonstrated by their model and calculations.


[^0]
## Explain your model and your calculations

- Students should explain that $1 / 2$ of the model is blue and $1 / 2$ is yellow. For example, the model shows that the area of each of the two yellow triangles is 4.5 units $^{2}$. Students should explain that both triangles will make a square that has an area of 9 square units. In addition, the yellow square has an area of 9 square units. The two yellow triangles and the yellow square equal to $1 / 2$ of the quilt. The students should explain the blue areas of the quilt using similar reasoning.
- Students should explain that their calculations for finding the area of the entire quilt support that $1 / 2$ of the quilt is blue and $1 / 2$ of the quilt is yellow. Students should explain that all of the shapes compose a quilt of 36 square feet.
- Students should explain that the sum of the blue and yellow areas on the quilt is equal to the area of the entire quilt. Students should support their responses with specific examples from their models and calculations.


## Student Self-Assessment

1) I can draw a model to represent composing or decomposing a figure.

Not at all
1

Sometimes
3

4
Absolutely
5

For example, in the lesson I $\qquad$
$\qquad$
$\qquad$ .
2) I can apply formulas for finding the area of a rectangle and a squares.

| Not at all | Sometimes | Absolutely |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 |

For example, in the lesson I $\qquad$
$\qquad$
$\qquad$ .
3) I can explain the models and how I calculated the area composed and decomposed figures.

| Not at all | Sometimes |  | Absolutely |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 |

For example, in the lesson I $\qquad$
$\qquad$
$\qquad$ .



| Standards Progression |  |  |  |
| :--- | :--- | :--- | :---: |
| Grade(s) below | Target grade | Grade(s) above |  |
| 4.MD.3 Apply the area and perimeter <br> formulas for rectangles in real world <br> and mathematical problems. For <br> example, find the width of a rectangular <br> room given the area of the flooring and <br> the length, by viewing the area formula <br> as a multiplication equation with an <br> unknown factor. | 6.G.3 Draw polygons in the <br> coordinate plane given <br> coordinates for the vertices; <br> use coordinates to find the <br> length of a side joining points <br> with the same first coordinate <br> or the same second <br> coordinate. Apply these <br> techniques in the context of <br> solving real-world and <br> mathematical problems. | 7.G.4 Know the formulas <br> for the area and <br> circumference of a circle <br> and use them to solve <br> problems; give an informal <br> derivation of the <br> relationship between the <br> circumference and area of <br> a circle |  |
|  | 6.G.4 Represent three- <br> dimensional figures using nets <br> made up of rectangles and <br> triangles, and use the nets to <br> find the surface area of these <br> figures. Apply these <br> techniques in the context of | 7.G.6 Solve real-world and <br> mathematical problems <br> involving area, volume and <br> surface area of two and <br> three-dimensional objects <br> composed of triangles, <br> quadrilaterals, polygons, <br> cubes and right prisms. |  |



What characteristics of this problem may confuse students?

- Students confuse the concepts of area with perimeter.
- Students may be confused that the labels for the sides of a rectangle are length and width, whereas the labels for a triangle are base and height.
- Students are unable to identify the appropriate formulas to use for calculating the area of a rectangle and the area of a right triangle.
- Students do not add the areas of the separate components of the polygon to find the area of the entire polygon.
- Students may not understand that, after decomposing a polygon, they may be required to use different formulas to find the area of each figure within the polygon.

What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?

- Students do not understand how to apply number values to a formula for the area of a rectangle.
- Students do not understand how to apply number values to a formula for the area of a right triangle.
- Students do not understand the meaning of the word compose as it applies to geometry.
- Students do not understand the meaning of the word decompose as it applies to geometry.
- Students experience difficulty with dividing by $1 / 2$.
- Students do not understand that area is measured in square units.
- Students do not understand how to apply number values to the formula for the area of a trapezoid.

What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?

- Students may believe that all composite shapes must be divided the same way.


[^0]:    Connecticut State Department of Education

