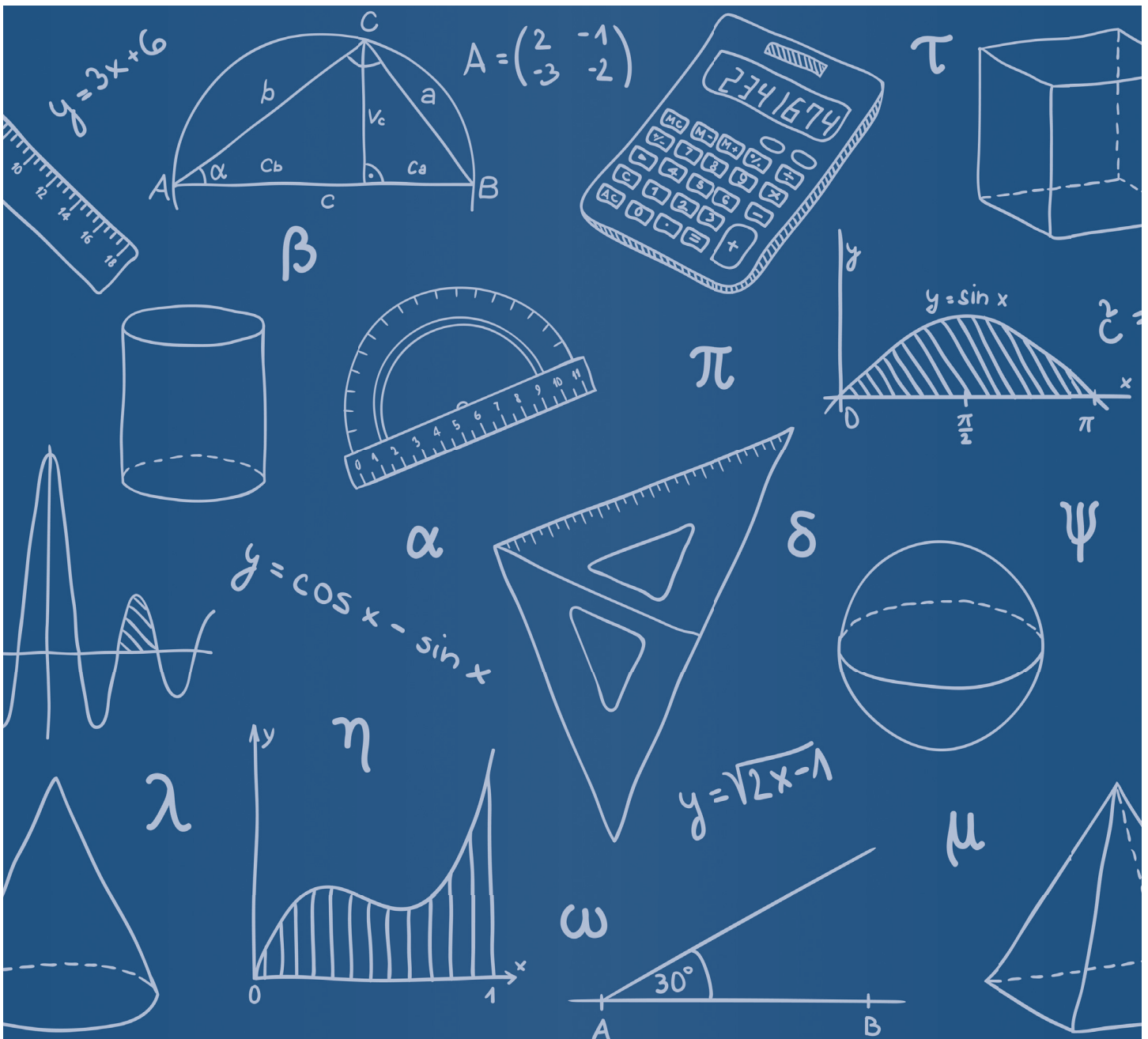


# LESSONS FOR LEARNING

FOR THE COMMON CORE STATE STANDARDS IN MATHEMATICS



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# Eighth Grade – Standards

- 1. Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations** – Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ( $y/x = m$  or  $y = mx$ ) as special linear equations ( $y = mx + b$ ), understanding that the constant of proportionality ( $m$ ) is the slope, and the graphs are lines through the origin. They understand that the slope ( $m$ ) of a line is a constant rate of change, so that if the input or  $x$ -coordinate changes by an amount  $A$ , the output or  $y$ -coordinate changes by the amount  $m \cdot A$ . Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and  $y$ -intercept) in terms of the situation. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.
- 2. Grasping the concept of a function and using functions to describe quantitative relationships** – Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another.

They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

- 3. Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem** – Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

## MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## THE NUMBER SYSTEM

**Know that there are numbers that are not rational, and approximate them by rational numbers.**

- 8.NS.1** Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat. Know that other numbers are called irrational.
- 8.NS.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g.,  $\pi^2$ ). For example, by truncating the decimal expansion of  $\sqrt{2}$ , show that  $\sqrt{2}$  is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

## EXPRESSIONS AND EQUATIONS

**Work with radicals and integer exponents.**

- 8.EE.1** Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example,  $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$ .
- 8.EE.2** Use square root and cube root symbols to represent solutions to equations of the form  $x^2 = p$  and  $x^3 = p$ , where  $p$  is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that  $\sqrt{2}$  is irrational.
- 8.EE.3** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as  $3 \times 10^8$  and the population of the world as  $7 \times 10^9$ , and determine that the world population is more than 20 times larger.
- 8.EE.4** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

**Understand the connections between proportional relationships, lines, and linear equations.**

- 8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
- 8.EE.6** Use similar triangles to explain why the slope  $m$  is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation  $y = mx$  for a line through the origin and the equation  $y = mx + b$  for a line intercepting the vertical axis at  $b$ .

**Analyze and solve linear equations and pairs of simultaneous linear equations.**

- 8.EE.7** Solve linear equations in one variable.
- a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form  $x = a$ ,  $a = a$ , or  $a = b$  results (where  $a$  and  $b$  are different numbers).
  - b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
- 8.EE.8** Analyze and solve pairs of simultaneous linear equations.
- a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
  - b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example,  $3x + 2y = 5$  and  $3x + 2y = 6$  have no solution because  $3x + 2y$  cannot simultaneously be 5 and 6.
  - c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

## **FUNCTIONS**

### **Define, evaluate, and compare functions.**

- 8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Note: Function notation is not required in Grade 8.)
- 8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
- 8.F.3** Interpret the equation  $y = mx + b$  as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function  $A = s^2$  giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

### **Use functions to model relationships between quantities.**

- 8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two  $(x, y)$  values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- 8.F.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

## **GEOMETRY**

### **Understand congruence and similarity using physical models, transparencies, or geometry software.**

- 8.G.1** Verify experimentally the properties of rotations, reflections, and translations:
- Lines are taken to lines, and line segments to line segments of the same length.
  - Angles are taken to angles of the same measure.
  - Parallel lines are taken to parallel lines.
- 8.G.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
- 8.G.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- 8.G.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
- 8.G.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

### **Understand and apply the Pythagorean Theorem.**

- 8.G.6** Explain a proof of the Pythagorean Theorem and its converse.
- 8.G.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
- 8.G.8** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

### **Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**

- 8.G.9** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

## **STATISTICS AND PROBABILITY**

### **Investigate patterns of association in bivariate data.**

- 8.SP.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
- 8.SP.2** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
- 8.SP.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
- 8.SP.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

# Real Number Race

## Common Core Standard:

**Know that there are numbers that are not rational, and approximate them by rational numbers.**

8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

## Additional/Supporting Standard(s): 8.NS.2

## Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
5. Use appropriate tools strategically
6. Attend to precision

## Student Outcomes:

- I can write a decimal approximation for an irrational number to a given decimal place
- I can convert either repeating or terminal decimals into a fraction
- I can explain the difference between a rational and an irrational number.

## Materials:

- One copy of the real number hexagon per team
- Spinner or cube
- Two different colors of pencils for each student

## Advance Preparation:

- Make copies of the real number hexagon
- Prepare the spinner or cube
- Put together colored pencils of different colors according to the number of students playing together, insuring that each student will use two different colors of pencils

## Directions:

1. S/he chooses one of their colors for rational and one for irrational
2. On each player's first turn, s/he will spin the spinner and get a real number, irrational number, rational number or lose a turn.
3. S/he colors a number on the hexagon that fits the category that they spun. If S/he spins a real number they can color either rational or irrational.
4. Students take turns with the spinner and marking their numbers.
5. The winner is the first player to get four in a diagonal row of one color. If a player colors an incorrect circle, the opponents should challenge her/him; a wrong move has the penalty of losing a spin and the color should be erased.

**Questions to Pose:**Before:

- What are the characteristics of a rational number?
- What are the characteristics of an irrational number?
- What is the definition of real numbers?

During:

- Does it matter which color you use for the real number choice?
- What strategy did you use to determine which number to choose?
- Are all square roots irrational?

After:

- What strategies did people in your group use to choose their numbers?
- Were there numbers that you disagreed with each other about their category and why?
- If you could remodel the task, what would you do to it?

**Possible Misconceptions/Suggestions:**

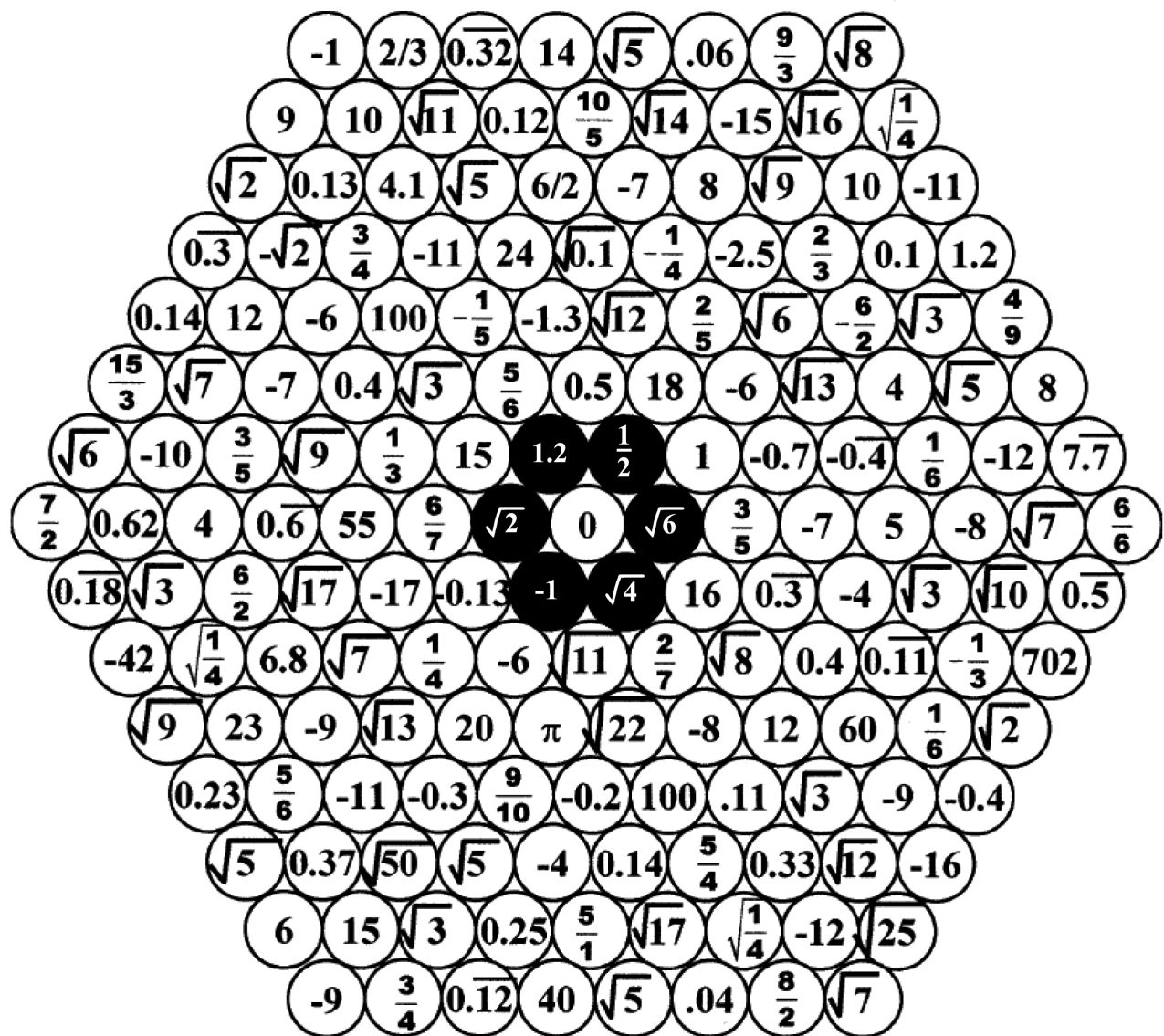
<b>Possible Misconceptions</b>	<b>Suggestions</b>
Students assume longer decimals are irrational	Work with rational numbers other than those that are common fractions.
Students do not recognize Pi as being irrational; the value is 3.14 (terminating)	

**Special Notes:**

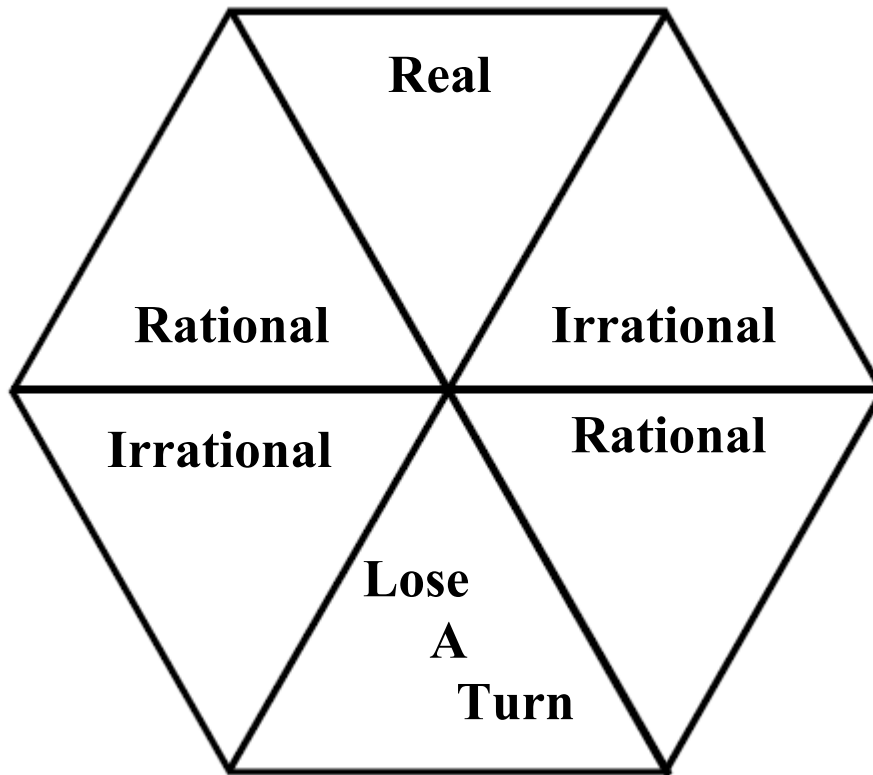
This activity can be used with various sized groups. Extension of this can be done by modifying the spinner to include natural numbers, whole numbers, etc. Categories can be put on a cube or students may roll a fair number cube with the following designations:

- 1 Rational
- 2 Irrational
- 3 Irrational
- 4 Rational
- 5 Lose a turn
- 6 Real

**Solutions:** NA



## Spinner for Real Number Race





# The Laundry Problem

## Common Core Standards:

**Know that there are numbers that are not rational, and approximate them by rational numbers.**

8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g.,  $\pi^2$ ). For example, by truncating the decimal expansion of  $\sqrt{2}$ , show that  $\sqrt{2}$  is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

## Additional/supporting standards: 8.NS.1

## Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

## Student Outcomes:

- I can explain the difference in a rational and an irrational number
- I can place rational and irrational numbers on a number line

## Materials:

- White paper
- Markers, crayons, colored pencils
- String or clothesline
- Glue sticks or tape
- Clothespins
- Number choices page

## Advance Preparation:

- Teacher should copy number choices and cut apart and place in a bag for students to choose from.
- Teacher should hang the string or clothesline in the front of the classroom and attach the 0 to the center of the clothesline.
- Depending on the level of ability of the students, the teacher can also add the integers evenly spaced.
- Students should know the definition of rational and irrational numbers and how to determine which category a number is in

**Directions:**

Teacher will introduce the activity with a quick review of rational and irrational numbers. After this, explain to the students that they will be creating “special laundry” to hang on the clothesline. Rational numbers will be placed on tee shirts and irrational numbers on pants. Each student chooses a number from the bag. Allow students time to create and cut out either a shirt or a pair of pants depending on their number. As students are working, the teacher should circulate around the room ask students to justify their clothing choice. Give students time to decorate their clothing piece and glue the number on the front. Once students have finished, it is time to hang the laundry. Allow students to come and pin their laundry items on the clothesline. As more clothing is added, students may need to move laundry to accurately place theirs on the line. While students are hanging out the laundry, allow student input on the accuracy of the clothesline. When all the laundry has been placed on the clothesline, lead a student discussion on the accuracy of the placement, allowing students to rearrange the clothing if necessary.

**Questions to Pose:**Before:

- What characteristics make a number rational?
- What characteristics make a number irrational?
- How do you determine whether your number is rational or irrational?

During:

- What mathematics is used to make this determination?
- What do you do if numbers are very close together to determine their placement on the clothesline?
- What steps do you take to place negative numbers on the clothesline?

After:

- How do you check to see if the laundry is placed correctly?
- Can you add another shirt or pants to the left or right of the clothesline?

**Possible Misconceptions/Suggestions:**

Possible misconceptions	Suggestions
Students assume longer decimals are irrational	Work with rational numbers other than those that are common fractions.
Students reverse the order of negative numbers	Relate greater than and less than to the direction you travel on the number line
Students do not recognize Pi as being irrational; the value is 3.14 (terminating)	

**Special Notes:**

To speed up the creation of shirts and pants, the teacher could create a template and give each student a handout with both patterns. If the class is large, smaller groups can work on several clotheslines and then check each others. Number sets be adjusted according to the ability of the students. Students need to have an understanding of perfect squares and other roots as well. Calculators may be needed to evaluate radicals.

**Solutions:** NA

$-\sqrt{21}$	5.00005	5.85	4.33.....
$-\sqrt[3]{101}$	-6.12	9.5791	$\pi$

**1.995**

$$\frac{17}{5}$$

$$\frac{-39}{9}$$

$$\frac{96}{10}$$

$$\frac{-2}{3}$$

$$\frac{-254}{255}$$

$$\frac{-24}{5}$$

$$\frac{48}{7}$$

$$\sqrt{98}$$

$$-\sqrt{10}$$

$$-\sqrt{14}$$

$$\sqrt{18}$$

$$\sqrt{80}$$

$$-\sqrt{100}$$

$$\sqrt{34}$$

$$-\sqrt{37}$$

$$-\sqrt{0.09}$$

$$\frac{-100}{11}$$

$$\sqrt{\frac{64}{4}}$$

$$\sqrt{3}$$

$$-\frac{\sqrt{49}}{2}$$

$$-\frac{\sqrt{50}}{2}$$

$$\sqrt{\frac{97}{3}}$$

$$\sqrt{\frac{26}{5}}$$

$$\frac{\sqrt{26}}{2}$$

# Perplexing Puzzle

## Common Core Standards

**8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

## Student Outcomes:

- I can dilate figures using a given ratio
- I can graph proportional relationships
- I can compare properties of a function represented in different ways
- I can interpret the equation of a linear function

**Additional/Supporting Standards:** 8.F.1, 8.F.2, 8.F.3, 8.F.4, 8.G.3

## Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## Materials:

- Puzzle (one per group)
- Activity Guide handout (one per group)
- Activity 1 & 2 handout (one per student)
- Colored construction paper (one per group)

## Advance Preparation:

- Puzzle pieces must be cut out and placed in envelopes for each group of students.
- Before doing this task, students should be familiar with the concept of slope.

## Directions for Activity:

Students are going to make a larger puzzle the same shape as the smaller puzzle, so that the edge of a piece that measures 4 cm in the old puzzle will measure 6 cm in the new puzzle.

1. Students work in groups. Each member of the group selects one or more of the pieces of the puzzle. The group works together to arrange the pieces to make a rectangle.
2. The task of each group is to make a puzzle larger than the one in the envelope using this clue: A side measuring 4 cm in the original puzzle must measure 6 cm in the larger puzzle.
3. Each student of the group works independently to enlarge his or her puzzle piece using one of the four colored sheets of construction paper.



4. When everyone in the group has completed making his or her enlargement, the group works together again to arrange the larger pieces to complete the puzzle. Students glue the puzzle on the light-colored piece of construction paper. If any pieces of the enlarged puzzle do not fit, students should discuss the strategies used to enlarge each piece.
5. Students will follow the Activity guide to complete the task.

**Questions to Pose:**

Before the Task:

- What do you know about similar figures?
- How could a ratio be related to similar figures?
- What is the meaning of scale factor?
- What strategies and/or tools could you use to find the missing puzzle dimensions?
- How will you know if your puzzle has been dilated correctly?

During the Task:

- How did you create your enlarged shape?
- Describe the new shape you created.
- How do the two shapes compare?
- What do you notice about the angles in the original shape and the angles in the enlarged shape?
- How do you know the two shapes are similar?
- What has to be true for the shapes to be mathematically similar?
- By examining your process of enlarging the figure, can you develop an equation that can be used to find dimensions of any puzzle piece?
- What do your variables in the equation represent?
- What patterns do you notice in the table in Activity 1?
- What proportional relationship do you notice?

After the Task:

- What is the scale factor used to enlarge the original puzzle?
- What do you notice about your graph in Activity 2?
- What does the constant ratio in the graph represent?
- What is the slope of the line formed when graphing your table on the coordinate plane?
- If you were to continue the graph, what might you observe?
- What is the relationship between the constant of proportionality, the slope and the scale factor?

**Possible Misconceptions/Suggestions:**

<b>Possible Misconceptions</b>	<b>Suggestions</b>
Students are likely to add 2 to 4 to get 6 and likewise add 2 to each measurement to get the larger measurement.	Students should notice this error when piecing the new puzzle together. Do not allow students to cut off excess of puzzle pieces that do not fit, rather, have the group work together to make a new piece that will fit.

**Special Notes:**

What students can learn from this lesson:

- The given ratio of 6:4 (image : pre-image) simplifies to 3:2 and can also be written as  $\frac{3}{2}$ .
- The scale factor can be found by dividing the new length of a dilated figure by the original length (image/pre-image). In this task, the scale factor is  $\frac{6}{4} = 1.5$ . This should be discovered with the function table if not already discovered when making the new puzzle piece(s).
- Students need to recognize that  $\frac{3}{2}$  is equivalent to 1.5 and that scale factor is also a ratio even when written in decimal form.
- If the puzzle has been correctly dilated and the points are plotted correctly, that ratio will always be 3:2 or  $\frac{3}{2}$  for this task. Thus the function also has a slope of  $\frac{3}{2}$ .
- Students should note that the plotted point (4, 6) represents the original length of 4 and the corresponding new length of 6. the y value is always 1.5 times larger than the x value.  $y/x = \frac{6}{4} = 1.5$ . Each correctly plotted point will have that same constant of proportionality.

**Solutions:**

Students should discover that the ratio, scale factor, slope, and rate-of-change are all 1.5 or  $\frac{3}{2}$  while completing the activity.

Adapted from [Mathematics TEKS Toolkit](#)

[http://www.utdanacenter.org/mathtoolkit/instruction/lessons/8\\_puzzle.php](http://www.utdanacenter.org/mathtoolkit/instruction/lessons/8_puzzle.php)

# Perplexing Puzzle Activity Guide

We are going to make a larger puzzle the same shape as the smaller puzzle, so that the edge of a piece that measures 4 cm on the old puzzle will measure 6 cm in the new puzzle.

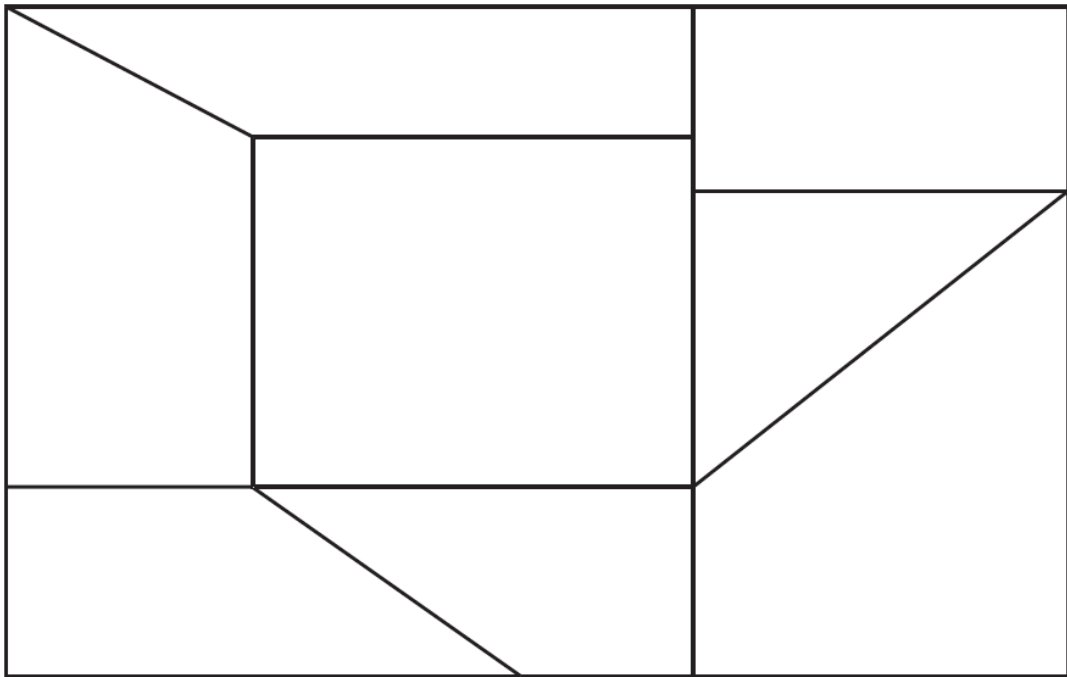
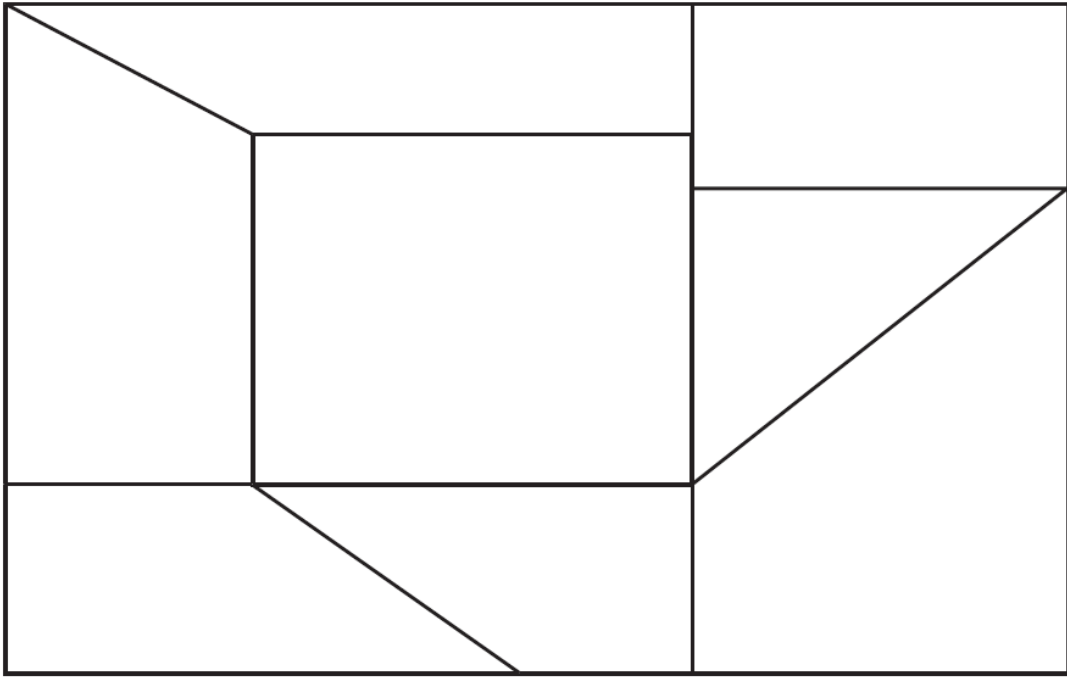
## **Directions:**

1. Assign a group leader. The group leader will ensure students stay on task, turn in work, and raise hand to ask questions.
2. Open the envelope containing the puzzle pieces and put it together. When completed, the puzzle will be a rectangle. (Use all the pieces.)
3. Sketch the completed puzzle in the area below.
  
4. Discuss how your group is going to enlarge each piece so that the edge of a piece that measures 4 cm will now measure 6 cm in the new puzzle.
5. Divide up the puzzle pieces so that each group member has at least one piece.
6. Enlarge your piece(s) independently based on the information given.
7. Once all individual pieces have been enlarged, use those pieces to put the puzzle back together. The puzzle should fit together perfectly, if not, you must make a new piece to replace those you think are incorrect.
8. Glue your enlarged puzzle to a piece of construction paper.
9. Complete Activity 1 as individuals.
10. Discuss Activity 1 and complete Activity 2 with group members.

Adapted from [Mathematics TEKS Toolkit](#)

[http://www.utdanacenter.org/mathtoolkit/instruction/lessons/8\\_puzzle.php](http://www.utdanacenter.org/mathtoolkit/instruction/lessons/8_puzzle.php)

# Perplexing Puzzle Templates



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## ACTIVITY 1

**Directions:** Complete the table and answer the questions.

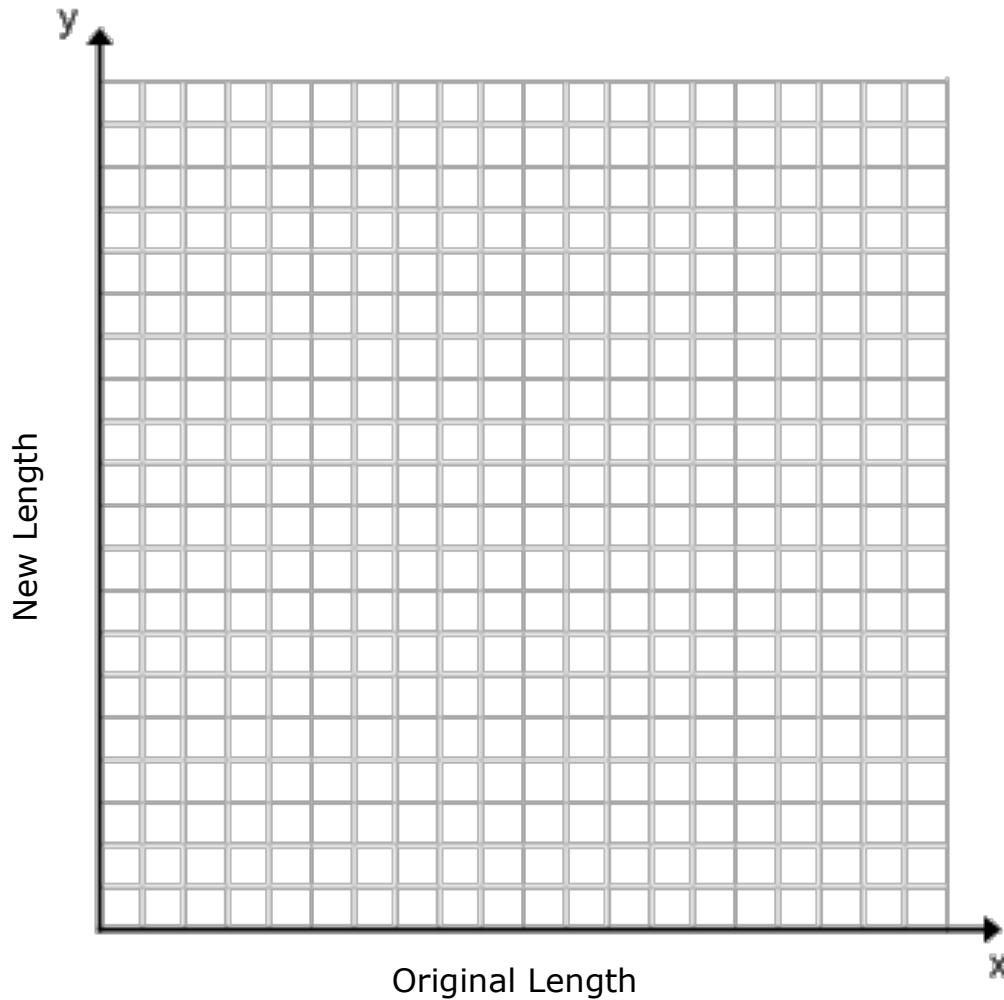
Original Length	Scale Factor	New Length
2		
3		
4		6
8		
13		
		30
$x$		$y$

1. What is the ratio of new length to original length for each row in the table? What does this ratio represent?
2. If a piece of the original puzzle had a side with length 45 cm, what would be the length of the side of the new puzzle piece?
3. If a piece of the original puzzle had a side with length 22.5 cm, what would be the length of
4. the side of the new puzzle piece?
5. If the new puzzle piece has a length of 22.5 cm, what was the length of the side of the original puzzle piece?

Adapted from [Mathematics TEKS Toolkit](http://www.utdanacenter.org/mathtoolkit/instruction/lessons/8_puzzle.php) [http://www.utdanacenter.org/mathtoolkit/instruction/lessons/8\\_puzzle.php](http://www.utdanacenter.org/mathtoolkit/instruction/lessons/8_puzzle.php)

## ACTIVITY 2

**Directions:** Plot the points from the table in Activity 1 on the graph. Next, each group member will place a point on the graph representing the measure of one side of the original length of a puzzle piece as the  $x$ -value and the measure of the corresponding side of the new length of a puzzle piece as the  $y$ -value (original length, new length). Do this for each puzzle piece so there will be 8 new points on your graph.



1. Describe the graph.
2. What is the slope of the line?
3. What is the equation of the line written in slope-intercept form?
4. How are the dilation, table, graph, and equation of the line related to each other?
5. Describe similarities among ratio, constant of proportionality, scale factor, slope, and rate-of-change?

Adapted from [Mathematics TEKS Toolkit](http://www.utdanacenter.org/mathtoolkit/instruction/lessons/8_puzzle.php) [http://www.utdanacenter.org/mathtoolkit/instruction/lessons/8\\_puzzle.php](http://www.utdanacenter.org/mathtoolkit/instruction/lessons/8_puzzle.php)

# Cookie Calorie Conundrum

## Common Core Standard:

**Analyze and solve linear equations and pairs of simultaneous linear equations.**

**8.EE.8** Analyze and solve pairs of simultaneous linear equations.

## Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

## Student Outcomes:

- I can solve systems of two linear equations with a model.
- I can solve systems of two linear equations algebraically.
- I can solve real-world problems leading to two linear equations in two variables.

## Materials:

- Warm Up
- Cookie Calorie Conundrum introduction (display for students to see)
- Cookie Calorie Conundrum Handout (one per student)
- Chocolate Sandwich Cookies (optional)

## Advance Preparation:

- Students must know how to solve multi-step equations.

## Directions:

1. Begin the task with the warm up. Before beginning the task, discuss strategies used to determine the unknown values.
2. Show students the Cookie Calorie Conundrum Introduction scenario.
3. Have students read the scenario and discuss possible ways to use the given information to solve the problem.

## Questions to Pose:

### Before:

- What strategies were successful or unsuccessful when working on the warm up?
- What was the key to solving the warm up?
- Were there particular equations that “gave away” the answers when used together?

### During:


- What do the nutritional facts tell you?
- How can you make the nutritional facts of the two cookie types easier to compare?
- How might the nutritional information help your classmate determine the number of calories she consumed?

- Why is the difference between the two types of cookies important information?
- What do the variables  $x$  and  $y$  represent?
- Can you explain your equations without using the variables?

After:

- If you were given the two equations representing both types of cookie, how would you have approached solving for  $x$  and  $y$ ?:  $2x + 2y = 68$  and  $x + 2y = 52$
- How can visuals help to solve a system of equations?
- Often times we are too quick to use algebra to solve a system without looking for other representations. What are some ways to represent a system of equations?
- How does the warm up relate to the task?

**Possible Misconceptions/Suggestions:**

Possible Misconceptions	Suggestions
The double cookie filling is actually two single fillings	Compare/contrast the two images used in the warm up and throughout the task. 
Not making a connection between what the variables represent.	The task explicitly tells students what $x$ and $y$ represent but it is important that students relay that information through discussion of the algebra.


**Special Notes:**


- When solving a system of equations, 8<sup>th</sup> grade students are only expected to solve algebraically through substitution.
- With equations like the two used in this lesson, it is acceptable to introduce elimination because no multiplication is required before adding/subtracting.

$$\begin{array}{r} 2x + 2y = 68 \\ (-) x + 2y = 52 \\ \hline x = 16 \end{array}$$

**Solutions:**

**WARM UP – What are the values?**

 = 32

 = 16

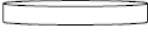
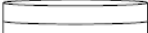


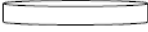







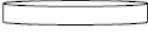
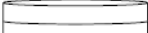


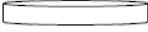







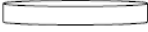
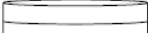


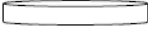







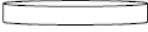
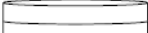


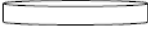







 = 18

**HANDOUT – See Key**




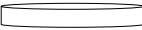
# Warm Up

**Directions:** The sums of combinations of three different figures are shown in the table below. What is the value of each figure?

 +  +  + 	+	 +  +  + 	+	 +  +  + 	=	50
 +  +  + 	+	 +  +  + 	+	 +  +  + 	=	68
 +  +  + 	+	 +  +  + 	+	 +  +  + 	=	52
 +  +  + 	+	 +  +  + 	+	 +  +  + 	=	66
=		=		=		
82		68		86		SUM

**What are the values?**

 = \_\_\_\_\_

 = \_\_\_\_\_

 = \_\_\_\_\_

# Cookie Calorie Conundrum Introduction

Your math teacher wants you to keep a precise record of the number of calories you consume each day. Your classmate has a strange habit of twisting the top wafer off a chocolate sandwich cookie and only eating the white filling and bottom wafer. After throwing away the top wafer, she realized she needs to know how many calories she consumed.



In attempt to help your classmate determine the number of calories in only the icing and bottom wafer, you searched the internet but could not find the answer. You did however find the Nutritional Facts for chocolate sandwich cookies and the Nutritional Facts for chocolate sandwich cookies with double the amount of white filling.

## Chocolate Sandwich Cookie with white filling.

<b>Nutrition Facts</b>	
Serving size	3 cookies
<hr/>	
<b>Amount Per Serving</b>	
Calories	156

## Chocolate Sandwich Cookie with double the amount of white filling.

<b>Nutrition Facts</b>	
Serving size	2 cookies
<hr/>	
<b>Amount Per Serving</b>	
Calories	136

What do the Nutritional Facts tell you?

How might this help your classmate figure out how many calories she consumed?

# Cookie Calorie Conundrum Handout

1. How many calories are in just one of each type of cookie?

Calories: \_\_\_\_\_



Calories: \_\_\_\_\_



2. What causes the difference in the amount of calories between the two types of cookies?  
What is the difference in calories?

3. Write an explanation of how the sums below describe the two types of cookies?

$$\text{Cookie with double filling} + \text{Cookie with single filling} + \text{Cookie with single filling} = 68 \text{ cal.}$$

$$\text{Cookie with single filling} + \text{Cookie with double filling} + \text{Cookie with single filling} = 52 \text{ cal.}$$

4. Let  $x$  represent the calories in a single white filling and  $y$  represents the calories in a single wafer. Write an equation that models the total amount of calories in each type of cookie.

Cookie with double filling equation: \_\_\_\_\_

Cookie with single filling equation: \_\_\_\_\_

5. What are the values? Explain how you got your answers.

$$\text{Cookie with double filling} = \underline{\hspace{2cm}} \quad \text{Cookie with single filling} = \underline{\hspace{2cm}} \quad \text{Wafer} = \underline{\hspace{2cm}}$$

**KEY**

**Cookie Calorie Conundrum**

1. How many calories are in just one of each type of cookie?

Calories: 52



Calories: 68



2. What causes the difference in the amount of calories between the two types of cookies?  
What is the difference in calories?

The difference in calories is attributed to the differences in the white filling. The difference in calories is 16.  $68 - 52 = 16$

3. Write an explanation of how the sums below describe the two types of cookies? **Answers will vary.**

$\text{White filling} + \text{Chocolate wafer} + \text{Chocolate wafer} = 68 \text{ cal.}$

$\text{White filling} + \text{Chocolate wafer} + \text{Chocolate wafer} = 52 \text{ cal.}$

4. Let  $x$  represent the calories in a single white filling and  $y$  represents the calories in a single chocolate wafer. Write an equation that models the total amount of calories in each type of cookie.

Cookie with double filling equation:  $2x + 2y = 68$

Cookie with single filling equation:  $x + 2y = 52$

5. What are the values of  $x$  and  $y$ ? Explain how you got your answers.

$x=16$  (calories in a single white filling)

$y=18$  (calories in a single chocolate wafer)

# Bow Wow Barkley

## Common Core Standards:

8.F.4 XXXX

## Student Outcomes:

- I can create a function to model a linear relationship between two quantities.
- I can determine the rate of change and initial value of a function from a description and by reading ordered pairs from a table or graph.
- I can describe a functional relationship between two quantities by analyzing tables and graphs.

## Materials:

- Bow Wow Barkley Student Handout
- Pattern Blocks
- Graphing Calculators (or graph paper and colored pencils)

## Advance Preparation:

- Large baggies or containers of pattern blocks can be created ahead of time to ensure that each small group of students has enough pattern blocks to complete the task. Each baggie or container will need a minimum of the following: 8 blue parallelograms, 22 yellow hexagons, 20 red trapezoids, 8 orange squares, 10 tan rhombi.
- Bookmark the virtual manipulative website on the desktop of classroom computer or Smartboard: [http://www.arcytech.org/java/patterns/patterns\\_j.shtml](http://www.arcytech.org/java/patterns/patterns_j.shtml)

## Directions for Activity:

Teacher will introduce the task by asking students if they have ever noticed how quickly puppies grow, particularly compared to humans. Don't belabor this discussion but use it to provide a segue into the Bow Wow Barkley task. Distribute the Bow Wow Barkley sheets and use the virtual manipulative website with pattern blocks to model how Bow Wow Barkley is growing from week one to week two. Ask students to share the patterns they notice. Each small group should have access to pattern blocks and based on the patterns observed, instruct them to build Bow Wow Barkley at Weeks 2 and 3 and complete the chart on their handout. As groups complete the chart, monitor their progress and instruct them to continue with the task by completing the remainder of the handout.

## Questions to Pose:

- What patterns did you notice with each shape as Bow Wow Barkley grew each week?
- How can you predict how many shapes will be needed to build Barkley at any given week?
- What relationships exist between each shape and Bow Wow Barkley's age in weeks?
- Given a verbal description of a function, how can we identify the y-intercept of its graph?
- Given a verbal description of a function, how can we identify the rate of change of its graph?

**Possible Misconceptions/Suggestions:**

<b>Possible Misconceptions</b>	<b>Suggestions</b>
Students cannot determine the number of blocks needed in chart for weeks 20 and 30 without using a recursive pattern.	Help students look for a relationship between the week number and the corresponding number of blocks.
Students reverse the rate of change and the intercept when writing the explicit equations in problem #4.	Refer students back to question #3 where an explanation of how the equations are derived is provided. Highlight the repeated additive pattern and why it is expressed with multiplication in the equation.
Students struggle with describing a change in the graph given only a verbal description (answering the question in Graph Analysis section).	Encourage students to graph the original function and then graph the new changed function on the same graph so the changes in graphs can easily be observed.

**Special Notes:**

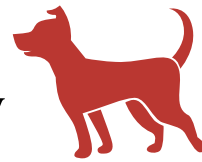
If graphing calculators are not available, the task can be completed without them. The items under section IV can be answered by using the equations created in questions 3 and 4 in section III. Using a different colored pencil for each graph is helpful if students are going to create graphs by hand.

**Solutions:**

Answer Key is available at the end of the activity sheets.

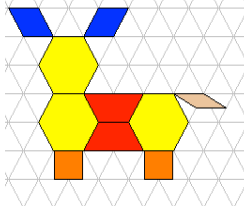


# Bow Wow Barkley

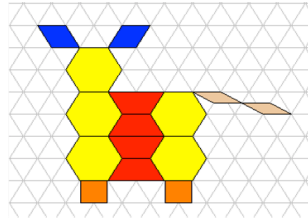


Bow Wow Barkley was born the smallest pup of all of his five brothers and four sisters. But Bow Wow Barkley soon became the biggest brown beagle in the litter. Just look at how quickly Bow Wow Barkley grew in a few short weeks.

Week 0



Week 1



- I. Build Bow Wow Barkley at Weeks 2 & 3 using pattern blocks then draw a picture of Bow Wow Barkley at Weeks 2 & 3.
- II. Record in the following table the number of total blocks used to create Barkley at various weeks. Then record the number of hexagons used in the pictures, the number of trapezoids and then the number of small, tan rhombi (tail only).

<u>Week #</u>	<u>Total blocks</u>	<u>Hexagons</u>	<u>Trapezoids</u>	<u>Rhombi</u>
0				
1				
2				
3				
4				
5				
6				
7				
20				
30				

### III. Table Analysis

1. What was the original number of blocks used to create Bow Wow Barkley? \_\_\_\_\_
2. As each week increases, what patterns do you observe with the total number of blocks?  
\_\_\_\_\_  
\_\_\_\_\_
3. An equation can be written to represent the number of total blocks used to create Bow Wow Barkley at any week.

$$y = \text{change} \cdot x + \text{initial amount}$$

$$y = \underline{\quad}x + \underline{\quad}$$

4. Write an equation that represents the different types of blocks used to create Bow Wow Barkley as the weeks increase.

Hexagons:  $y = \underline{\quad}x + \underline{\quad}$

Trapezoids:  $y = \underline{\quad}x + \underline{\quad}$

Rhombi:  $y = \underline{\quad}x + \underline{\quad}$

#### IV. Graphs

Input into your calculator the equations that represent the total number of blocks, hexagons, trapezoids and rhombi used to create Bow Wow Barkley as the weeks increase.

- A. Use the table on the calculator to determine the number of blocks needed to create a picture of Bow Wow Barkley when he is 1 year old. \_\_\_\_\_
- B. How old will Bow Wow Barkley be when his picture contains 75 hexagons? \_\_\_\_\_
- C. How many trapezoids are needed to create Bow Wow Barkley when he is 38 weeks old? \_\_\_\_\_
- D. How old will Bow Wow Barkley be when his picture contains 100 rhombi? \_\_\_\_\_

#### V. Graph Analysis

1. If Bow Wow Barkley's picture originally consisted of 6 blocks instead of 10, with his growth rate remaining the same, how would the graph representing the total number of blocks change?  
\_\_\_\_\_  
\_\_\_\_\_
2. If the original picture had 10 blocks but increased by 8 blocks each week versus the current 5 blocks, how would the graph change?  
\_\_\_\_\_  
\_\_\_\_\_
3. If Bow Wow Barkley's original picture consisted of 4 hexagons, with the growth rate remaining the same, how would the graph representing the number of hexagons change?  
\_\_\_\_\_  
\_\_\_\_\_
4. If the original picture of Bow Wow Barkley had 2 trapezoids but it increased by 4 trapezoids each week versus the current 2 trapezoids, keeping all other shapes the same, how would the graph change?  
\_\_\_\_\_  
\_\_\_\_\_
5. Write the equation of a graph that represents the change in Bow Wow Barkley's tail if his tail was originally 3 rhombi long. \_\_\_\_\_  
\_\_\_\_\_
6. Compare this equation with that of his original tail. How do the graphs differ?  
\_\_\_\_\_  
\_\_\_\_\_
7. How does the initial value affect the function?  
\_\_\_\_\_  
\_\_\_\_\_



8. How does the growth pattern affect the function?

---

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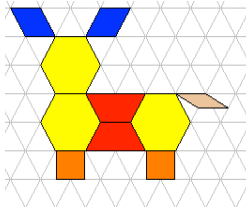
# Bow Wow Barkley

## ANSWER KEY

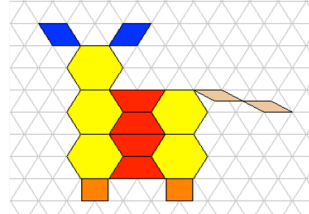


Bow Wow Barkley was born the smallest pup of all of his five brothers and four sisters. But Bow Wow Barkley soon became the biggest brown beagle in the litter. Just look at how quickly Bow Wow Barkley grew in a few short weeks.

Week 0



Week 1



- I. Build Bow Wow Barkley at Weeks 2 & 3 using pattern blocks then draw a picture of Bow Wow Barkley at Weeks 2 & 3.
- II. Record in the following table the number of total blocks used to create Barkley at various weeks. Then record the number of hexagons used in the pictures, the number of trapezoids and then the number of small, tan rhombi (tail only).

<u>Week #</u>	<u>Total blocks</u>	<u>Hexagons</u>	<u>Trapezoids</u>	<u>Rhombi</u>
0	10	3	2	1
1	15	5	4	2
2	20	7	6	3
3	25	9	8	4
4	30	11	10	5
5	35	13	12	6
6	40	15	14	7
7	45	17	16	8
20	110	43	42	21
30	160	63	62	31

III. Table Analysis

1. What was the original number of blocks used to create Bow Wow Barkley? **10**
2. As each week increases, what patterns do you observe with the total number of blocks?  
**Total blocks increase by 5 each week.**
3. An equation can be written to represent the number of total blocks used to create Bow Wow Barkley at any week. **Blocks** = change in blocks per **week** + initial # of blocks  
**Output** = change • **input** + initial amount  
**y** = change • **x** + initial amount  
**y = 5x + 10**

4. Write an equation that represents the different types of blocks used to create Bow Wow Barkley as the weeks increase.

Hexagons:  $y = 2x + 3$

Trapezoids:  $y = 2x + 2$

Rhombi:  $y = 1x + 1$  Students may also write  $y = x + 1$

#### IV. Graphs

Input into your calculator the equations that represent the total number of blocks, hexagons, trapezoids and rhombi used to create Bow Wow Barkley as the weeks increase.

- A. Use the table on the calculator to determine the number of blocks needed to create a picture of Bow Wow Barkley when he is 1 year old. **270**
- B. How old will Bow Wow Barkley be when his picture contains 75 hexagons? **36 weeks**
- C. How many trapezoids are needed to create Bow Wow Barkley when he is 38 weeks old? **78**
- D. How old will Bow Wow Barkley be when his picture contains 100 rhombi? **99 weeks**

#### V. Graph Analysis

1. If Bow Wow Barkley's picture originally consisted of 6 blocks instead of 10, how would the graph representing the total number of blocks change?  
**The graph would be translated down 4 units. It will now cross the y-axis at 6 instead of 10.**
2. If the original picture had 10 blocks but increased by 8 blocks each week versus the current 5 blocks, how would the graph change?  
**The graph would be steeper because the slope increased from 5/1 to 8/1. The y-intercept would not change.**
3. If Bow Wow Barkley's original picture consisted of 4 hexagons, how would the graph representing the number of hexagons change?  
**The graph would be translated up 2 units on the y-axis resulting in a y-intercept of 4 instead of 2. The slope of the line would not change.**
4. If the original picture of Bow Wow Barkley had 2 trapezoids but it increased by 4 trapezoids each week versus the current 2 trapezoids, how would the graph change?  
**The slope of the line would become steeper with a rate increase from 2 trapezoids per week to a rate of 4 trapezoids per week. The y-intercept of 2 would not change.**
5. Write the equation of a graph that represents the change in Bow Wow Barkley's tail if his tail was originally 3 rhombi long.  $y = 1x + 3$
6. Compare this equation with that of his original tail. How do the graphs differ? **The y-intercept of the graph would change to 3. Thus the line would have been translated up 2 units. The slope of the line would not change as the rate of increase per week remained constant.**
7. How does the initial value affect the function?  
**The initial value determines the y-intercept of the function. In the equation,  $y = mx + b$ , the initial value is "b". On the graph, it will always be y-coordinate of the point where the line crosses the y-axis.**
8. How does the growth pattern affect the function?  
**The additive growth is the rate of change of the function. In the equation, the additive growth pattern is represented as the coefficient of the input (independent) variable. The growth pattern is defined as the slope when looking at the graph of the function.**

# Nonlinear Functions

## Common Core Standard:

### Define, evaluate, and compare functions.

8.F.3 Interpret the equation  $y = mx + b$  as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function  $A = s^2$  giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4), and (3, 9), which are not on a straight line.

## Additional/Supporting Standard(s):

8.F.4 Use functions to model relationships between quantities.

## Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
8. Look for and express regularity in repeated reasoning.

## Student Outcomes:

- I can determine whether a given pattern represents a linear or non-linear relationship between two quantities.
- I can recognize that a constant rate of change results in a linear function that creates a straight line graph.
- I can recognize that a non-constant rate of change results in a non-linear function that creates a graph that is not a straight line.

## Materials:

- 1 copy of each function, Pencils, Diamonds, Houses, Growing Rectangles, and Stair Steps

## Advance Preparation:

- Make a copy of each of the function blacklines
- Graph paper
- Colored pencils
- Rulers

## Directions:

1. Allow students to work in small groups or individually.
2. Pass out graph paper, rulers, and colored pencils.
3. Instruct students to create a set of axes along the bottom and left sides of the grid. They should label the axes with scales of 1.
4. Display the Pencil Function to the class.
5. Ask students to construct a T-table for the function that compares the steps in the pattern to the number of toothpicks and graph steps 1 through 3. Encourage students to add a few more values to their T-table based on the pattern they observe and add points to the graph.
6. The T-tables should be constructed on a separate sheet of paper or on the back of their graph paper. Make sure the T-table is labeled with the title of the function.

7. Students should then plot the points on their graph in one color. Label the function with its name (Pencils).
8. Facilitate a class discussion by having students share their observations and justify their reasoning. Important points to discuss:
  - The points fall in a straight line. Why? (There is constant growth pattern.)
  - There is a pattern within the values in the table from one step to the next. What is it? (As the x-values increase by 1, the y-values increase by 3.)
  - Predict the y-value when x is 0. (By reversing the pattern of adding 3, students will determine that when x is 0, y will be 3. Students could also extend the line on their graph through the y-intercept and observe that the graph will cross the y-axis at 3, translating to the point (0, 3).)
  - Since this is a linear pattern with a constant additive growth rate, encourage the students to write a function rule that describes the pattern. ( $y = 3x + 3$ )
  - Ask the students to connect their rule to the Pencil toothpicks. (One explanation is that the original step would be a triangle and then as each square is connected to it, the pattern increases by 3 each time since there is a shared side.)
9. Next display the Diamonds Function to the class.
10. Repeat steps 5 through 8 above, having the students use a different color pencil to plot their points on the same graph as the Pencil Functions. Students should label their new graph. (Diamonds) and also label their T-Table.
11. Again, facilitate a discussion by having students share their observations. Important points to discuss:
  - The points do not fall in a straight line. Why? (There is not a constant rate of change in the y-values as the x-values increase by 1 each time.)
  - Predict the y-value when x is 0. (This may be a bit more challenging for the students since it is not a constant additive pattern. Several options include using the pattern that the y-value is simply the square of the corresponding x-value, hence an x-value of 0 will result in a y-value of 0. Another observation from the table could be that the 1<sup>st</sup> difference among consecutive y-values is not constant but are increasing by odd numbers. However, the 2<sup>nd</sup> difference (difference between the 1<sup>st</sup> difference values) is constant with an increase of 2 each time. Reversing this pattern will result in a y-value of 0 when the x-value is 0.)
  - Plot the point (0, 0) that has been added to the table and extend the graph to the origin, relating the y-intercept of the graph to the table.
  - Since this graph is non-linear, it is not expected that students write a rule for the function but this can certainly be encouraged. ( $y = x^2$ )
12. Repeat the above procedures with the last three functions, Houses, Growing Rectangles, and Stair Steps. (see solutions for details)

### Questions to Pose:

#### Before:

- How do you construct a T-table?
- How do you use a T-table to create a graph?

#### During:

- What patterns do you notice?
- How does the rule for the function connect to the picture?
- Do all patterns result in a straight line graph? Explain.
- Using the observed patterns or rules written for a function, predict the number of “toothpicks” or “diamonds” in steps 4, 5 and 10.

After:

- Given a table of value, how can you predict whether the graph will be linear or non-linear?  
(If the pattern varies from step to step, the graph will be a curve)
- What can we conclude about the graphs of patterns that have a constant rate of change?
- What can we conclude about the graphs of patterns that do not have a constant rate of change?
- Why is it more difficult to find formulas for nonlinear functions?

**Possible Misconceptions/Suggestions:**

Possible Misconceptions	Suggestions
Students may assume that a graph is linear if they do not plot enough points on the graph.	Encourage students to plot more points or reduce the scale on their grid.

**Special Notes:** N/A

**Solutions:**

**Pencils**

0	3	} +3 } +3 } +3 } +3
1	6	
2	9	
3	12	
4	15	
5	18	
10	27	
x	$3x + 3$	

linear

**Diamonds**

0	0	} +1 } +3 } +5 } +7	} +2 } +2 } +2
1	1		
2	4		
3	9		
4	16		
5	25		
10	100		
x	$x^2$		

nonlinear

**Houses**

0	1	} +4 } +4 } +4 } +4
1	5	
2	9	
3	13	
4	17	
5	21	
10	41	
x	$4x + 1$	

linear

**Growing Rectangles**

0	0	} +2 } +4 } +6 } +8	} +2 } +2 } +2
1	2		
2	6		
3	12		
4	20		
5	30		
10	110		
x	$x^2 + x$		

nonlinear

**Stair Steps**

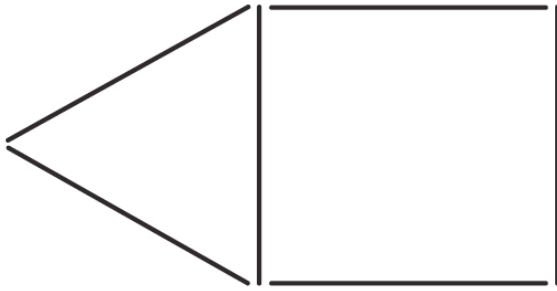
0	0	} +4 } +6 } +8 } +10	} +2 } +2 } +2
1	4		
2	10		
3	18		
4	28		
5	40		
10	130		
x	$x^2 + 3x$		

nonlinear

*Adapted from The Pattern and Function Connection, Brad Fulton and Bill Lombard*

# Pencils

1)



2)



3)



# Diamonds

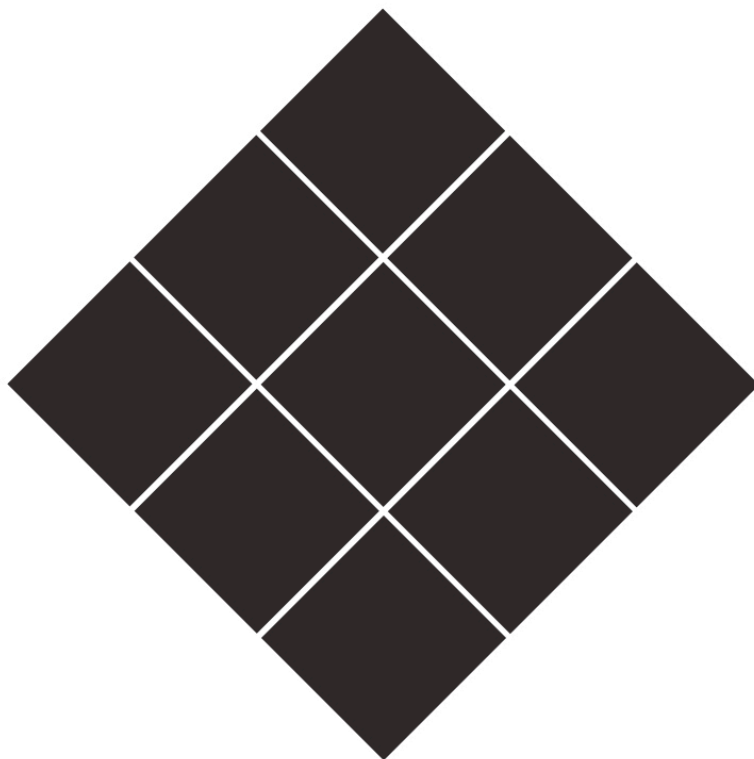
1)



2)



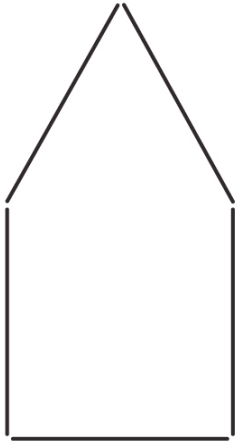
3)



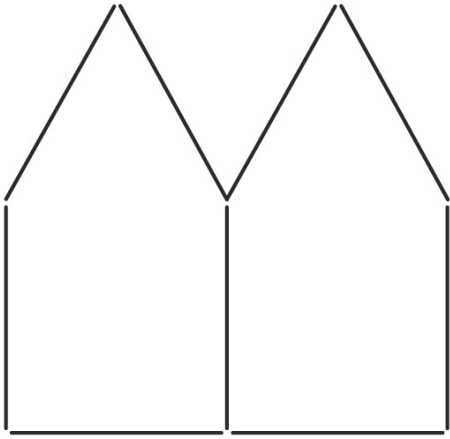


# Houses

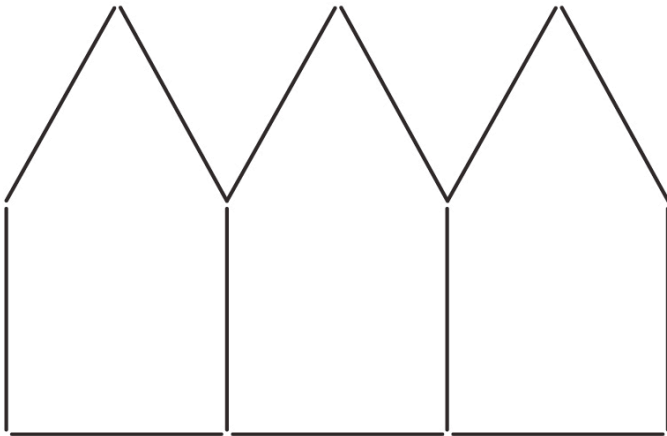
1)



2)



3)

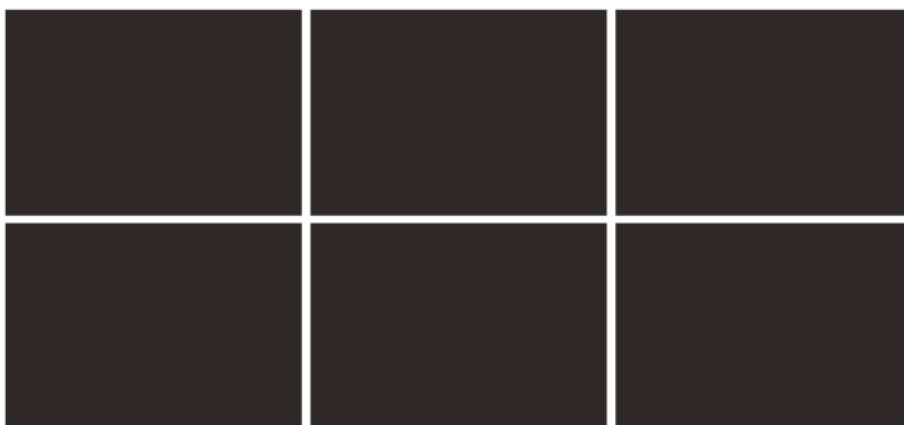


# Growing Rectangles

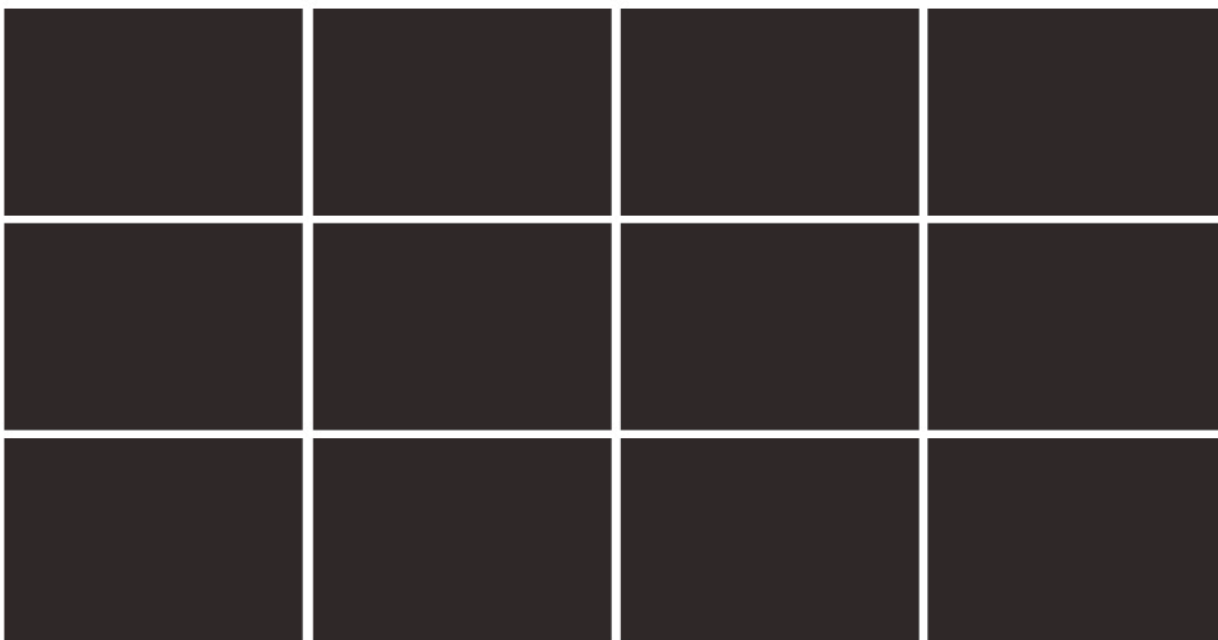
1)



2)



3)

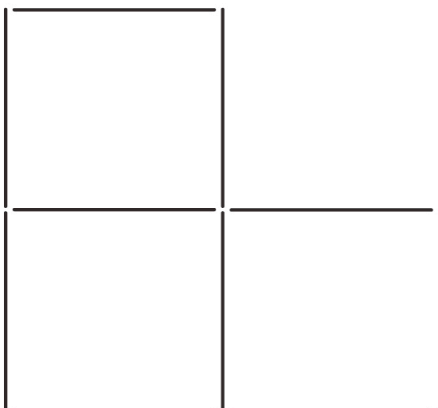


# Stair Steps

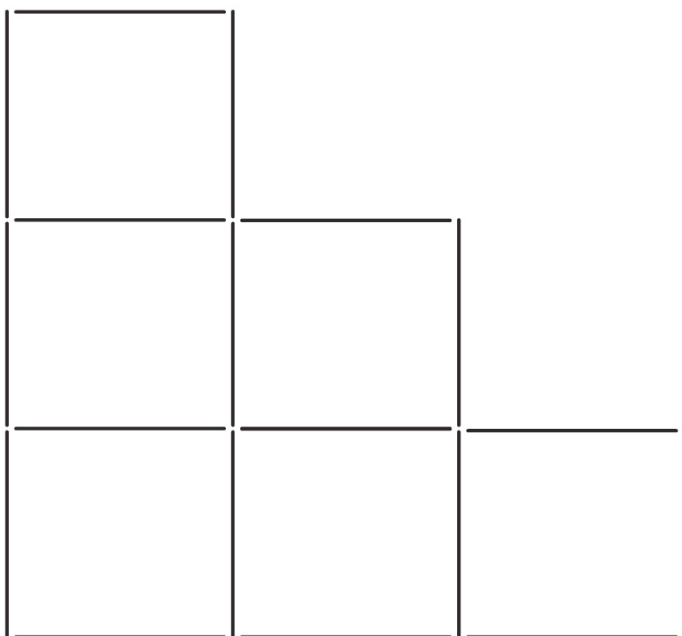
1)

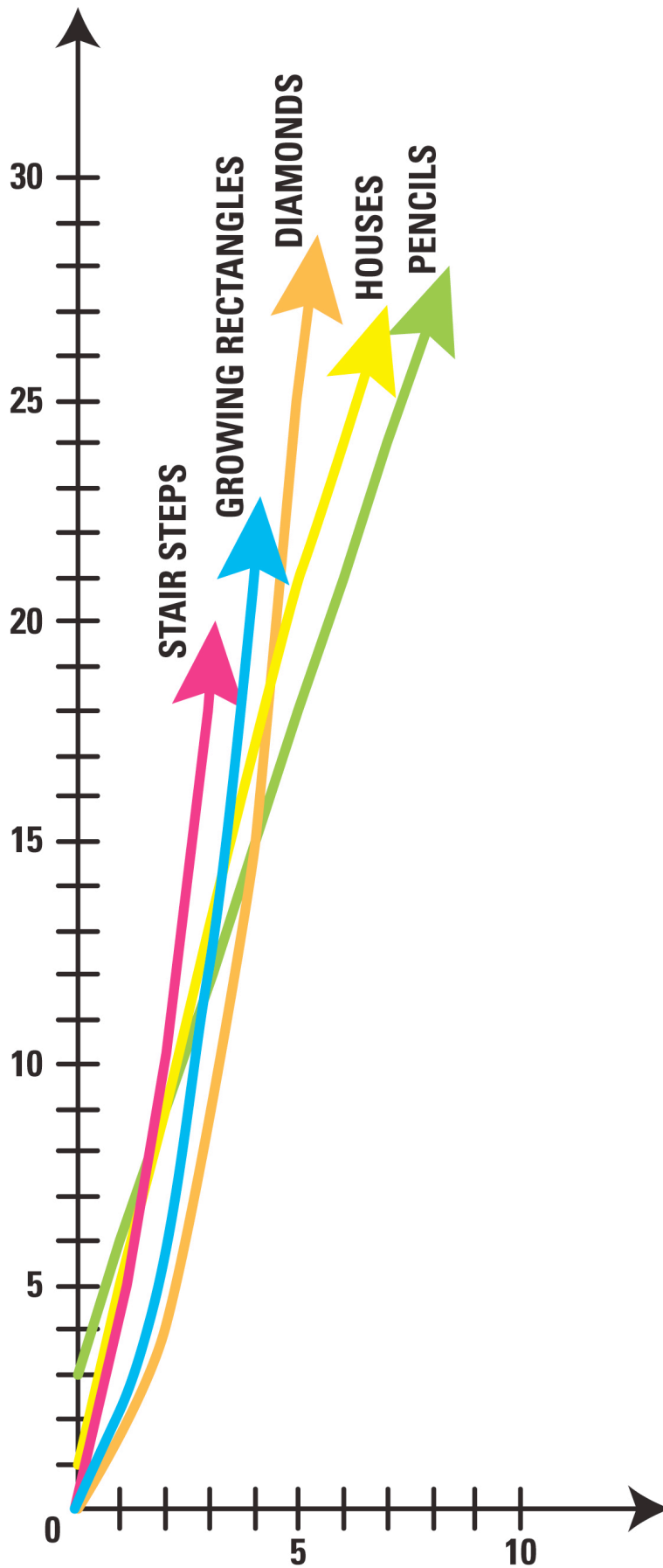


2)



3)





# Sandy's Candy Corporation

## Common Core Standard:

### Use functions to model relationships between quantities.

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two  $(x, y)$  values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

## Additional/Supporting Standard(s): 8.F.3

## Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
6. Attend to precision.

## Student Outcomes:

- I can create a function to model a linear relationship.
- I can determine the rate of change and initial value of the function from a description of a relationship or from two  $(x, y)$  values, a table or a graph.
- Interpret the rate of change and initial value in the context of the situation in models.
- I can determine whether a function will be linear or not.

## Materials:

- Sandy's Candy Corporation task
- Graphing Calculators (optional)

## Directions:

1. Begin the lesson by hooking the students. Suggested hooks might include providing each student with a piece of caramel candy or showing a quick video on how candy is made.
2. Have students read the initial scenario and then discuss it as a whole group to ensure everyone has a common understanding.
3. Students should complete the task (in pairs).

## Questions to Pose:

### Before:

- What is the difference between increasing an amount by \$0.05 and increasing an amount by 5%?

### During:

- How will you determine the scale on your graph?
- Which option do you predict will be the best for Sandy?
- How can the y-intercept of a graph be identified from a verbal description of a pattern?
- How can the slope of the function be identified from a verbal description of a pattern?

After:

- Which functions were linear and which were not?
- What characteristics of a function determine whether it is linear or non-linear?
- Which option did you choose and why?

**Possible Misconceptions/Suggestions:**

Possible Misconceptions	Suggestions
Students might initially mistake that increasing \$0.05 each week is the same as increasing 5% each week.	Emphasize that increasing by 5% each week is found by multiplying 1.05 times the previous week. Ask: why 1.05 and not just 0.05?

**Special Notes:**

Option 2 will reach \$2.00 at week 7. At first glance, students might assume the graph is linear. It is helpful to ask the students to continue a table so they can extend the graph so that its exponential behavior is more apparent. Graphing calculators might also be useful to see the differences in the three graphs.

**Solutions:**

Student solutions as to the best option may vary. The rationale they provide is the focus of their understanding.



# Sandy's Candy Corporation



Sandy's Candy Corporation has recently had some bad luck. The machine that makes their famous caramel candy has just broken. Sandy must purchase a new caramel candy machine immediately. However, new machines are very, very expensive. In order to pay for the new caramel candy machine, Sandy must increase the price of her caramel candy. Sandy currently sells the caramel candy packages for \$1.00 per bag. She needs to increase the price to \$2.00 per bag. She wants the increase to be gradual as not to surprise her customers. Sandy has decided on one of three options.

Option 1: Increase the price per bag of caramel candy \$0.05 per week until the price of \$2.00 is reached.

Option 2: Increase the price per bag of caramel candy 5% each week until the price of \$2.00 is reached.

Option 3: Increase the price per bag of candy each week equally for 10 weeks until the price of \$2.00 is obtained.

Sandy has come to you for advice. She wants to know which option you feel she should implement in her store.

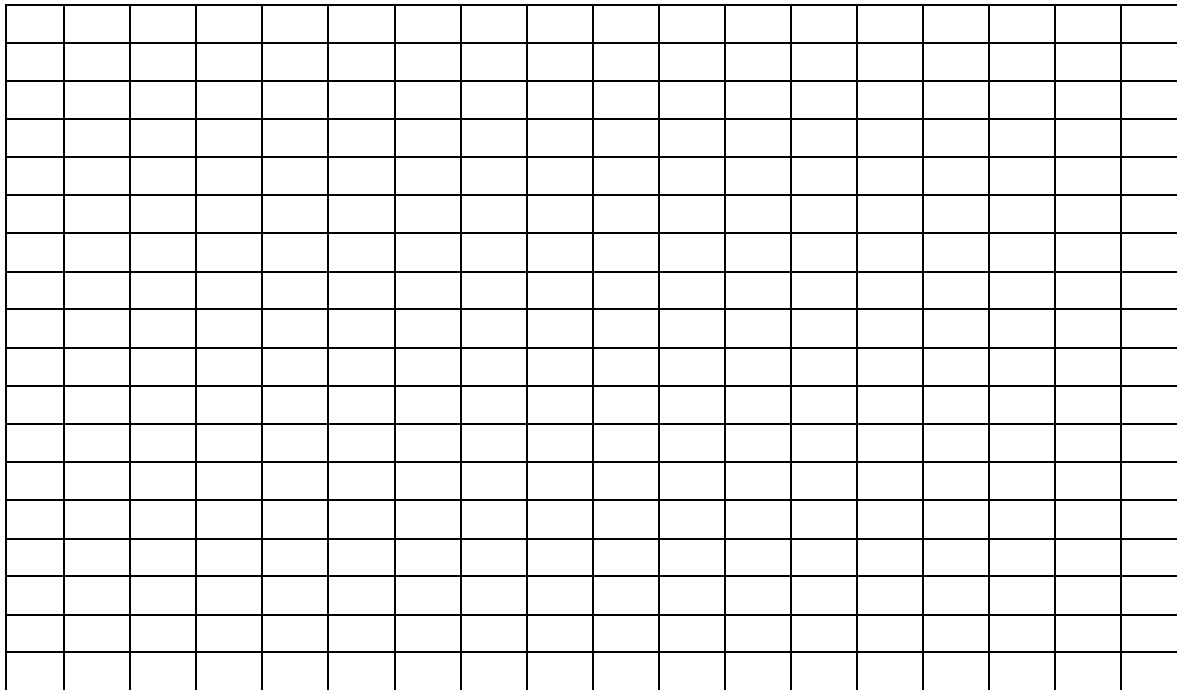
In order to make an informed decision, you must research and analyze the three options very carefully. In general, you will:

- 1) Make a table of values for each option.
- 2) Graph the data for each option. (Color code the different graphs for ease in reading the graph.)
- 3) Write an equation that models each option in order to make accurate predictions.

I. Make a table of values for each option.

	<b>Option 1 Cost</b>	<b>Option 2 Cost</b>	<b>Option 3 Cost</b>
Initial cost per bag	\$1.00	\$1.00	\$1.00
Week 1	\$1.05		
Week 2			
Week 3			
Week 4			
Week 5			
Week 6			
Week 7			
Week 8			
Week 9			
Week 10			

II. Graph the data for each option. The independent variable is the week number and the dependent variable is the cost. Label the axes and give your graph a title. Color code each option and make a legend.



III. How many weeks will it take the price to reach \$2.00 under each option?

- Option 1: \_\_\_\_\_
- Option 2: \_\_\_\_\_
- Option 3: \_\_\_\_\_

IV. Analyze Option 1:

- A. Is the graph of option 1 linear? \_\_\_\_\_
- B. The cost under option 1 increases constantly at a rate of \_\_\_\_\_ Per week.  
Thus, \_\_\_\_\_ is the slope.
- C. Option 1 intercepts the y-axis at the point \_\_\_\_\_.
- D. What is the original price of the caramel candy before the price is increased? \_\_\_\_\_
- E. Model option 1 with an equation: \_\_\_\_\_

V. Analyze Option 2:

- A. Is the graph of option 2 linear? \_\_\_\_\_
- B. Does the cost increase at a constant rate each week?
- C. Option 1 intercepts the y-axis at the point \_\_\_\_\_.
- D. What is the original price of the caramel candy before the price is increased? \_\_\_\_\_
- E. Mode option 2 with an equation: \_\_\_\_\_



VI. Analyze Option 3:

- A. Is the graph of option 3 linear? \_\_\_\_\_
- B. The cost under option 3 increases constantly at a rate of \_\_\_\_\_  
Per week. Thus, \_\_\_\_\_ is the slope.
- C. Option 3 intercepts the y-axis at the point \_\_\_\_\_.
- D. What is the original price of the caramel candy before the price is increased? \_\_\_\_\_
- E. Model option 3 with an equation: \_\_\_\_\_

VII. Which option do you feel Sandy should implement in order to raise the cost of caramel candy from \$1.00 to \$2.00? Explain your reasoning.

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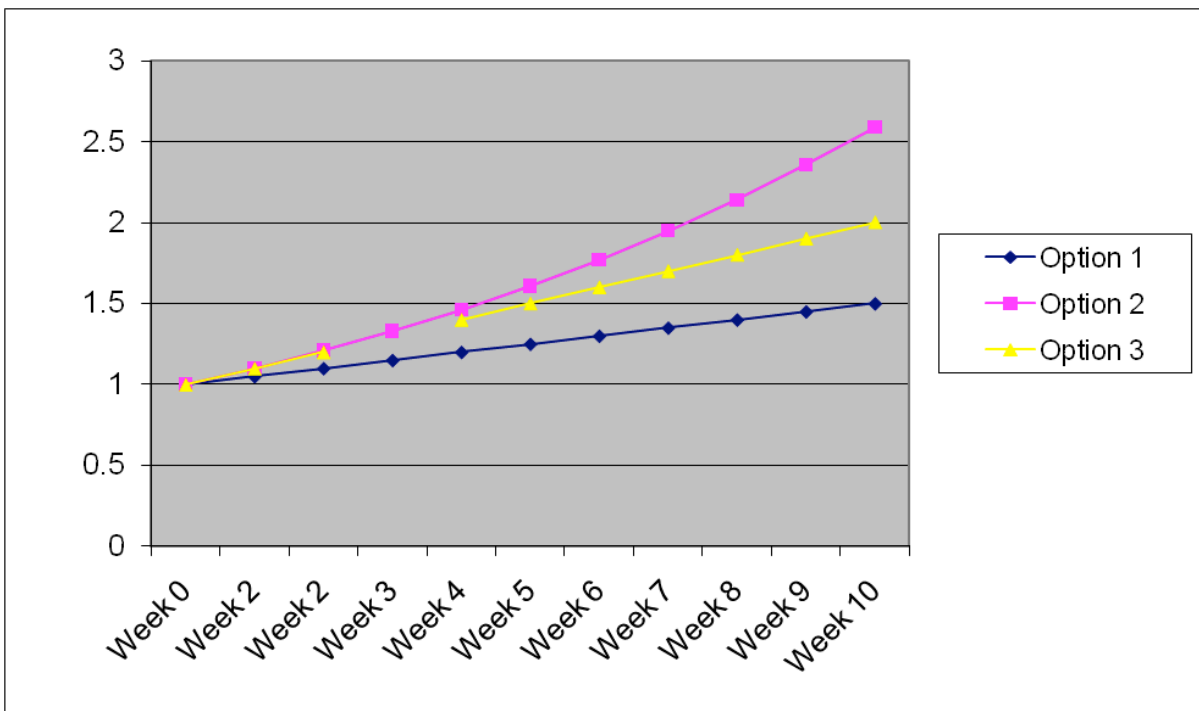
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Solutions to table and graphs:

	Option 1	Option 2	Option 3
Week 0	1	1	1
Week 2	1.05	1.1	1.1
Week 2	1.1	1.21	1.2
Week 3	1.15	1.33	
Week 4	1.2	1.46	1.4
Week 5	1.25	1.61	1.5
Week 6	1.3	1.77	1.6
Week 7	1.35	1.95	1.7
Week 8	1.4	2.14	1.8
Week 9	1.45	2.36	1.9
Week 10	1.5	2.59	2



# The Case of the Vase

## Common Core Standard:

### Use functions to model relationships between quantities

**8.F.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

## Additional/Supporting Standard(s):

8.F.1, 8.F.2 Define, evaluate and compare functions.

## Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
8. Look for and express regularity in repeated reasoning

## Student Outcomes:

- I can distinguish between linear and nonlinear functions given a table and graph.
- I can create a table and graph to represent a real-life situation.
- I can use tables and graphs of functions to describe real-world situations.

## Materials:

- Enough vases of different sizes for each group of four students to have at least one vase. Half of the vases should be cylinders or have straight sides perpendicular to the base (which will create a linear function/graph); the other half should not (nonlinear function/graph)
- Graduated cylinders, one for each group
- Pitcher of water for each group
- Food coloring (not required, but colored water is easier to measure)
- Centimeter rulers
- Handout
- Materials to clean up spills

## Advance Preparation:

- Students should have had prior experience with creating function tables, graphing functions, and determining if the function is linear or nonlinear
- Determine student assignments for groups
- Make three copies of the station handout for each group
- Prepare the materials listed above
- Set up stations, making sure to alternate types of vases so that every other station has a vase that produces a linear function

**Directions:**

Students will compare linear and nonlinear functions that are created through an investigation involving volume and height of water in vases.

1. Distribute handouts to each group.
2. For the initial station, guide students through the process of adding water to their vases. Each group will add water to their vase in 25ml increments and measure the height of the water in cm (to the nearest tenth) each time. If the vase is small you may need to adjust the amount of water to add each time.
3. Be sure that students follow the guidelines for measuring the height of the water. Measurements should be taken from the *outside* of the vase, and should include the thickness of the glass base.
4. Students should continue adding water until their vase is as full as possible. Note that the number of times water is added will vary with the size of the vase.
5. Students should record their data in a table each time that they repeat this procedure.
6. Students should then graph the resulting data (ml of water, height of water line) and complete the handout tasks.
7. Allow time for groups to complete three stations, leaving time for class discussion.
8. After the third stations, groups will present their findings from the third station. Use the questions below (in the *After* section) to guide discussion.

**Questions to Pose:**During:

- How did you decide what to use as the independent and dependent variables?
- Should the points on the graph be connected?
- Is there a relationship between the shape of the vase and the graph it produces?

After:

- What are the characteristics of vases that produce linear functions? Nonlinear functions? Explain the connections.
- Looking at the vases that produced linear functions, what attributes affected the rate of change?
- How can you use the graph to predict the height of the waterline after adding 25 ml of water 10 times? 20 times? (This question may need to be modified depending on the volume of the vase.)
- By looking only at the linear graphs, would it be possible to put the corresponding vases in order of volume? (No, because the graph only shows rate of change.)
- Why were there no horizontal lines in the graphs? What would a horizontal line mean? (As you add water there is no change in the height of the water.) What could you do to make that happen? (If there is a leak in the vase and the water leaves at the same rate that it is poured into the vase.)
- How could you change the task so that the graph has a negative slope? (Start with water in the vase and pour it out.)

**Possible Misconceptions/Suggestions:**

<b>Misconception</b>	<b>Suggestion</b>
Students may record the amount of water they add each time rather than the cumulative volume of water.	Monitor groups as they work and use questioning to correct their mistakes.
Students may inaccurately predict that the shape of their graph will match the shape of the vase.	Do not correct this misconception at the beginning of the task. Allow students to discover the relationship between the volume and the height of the water as they work.
Students may struggle with creating their graphs from scratch.	Watch for issues with determining appropriate scales for the axes and appropriate labeling.
Students may not account for the leading edge of ruler when taking their measurements.	This is okay as long as all measurements are taken the same way.

**Special Notes:**

In the group discussion at the end of the task, students should come to the conclusion that straight-sided vases (with consistent horizontal cross-sections) will produce linear functions.

## Possible extensions:

- Ask students to sketch a vase that would produce a linear graph and another vase that would produce a nonlinear graph.
- Show students vases different from the ones used in the group task and ask them to sketch the graph if water was poured in at a constant rate.
- Write a function rule for the height of the water as the volume of water increases. (Note that the thickness of the base of the vase will affect the y-intercept.)

**Solutions:**

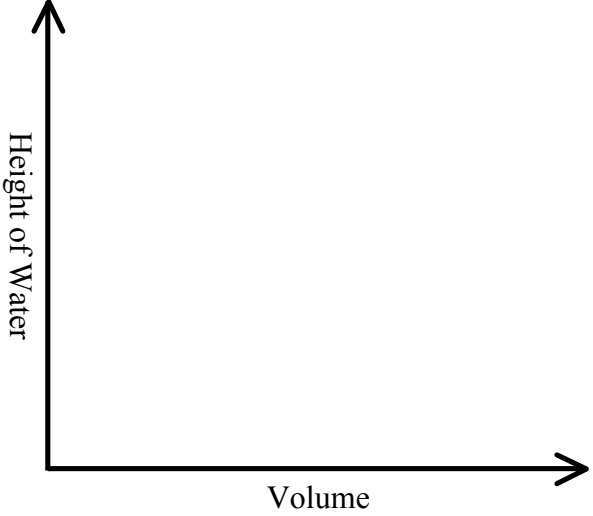
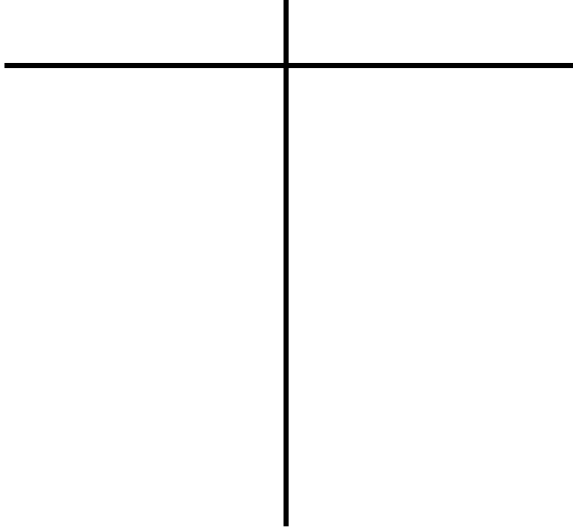
Student work will vary depending on the vases used.

Adapted from *Functions and the Volume of Vases*, Mathematics Teaching in the Middle School, May 2012

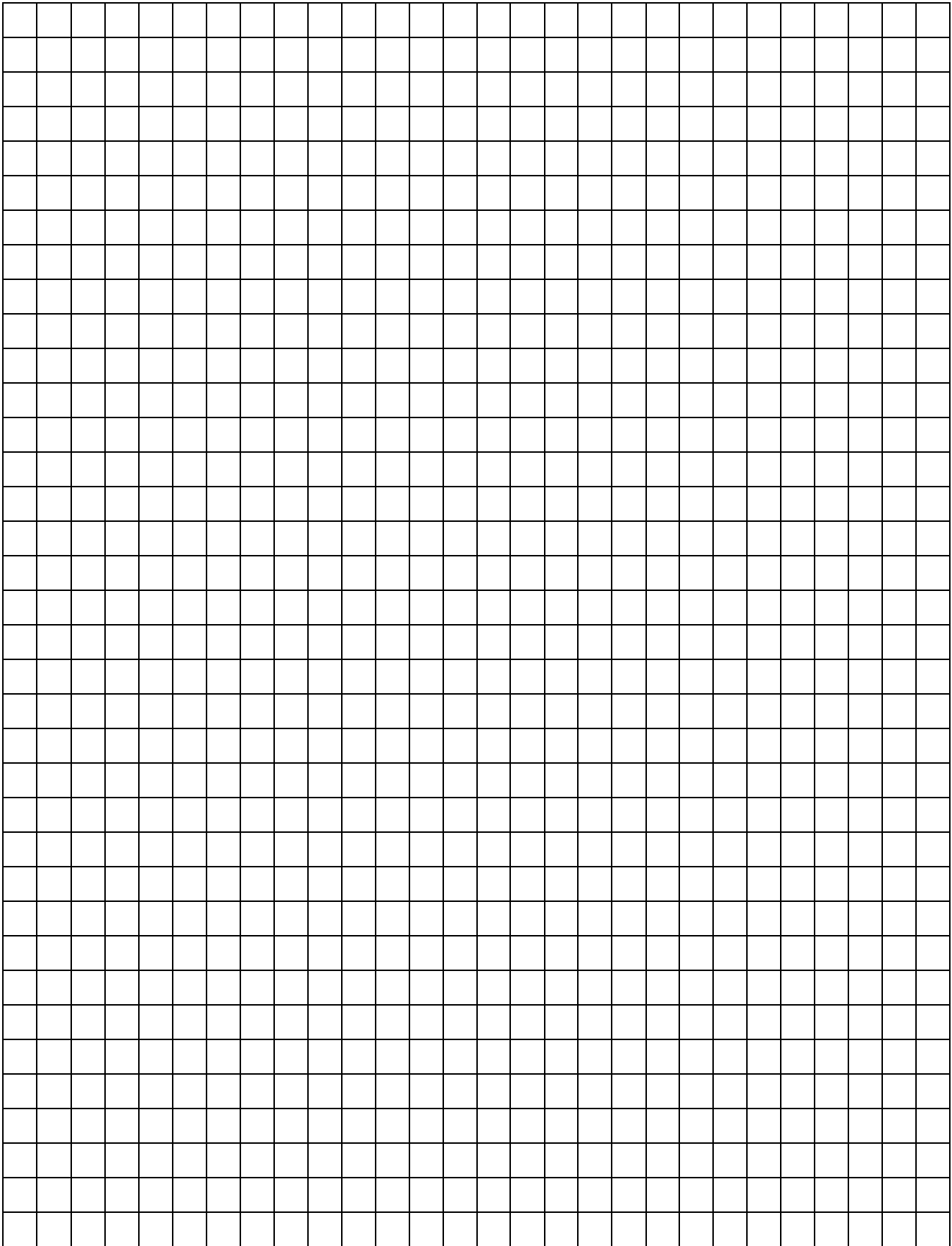
# The Case of the Vase

## STATION DIRECTIONS

- 1) Sketch a picture of the vase
- 2) Sketch a graph that represents how you think the height of the water will change as water is poured in
- 3) Add water to the vase in 25 ml increments
- 4) Use a ruler on the outside of the vase to measure the height of the water to the nearest tenth of a cm
- 5) Record your data in a table
- 6) Graph the data on grid paper. Be sure to label appropriately.
- 7) Verbally describe how your prediction graph compares with the actual graph

<b>Sketch the Vase</b>          	<b>Sketch of the graph (your prediction)</b> 
<b>Data Table</b> 	<b>Describe how the actual graph compares to your prediction.</b>          

Adapted from *Functions and the Volume of Vases*, Mathematics Teaching in the Middle School, May 2012



# Gift Box Dilemma

## Common Core Standard:

**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**

8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

## Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.

## Student Outcomes:

- I can use the volume of cylinders to solve problems.
- I can use the volume of rectangular prisms to solve problems.
- I can use the surface area of rectangular prisms to solve problems.
- I can find the appropriate size box to hold a cylinder with a given radius and height.
- I can reason about the arrangement of cylinders and their effects on the volume and surface area of the boxes that will hold them.

## Materials:

- Cylindrical candle (optional... used as a visual for the students)
- Foam packing peanuts (optional... used as a visual for students who may not be familiar with them)
- Flat cardboard or poster board

## Advance Preparation:

- Students should be familiar with how to find the surface area of rectangular prism from 6<sup>th</sup> grade. They will also need to have experience working with volume of rectangular prisms and cylinders.

## Directions:

1. Read the Gift Box Dilemma aloud with the students.
2. Model the scenario with a cylindrical candle and a piece of poster board that you want to use to create a box that will hold the candle. It will be helpful if you have a cut-out net of the box you will create to show the students how the box will be designed and taped together to hold the candle.
3. Allow the students to work in pairs to complete the task.

## Questions to Pose:

Before:

- Have you ever wrapped a present?
- What does the amount of wrapping paper needed tell us about the box?
- How could we determine how much a box holds?
- Have you ever made your own box? Did you use a template (net)?

During:

- How do you know you have designed the smallest possible box for the candle?
- Would there be other size boxes that are possible to design that would hold the candle?
- What effect would the box size have on the amount of foam packing peanuts needed?

After:

- Could there be other arrangements of the four candles? Why do you think Sam only choose to consider the two arrangements shown?
- Why do you believe that doubling the candle radius resulted in a greater change in volume than doubling its height?

**Possible Misconceptions/Suggestions:**

<b>Possible Misconceptions</b>	<b>Suggestions</b>
Students may assume that doubling the radius of the candle will result in its volume being doubled (as it does when the height is doubled).	A visual is often helpful: Using soup cans, show how simply stacking the cans will result in twice the volume. However, using a wider can with the same height, students will visually observe that it obviously holds more than double the amount.

**Solutions:**

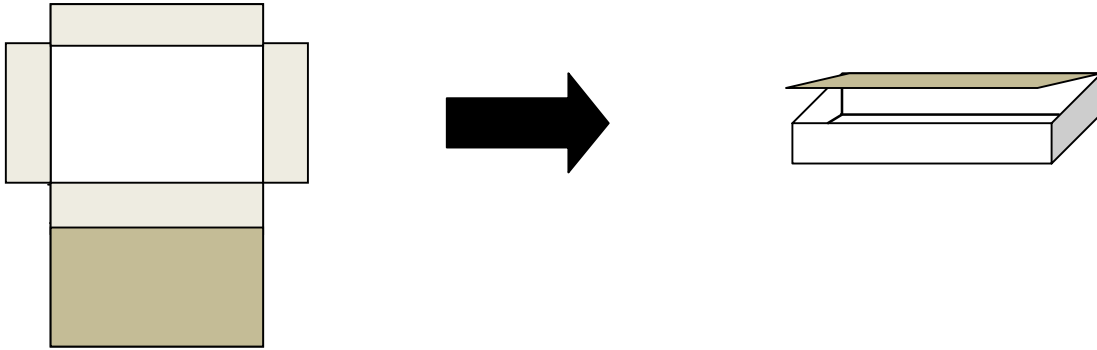
Attached on separate paper.



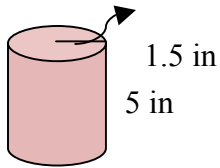
# Gift Box Dilemma

Sam just purchased a candle to give to his mom as a birthday gift. Unfortunately, he has no box to put it in. Sam knows that he can make a box using tape and cardboard.

If he cuts the cardboard as shown below, he can fold and tape to make this box.



A) Sam's candle is 5 inches tall and has a radius of 1.5 inches. Draw a plan for his box below.



B) What is the volume of the smallest box Sam could build to hold the candle?  
Show the mathematics you used to determine your answer.

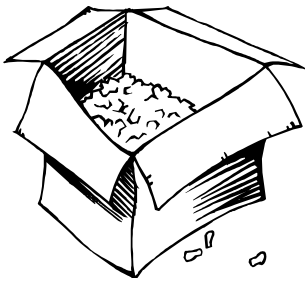
C) What is the surface area for this box? Show the mathematics you used to determine your answer.

D) Sam wants to purchase some velvet coated cardboard to use for his box. The cardboard is sold at a craft store for 5 cents per square inch. What is the minimum Sam will have to pay for the cardboard? Show your work.



E) Approximately how much extra space will there be in the box that could be filled with foam packing peanuts? Show your work.

F) If Sam finds the foam packing peanuts at a local office supply store for \$0.02 per cubic inch, what is the minimum he must pay for the foam packing peanuts? Show your work.



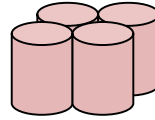
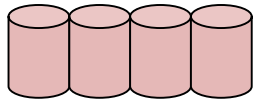
G) If Sam finds a candle twice as tall, how will the volume of the candle change?

- How will this larger candle affect the surface area of the box needed to hold it?
- How will this larger candle affect the volume of the box needed to hold it?

H) If Sam finds a candle with twice the radius and the same height as the original, how will the volume of the candle change?

- How will this larger candle affect the surface area of the box?
- How will the larger candle affect the volume of the box?

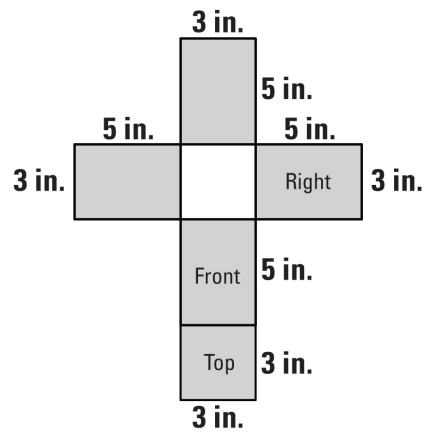
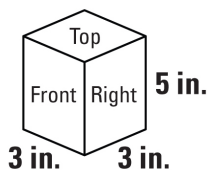
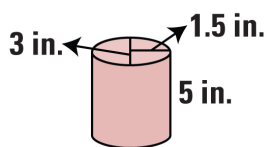
- I) Sam decides to give his mom a set of four matching candles with the original dimensions. He can pack them in either of the ways shown below.



- Calculate the surface area of the boxes needed to fit each configuration. Draw a plan for each of the boxes and label its dimensions.
- Will the candle arrangements make a difference as to how much of the foam packing peanuts will be needed? Justify your reasoning.
- Summarize your conclusions about how the arrangements of the candles affect the volume and surface area of the boxes needed to hold them.

# Gift Box Dilemma Answer Key

- A) Sam's candle is 5 inches tall and has a radius of 1.5 inches. Draw a plan for his box below.



- B) What is the volume of the smallest box Sam could build to hold the candle? Show the mathematics you used to determine your answer.

$$\begin{aligned}
 V &= Bh \\
 V &= (3 \cdot 3)5 \\
 V &= 9 \cdot 5 \\
 V &= 45 \text{ in.}^3
 \end{aligned}$$

- C) What is the surface area for this box? Show the mathematics you used to determine your answer.

$$\begin{aligned}
 SA &= \text{Top Area} \cdot 2 + \text{Front Area} \cdot 2 + \text{Right Side Area} \cdot 2 \\
 SA &= (3 \cdot 3) \cdot 2 + (3 \cdot 5) \cdot 2 + (3 \cdot 5) \cdot 2 \\
 SA &= 9 \cdot 2 + 15 \cdot 2 + 15 \cdot 2 \\
 SA &= 18 + 30 + 30 \\
 SA &= 18 + 60 \\
 SA &= 78 \text{ in.}^2
 \end{aligned}$$

- D) Sam wants to purchase some velvet coated cardboard to use for his box. The cardboard is sold at a craft store for 5 cents per square inch. What is the minimum Sam will have to pay for the cardboard? Show your work.

$$\begin{aligned}
 &\$0.05 \text{ per sq inch} \qquad 78 \times 0.05 = \$3.90 \text{ for velvet cardboard}
 \end{aligned}$$

- E) Approximately how much extra space will there be in the box that could be filled with foam packing peanuts? Show your work.

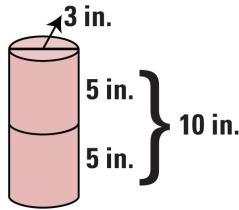
$$\begin{aligned}
 V_{\text{candle}} &= \pi r^2 h \\
 V &= \pi \cdot 1.5^2 \cdot 5 \\
 V &= 11.25 \pi \\
 V &= 35.325 \text{ in.}^3 \\
 V_{\text{box}} - V_{\text{candle}} &= V_{\text{extra space}} \\
 45 \text{ in.}^3 - 35.325 \text{ in.}^3 &= 9.675 \text{ in.}^3 \text{ extra space}
 \end{aligned}$$

- F) If Sam finds the foam packing peanuts at a local office supply store for \$0.02 per cubic inch, what is the minimum he must pay for the foam packing peanuts? Show your work.

$$9.675 \text{ in.}^3 \times .02 = .1935$$

$$\approx 0.20 \text{ for foam packing peanuts}$$

- G) If Sam finds a candle twice as tall, how will the volume of the candle change?



$$V = \pi r^2 h$$

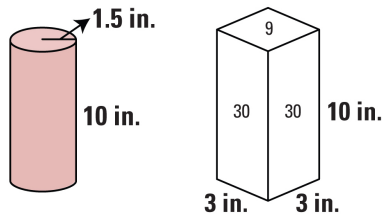
$$V = \pi (1.5)^2 10$$

$$V = \pi (2.25) 10$$

$$V = 70.65 \text{ in.}^3$$

Volume of new candle is twice as much as original

- How will this larger candle affect the surface area of the box needed to hold it?



$$SA \text{ new box} = (9 \cdot 2) + 30(4)$$

$$SA = 18 + 120$$

$$SA = 138 \text{ in.}^2$$

Surface Area increases by  $60 \text{ in.}^2$

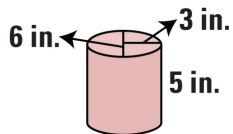
- How will this larger candle affect the volume of the box needed to hold it?

$$V \text{ new box} = (3 \cdot 3) 10$$

$$V = 90 \text{ in.}^3$$

Volume of box doubles with a candle twice as tall.

- H) If Sam finds a candle with twice the radius and the same height as the original, how will the volume of the candle change?



$$V = \pi r^2 h$$

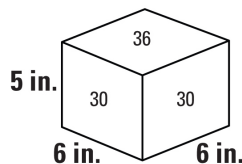
$$V = \pi 3^2 5$$

$$V = 141.3 \text{ in.}^3$$

New candle has a volume 4 times greater than original

- How will this larger candle affect the surface area of the box?

New Box



$$SA \text{ new box} = 36 \cdot 2 + 30 \cdot 4$$

$$SA = 72 + 120$$

$$SA = 192 \text{ in.}^2$$

The surface area increases by  $114 \text{ in.}^2$  when compared to original box.

- How will the larger candle affect the volume of the box?

$$V \text{ new box} = (6 \cdot 6) 5$$

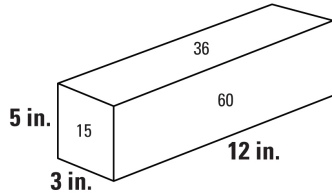
$$V = 36 \cdot 5$$

$$V = 180 \text{ in.}^3$$

The volume of new box is 4 times the volume of original.

I) Sam decides to give his mom a set of four matching candles with the original dimensions. He can pack them in either of the ways shown below.

- Calculate the surface area of the boxes needed to fit each configuration. Draw a plan for each of the boxes and label its dimensions.

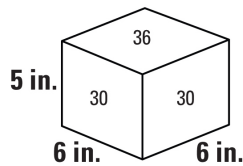


$$SA = 36 \cdot 2 + 15 \cdot 2 + 60 \cdot 2$$

$$SA = 72 + 30 + 120$$

$$SA = 72 + 150$$

$$SA = 222 \text{ in.}^2$$



$$SA = 36 \cdot 2 + 30(4)$$

$$SA = 72 + 120$$

$$SA = 192 \text{ in.}^2$$

- Will the candle arrangements make a difference as to how much of the foam packing peanuts will be needed? Justify your reasoning.

Need to find amount of extra space in each arrangement by subtracting volume of 4 candles from volume of each box.

$$V_1 \text{ candle} = 35.325 \text{ in.}^3$$

$$V_4 \text{ candles} = 141.3 \text{ in.}^3$$

$$V \text{ long box} = (3 \cdot 12) 5$$

$$V \text{ long box} = 36 \cdot 5$$

$$V \text{ long box} = 180 \text{ in.}^3$$

$$V \text{ square like box} = (6 \cdot 6) 5$$

$$V \text{ square like box} = 180 \text{ in.}^3$$

$$\text{Extra space: } 180 \text{ in.}^3 - 141.3 \text{ in.}^3 = 38.7 \text{ in.}^3$$

$$\text{Extra Space: } 180 \text{ in.}^3 - 141.3 \text{ in.}^3 = 38.7 \text{ in.}^3$$

The amount of extra space is the same in each box for the two different arrangements. Thus, the same amount of foam packing peanuts is required for both boxes.

- Summarize your conclusions about how the arrangements of the candles affect the volume and surface area of the boxes needed to hold them.

The arrangement of the four candles has no effect on the volume for the boxes needed to hold them. The surface area does vary based on the arrangements. (The longer the arrangement – the larger the surface area. The more square-like the arrangement – the smaller the surface area.)

# Meltdown

## Common Core Standard:

**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**

8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

## Additional/Supporting Standard(s):

## Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.

## Student Outcomes:

- I can use the formulas for finding volumes of cones and spheres to solve real-world mathematical problems.
- I can justify my reasoning for determining the height of a cone with a given volume.

## Materials:

- Meltdown student task

## Advance Preparation:

- Students should have an understanding of the volume formulas for cones and spheres.

## Directions:

1. Show students the video clip of a melting ice cream cone (<http://www.youtube.com/watch?v=BDnu8Eru6QE>) to focus their attention for the lesson.
2. Provide students with a copy of the Meltdown task and read the scenario aloud.
3. Instruct students to work in pairs on the task and make sure they provide justification for their reasoning.

## Questions to Pose:

### During:

- How do we take into account that the scoops can range in their diameter length?
- How might you find the height of a cone given its volume?

### After:

- Why is there a range of suitable heights for your single scoop cones?
- Is there a way we could rewrite the volume formula of a cone so that we can easily determine its height?
- How can we express the resulting minimum and maximum cone heights in terms of an inequality? (i.e. minimum height  $\leq h \leq$  maximum height)

**Possible Misconceptions/Suggestions:**

Possible Misconceptions	Suggestions
Students may struggle with determining the height of their cones as a whole number if they try to manipulate the formulas using the decimal representation of pi.	Allow students to manipulate the formulas by working in terms of pi (using the pi symbols).
Students may be uncertain as to how to address the range in diameter lengths of the ice cream scoops.	Have the students think in terms of the minimum and maximum diameter lengths and thus the cone will have a minimum and maximum height.

**Special Notes:**

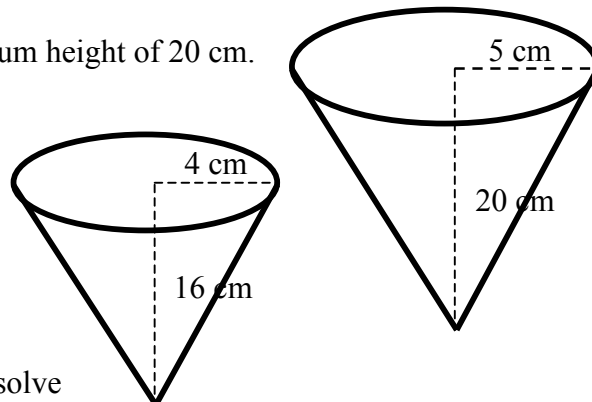
Encourage the students to express their results in terms of a compound inequality since there is a range of cone heights that would meet Izzy’s criteria.

**Solutions:**

The cone will have a minimum height of 16 cm and a maximum height of 20 cm.

Volume of ice cream scoops with diameters of 8 and 10:

$$\begin{aligned}
 V &= (4/3)\pi r^3 & V &= (4/3)\pi r^3 \\
 V &= (4/3)\pi (4^3) & V &= (4/3)\pi (5^3) \\
 V &= (4/3)\pi(64) & V &= (4/3)\pi(125) \\
 V &= (256/3)\pi & V &= (500/3)\pi
 \end{aligned}$$



Set the scoop (sphere) volume equal to the cone volume and solve for the missing height.

<p>8 cm scoop needs a cone with a height of <u>16 cm</u>.</p> $  \begin{aligned}  (256/3)\pi &= (1/3)\pi(4^2)h \\  (256/3)\pi &= (1/3)\pi(16)h \\  \underline{256\pi} &= \underline{16\pi h} \\  16\pi & \quad 16\pi \\  16 &= h  \end{aligned}  $	<p>10 cm scoop needs a cone with a height of <u>20 cm</u>.</p> $  \begin{aligned}  (500/3)\pi &= (1/3)\pi(5^2)h \\  (500/3)\pi &= (1/3)\pi(25)h \\  \underline{500\pi} &= \underline{25\pi h} \\  25\pi & \quad 25\pi \\  20 &= h  \end{aligned}  $
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The same process works for double and triple scooped ice cream cones.

When there are two scoops of ice cream with a diameter of 8 cm each, the combined volume of the scoops are  $(512/3)\pi$ . Since the diameters of the scoops are still 8 cm and the scoops are stacked, the diameter of the cone will remain 8 cm. Thus, the height of the cone needs to be 32 cm tall.

When there are two scoops of ice cream with a diameter of 10 cm each, the combined volume of the scoops are  $(1000/3)\pi$ . Since the diameters of the scoops remain 10 cm and the scoops are stacked, the diameter of the cone will remain 10 cm. Thus, the height of the cone needs to be 40 cm.

Triple scooped cones with a diameter of 8 cm must be 48 cm tall.  
 Triple scooped cones with a diameter of 10 cm must be 60 cm tall.



Students should summarize their observations by stating that as the volume of the scoops are doubled, the cones must double in height. As the volume of the scoops are tripled, the cones height must be tripled as well.

Findings can be expressed as inequalities:

Single scoop cones must range in height from 16 to 20 cm tall. Or,  $16 \text{ cm} \leq h \leq 20 \text{ cm}$ .

Double scoop cones must range in height from 32 to 40 cm tall. Or,  $32 \text{ cm} \leq h \leq 40 \text{ cm}$ .

Triple scoop cones must range in height from 48 to 60 cm tall. Or,  $48 \text{ cm} \leq h \leq 60 \text{ cm}$ .

Adapted from TAP Math

