RVRFLO

A hydraulic simulator of water quality
in the Housatonic River in Connecticut

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I — INTRODUCTION

The Housatonic River in western Connecticut has long received pollutants from municipal and industrial discharges, as well as runoff from forests and fields. As the river was impounded, pollutants accumulated and water quality was impaired. For example, Lakes Lillinonah and Zoar on the Housatonic River detain plant nutrients so that they have become highly eutrophic. Accumulation of polychlorinated biphenyls (PCBs) in the fish and sediments of these lakes has further reduced their recreational value. Effective management of water quality in these lakes requires a thorough knowledge of the fate of pollutants once they are introduced into the river. This requires that we first understand the movement of water, including its ability to transport and to dilute solutes.

The purpose of this study is to develop a computer simulation model for the transport of chemicals in the Housatonic River between Bulls Bridge, CT and Stevenson Dam, CT. The model, called RVRFLO, is based on the physical principles of hydraulics and uses measurements of water velocity, temperature and solute concentration at selected sites on the river as well as weekly mean flow and concentration data for major tributaries. RVRFLO is written in Fortran and utilizes many acronyms, which we shall introduce as we proceed. With RVRFLO, we can calculate changes in the quality of a volume of water that traverses the length of the reservoirs. Our goal is to predict seasonal losses or gains of nutrients or other pollutants by the lakes. Having now stated the promise of our labors, we embark at once upon the details that will give our model substance and merit.

Our study begins with a detailed analysis of the geometry of the 33.5 mile stretch of the Housatonic River from Bulls Bridge to Stevenson Dam. Section II locates and describes the river and its impoundments, contains mathematical functions for the cross-sectional area of the channel, and describes our treatment of the storage volume and change in content of the reservoirs. Section III contains a calculated water balance and examines possible sources of error. Important elements included in the water balance are ungaged flow, time of travel of a hydraulic signal carrying information about flow, and changes in storage in the reservoirs.

The model, developed in Section IV, begins with a discussion of the one-dimensional advective dispersion equation. The seasonal variation of flow in the Housatonic River is large, but for much of the year advection is much more important for determining concentration than is dispersion. Thus, we obtained an initial estimate of concentration by first neglecting dispersion. Later, this estimate is corrected by introducing an empirical dispersion coefficient. Finally, the computer algorithm is described.

The model is tested in Section V by comparison of predicted and observed concentrations of the conservative element Cl. The model is then used to estimate the attenuation of the non-conservative elements P and N. The attenuation of these non-conservative elements is compared with measured accumulation in lake sediments and with observations in other lakes and reservoirs.

II — GEOMETRY

A. Location and Description

The Housatonic River drains about 1232 square miles in western Connecticut, or approximately 25% of the area of the state (Fig. II-1). The river arises near Pittsfield, Massachusetts and flows south 132 miles to Long Island Sound. The drainage area in Massachusetts is about 497 square miles, with another 217 square miles in New York. Smith (1946) gives a highly readable account of the

Fig. II-1. The Housatonic River with a combined watershed of 1946 square miles arises near Pittsfield, Massachusetts, flows southerly through Connecticut and empties into Long Island Sound at Stratford. The area of study, indicated by the circle, is bounded on the north by Bulls Bridge and on the south by Stevenson Dam and drains an area of approximately 750 square miles.

river valley since the arrival of the Mohicans in the first half of the 17th Century.

Like many other rivers in New England, the Housatonic has long been impounded for water supplies, water power, and, more recently, for hydroelectric power. There are 13 dams along 70 river miles in Massachusetts where the river drops nearly 900 feet from its headwaters to the Connecticut state line. This portion of the river is described in reports of the Massachusetts Water Resources Commission (1974, Parts A, B, C).

As the river enters Connecticut, it encounters its first impoundment, Falls Village Reservoir, constructed in 1914 and used for hydroelectric power by the Hartford Electric Light Company. This is followed by an impoundment at Bulls Bridge, constructed for hydroelectric power in 1903 by the Connecticut Light and Power Co. (CL&P). These two plants employ a small dam located some distance upstream from the actual power plant, with the water conveyed to the turbines via an aqueduct. In this way, a substantial head can be maintained without the large dams typical of many hydroelectric plants.

The impoundment at Bulls Bridge marks the upper boundary of the section of the river under study; we define its location as river mile 0.0 (Table II-1). Proceeding downstream, the Ten Mile River enters at mile 0.9 and marks the end of the first of nine reaches, defined by distance between major tributaries. The United States Geological Survey (USGS) gaging station at Gaylords-

ville is located just below this confluence, allowing us to calculate the flow at Bulls Bridge by difference. At about 8.9 miles, CL&P constructed Lake Candlewood by impounding the Rocky River before it reaches the Housatonic. This reservoir, with a surface elevation of 429 feet and an area of 5,420 acres, impounds water for generation of power at the Rocky River Station. Some of this water is pumped from the Housatonic River. Constructed in 1928, Candlewood is believed to be the first pumped storage reservoir in the United States. We have arbitrarily separated the pumping from the generation (Table II-1) for ease of computing, but in reality they are at the same physical location. Although in principle this artificial separation is not required, we prefer to maintain it to help us think of these stations as two tributaries. The pumping, of course, is out of the river; hence, its flow is negative.

The next major tributary, the West Aspetuck River, enters at mile 9.7 and the river continues to the backwater of Lake Lillinonah at mile 10.9 at the Robertson Bleachery Company Dam. Lake Lillinonah, with an approximate surface elevation of 198 feet and a surface area of 1900 acres, was created by construction of the Shepaug Dam in 1955 and is used for hydroelectric power by CL&P and for recreation. This reach ends at the confluence with the Still River (mile 12.5) at the site of the former Silex Mill. Continuing down the river, the next confluence is at mile 21.3 at the flooded mouth of the Shepaug River. Lake Lillinonah widens at this point and forms the Shepaug Arm, with the Shepaug River actually entering some 3.25 miles upstream. The Housatonic River continues to the Shepaug Dam at mile 23.5, with a normal surface elevation of 198 feet (USGS) above mean sea level.

This marks the backwater of Lake Zoar, with a surface elevation of about 98 feet above mean sea level and a surface area of 975 acres. Zoar, impounded in 1919 by

Table II-1. Distance in river miles of various geographic features of the Housatonic River.

End of Reach	Feature	River Mile	Drainage Area (sq. mi.)
—	Bulls Bridge Reservoir	0.0	791.0
1	Ten Mile River	0.9	203.0
	Gaylordsville Gaging Station (USGS mile 50.6)	2.3	994.0
2	Candlewood Pump	8.85	—
3	Candlewood Generate	8.95	40.7
4	West Aspetuck River	9.7	23.3
	Backwater Lake Lillinonah	10.9	—
	Bleachery Dam	10.9	—
5	Still River	12.5	69.8
6	Shepaug River	21.3	133.0
	Shepaug Dam (Lake Lillinonah)	23.5	—
	Backwater Lake Zoar	23.5	—
7	Pootatuck River	25.8	24.2
8	Pomperaug River	26.9	75.3
9	Stevenson Dam (Lake Zoar)	33.5	—
	Stevenson Gaging Station (USGS mile 19.2)	33.7	1545.0

CL&P, is used for hydroelectric power and for recreation. The next confluence is with the Pootatuck River at mile 25.8, and the next is at mile 26.9 with the Pomperaug River. The river continues to mile 33.5 at Stevenson Dam. Figure II-2 shows the monthly mean flow at Stevenson Dam for the 10 water years (October 1 to September 30) 1966-1975. The variability from month to month and from year to year is great, reflecting the variability of our New England weather. Although the largest flows generally occur during melting of snow and spring runoff, the highest daily flow recorded at Stevenson Dam was 75,800 cubic feet per second (cfs) on October 16, 1955 during a disastrous flood.

The USGS gaging station on the river is 0.2 mile downstream from this dam, where the river surface is about 25 feet above sea level. We use this station as well as data from CL&P for flow at the downstream end of the section of the river selected for study. More detail concerning the upper Housatonic River Basin is contained in reports by Cervione, et al. (1972), while the section from the Shepaug Dam to Long Island Sound is described by Wilson, et al. (1974).

A sketch of the river section under study (Fig. II-3) shows the elevation of the river surface versus river miles. The total drop is about 330 feet; much is harnessed by the generating stations at Bulls Bridge, Shepaug Dam and Stevenson Dam. Bleachery Dam below the West Aspetuck, with an approximate elevation of 198 feet, may impound some water, but its effect has been ignored because it has been breached. The approximate size and location of the penstocks are also shown since these could affect drawdown during the summer when the reservoirs became thermally stratified. The effect of thermal stratification is discussed in Section IV.B.1.
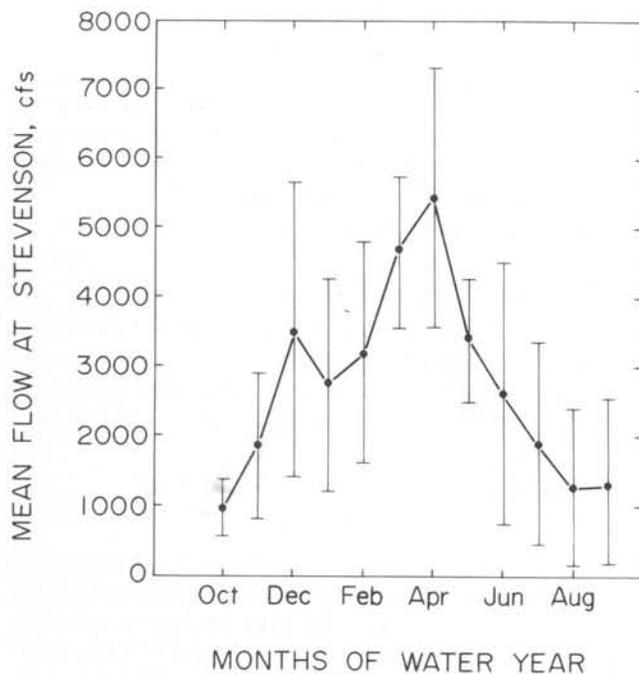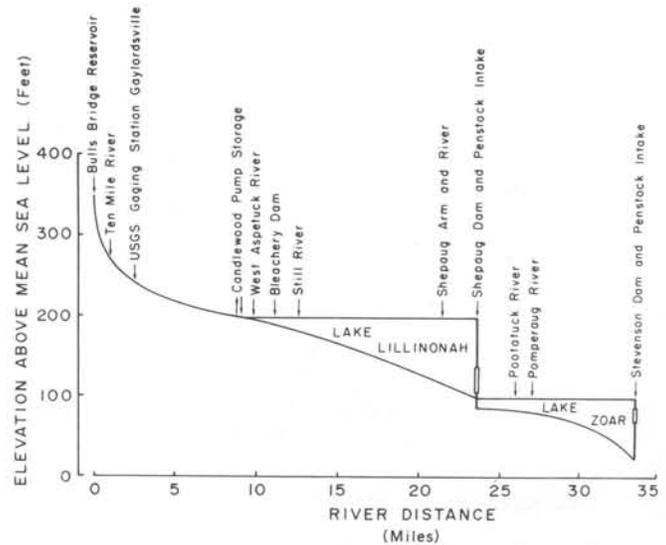

Fig. II-3. The elevation of the river bed and the water surface for the section of the Housatonic River being studied showing the major tributaries and dams.

B. Channel Cross-Sectional Area

In the model that we will develop, we need to determine the average speed of water moving through the river system. This speed is obtained from the flow (cfs) and the cross-sectional area (ft^2) through which the water moves. In this section we estimate these areas.

Separate area functions were derived for three sections of the river, namely the river itself from $x = 0.0$ to $x = 10.9$, for Lake Lillinonah for $x = 10.9$ to $x = 23.5$ and for Lake Zoar for $x = 23.5$ to $x = 33.5$ miles. In addition, an area function was determined for the 3.25-mile-long Shepaug Arm at the mouth of the Shepaug River.

The cross-sectional area of the river from $x = 0.0$ to $x = 10.9$ was assumed to be the same as the control area at the USGS gaging station at Gaylordsville. While this is a simplification, the time of travel in this section of the river is short relative to the time of travel in the impoundments. Hence, this approximation introduces little error. More precise information on this section of the river could be readily obtained from additional measurements in the field. The USGS supplied their calibration curve of gage height versus cross-sectional area at Gaylordsville, and a table of flow versus gage height. From these data we derived a relationship between flow and cross-sectional area by regression analysis. For 17 pairs of data points with areas between 240 and 2230 ft^2 and flows between 200 and 10,600 cfs, the function is:

$$A = 18.4 * RQ ** 0.52$$

where A = cross-sectional area in ft^2, RQ = flow in cfs at Gaylordsville and $r^2 = 0.99$. The * symbolizes multiplication and the ** exponentiation. The r^2 is the coefficient of determination.

Because bathymetric data for Lake Lillinonah were not available, we measured the width and depth from USGS topographic sheets (Scale 1:24000). We established transects across the lake approximately 1 mile apart and measured the width of the lake at these transects at 10 ft. intervals of depth with vernier calipers. The Shepaug arm

Fig. II-2. Mean monthly flow at Stevenson Dam, CT during 1966-1975. The vertical bars indicate the standard deviation of the monthly means.

was measured similarly, but was treated as a separate body of water in the analyses that follow.

The area of each transect was plotted as a function of river miles; a smooth function was fitted. Although a polynomial of sufficient order will describe any irregularly shaped area, we instead use a smooth and well-behaved exponential function:

$$A = 1000 * A2 * exp(B2 * x)$$

where A is area in ft^2, $A2$ (ft^2) and $B2$ $(mile^{-1})$ are functions of depth, and x is distance in river miles.

The coefficients $A2$ and $B2$ were plotted as a function of depth, and curves were fitted by regression analysis. Because there was a distinct change in slope at HL (surface elevation of Lillinonah) = 170 ft, two sets of equations were necessary. Above 170 ft, the equations are:

$$ln\ A2 = -11.8936 + 0.06268 * HL$$
$$B2 = .5438 - 0.001878 * HL$$

with $r^2 = 0.99$ and 0.66 respectively. Below 170 ft, the corresponding equations are:

$$ln\ A2 = -38.8504 + 0.2205 * HL$$
$$B2 = 1.5323 - 0.007563 * HL$$

with $r^2 = 0.98$ and 0.94 respectively. If the lake surface never goes below 170 ft or if thermal stratification does not cause selective withdrawal, this second set of coefficients of course would not be needed.

The Shepaug Arm was treated in a similar fashion, with somewhat simpler linear functions. The area function is:

$$A = (A4 - B4 * x_s) * 1000$$

where x_s in miles is 0 at the mouth of the Shepaug Arm and 3.25 at the confluence with the Shepaug River. The coefficients are:

$$A4 = -195.10 + 1.235 * HL$$
$$B4 = -50.80 + 0.325 * HL$$

with $r^2 = 0.995$ and 0.996 respectively.

The volume of the lake was calculated from these expressions at full pond and at various elevations below full pond and compared with data obtained from CL&P in Table II-2. After correcting for the difference in datum (1.73 ft) we estimate the volume at full pond to be $2.610 \times 10^9\ ft^3$ in the main arm and $0.474 \times 10^9\ ft^3$ in the Shepaug Arm for a total of $3.084 \times 10^9\ ft^3$. This is only 3.7% less than the volume reported by CL&P, which we consider good agreement. Of equal importance, our calculated change in volume with change in height is in good agreement with that obtained from Table II-2.

Bathymetric data were available for Lake Zoar from publications of the Conn. State Board of Fisheries and Game (1959). Twelve transects were used to construct a plot of cross-sectional area versus river miles, and this was fitted by the regression equation:

$$A = 1000 * A3 * exp(B3 * x).$$

For the twelve stations with surface elevations HZ (CL&P datum) from 40 to 110 ft, the average r^2 for this equation was 0.82, an improvement over our prediction for Lake Lillinonah. Again, two sets of equations are required. Above 80 ft they are:

$$ln\ A3 = -56.3500 + 0.4898 * HZ$$
$$B3 = 1.7366 - 0.01391 * HZ$$

with $r^2 = 0.999$ for both equations. Below HZ = 80, the equations are:

$$ln\ A3 = -66.6098 + 0.5753 * HZ$$
$$B3 = 1.9791 - 0.01559 * HZ$$

with $r^2 = 0.85$ and 0.89 respectively. Again, if the water surface never falls below 80 ft or if thermal stratification does not cause selective withdrawal, this second set of coefficients would not be needed.

Using these equations, at CL&P elevation 100 ft, we calculated the volume to be $0.994 \times 10^9\ ft^3$, or 3.1% less than reported by CL&P. In the next section we shall use data supplied by CL&P, although our calculated volumes could be used in the absence of more precise information.

C. Storage Volume and Change in Contents

The total volume of water impounded in Lake Lillinonah, according to CL&P, is $3.200 \times 10^9\ ft^3$. In our model we shall divide this into two volumes: 1) the part delineated by the Housatonic river channel that we call Lillinonah proper; and 2) the part delineated by the Shepaug river channel that we call the Shepaug Arm. From our data we estimate that about 15.4% of the total volume or $0.492 \times 10^9\ ft^3$ is in the Shepaug Arm; the remaining 84.6% or $2.708 \times 10^9\ ft^3$ is in Lillinonah proper.

From data supplied by CL&P on storage volume vs. water surface elevation (Table II-2) we derived DVTDH, the change in content for a change in water elevation of the total lake. That is, DVTDH is the slope of the volume vs. elevation curve. Since this slope changes with drawdown over the usual range of 20 ft, we determined the slope as a function of surface elevation HL. In this case:

$$DVTDH = .312 * HL ** 3.65.$$

At full pond, HL = 200 ft (CL&P datum), and thus DVTDH $= .0781 \times 10^9$ (ft^3/ft). This total change in content was also split into two parts: 1) DVLDH for Lillinonah proper; and 2) DVSDH for the Shepaug arm. Using the cross-sectional area of the channel from topographic maps we obtain at full pond:

$$DVLDH = .0658 \times 10^9\ (ft^3/ft)$$
and
$$DVSDH = .0123 \times 10^9\ (ft^3/ft).$$

The total volume of water impounded in Lake Zoar, according to CL&P, is $1.026 \times 10^9\ ft^3$ at full pond (cf. Table II-2). Using the method described above, the change in contents in Zoar with change in water surface elevation is:

$$DVZDH = -62.38 \times 10^6 + 1.057 \times 10^6 * HZ$$

or $0.0433 \times 10^9\ ft^3/ft$ at full pond.

In every case, the change in contents can be converted to an equivalent flow in cfs by dividing by the appropriate time (in seconds) during which the change in reservoir height is observed. Therefore, if the level of Lake Lillinonah dropped 1 ft from full pond in 1 day the *average net* rate of outflow Δ STOR (cfs) for the day due to this change in storage would be:

Table II-2. **Amount of water stored in Lakes Lillinonah and Zoar as a function of lake surface elevation. Data supplied by CL&P.**

LAKE LILLINONAH

Water Surface Elevation Feet (CL&P datum[1])	Total Volume Million Cubic Feet
180.0	1871
181.0	1928
182.0	1985
183.0	2044
184.0	2103
185.0	2163
186.0	2223
187.0	2285
188.0	2348
189.0	2412
190.0	2478
191.0	2544
192.0	2611
193.0	2680
194.0	2749
195.0	2819
196.0	2891
197.0	2964
198.0	3041
199.0	3119
200.0 (Full Pond)	3200

LAKE ZOAR

Feet (CL&P datum[2])	Million Cubic Feet
90.0	650
95.0	821
96.0	859
97.0	899
98.0	940
99.0	982
100.0 (Full Pond)	1025
101.0	1070
102.0	1115
103.0 (Top of Flashboards)	1160
104.0	1210
105.0	1255

[1]To obtain elevations on USGS mean sea level datum subtract 1.73 ft from elevations given on CL&P datum for Lake Lillinonah.

[2]To obtain elevations on USGS mean sea level datum subtract 1.70 ft from elevations given on CL&P datum for Lake Zoar.

$$\Delta STOR \text{ (cfs)} = 0.0781 \times 10^9 / (24 * 3600) = 904 \text{ cfs.}$$

Similarly, if the level of Lake Zoar dropped 1 ft from full pond in 1 day, the *average net* rate of outflow (cfs) for the day would be 501 cfs.

The change in storage that we computed above is due to the change in elevation along the entire length of the reservoir. A simple balance of inflow, outflow, and change in storage for an idealized rectangular box is shown in Fig. II-4. Writing a mass balance in terms of this notation, we can compute the outflow OUT(X) at any cross section along the length of the box as affected by the change in contents, using inflow at the backwater IN(0) as reference:

$$OUT(X) = IN(0) - \Delta STOR(X)$$

where $\Delta STOR(X) = w \cdot (dH/dt) \cdot X.$

Here, IN(0) is the total upstream flow, H is the height of the lake surface and X is distance measured downstream

from the end wall of the box. For a rectangular box, the width w is a constant and Δ STOR increases linearly with X. For the actual reservoir, w is a function of X. Again, we used the geometric data to derive the fractional change in contents vs. distance from the backwater X, or, in the case of the Shepaug arm, distance from the confluence with the Housatonic river channel. This fractional change in storage volume with distance is then used to multiply the change in contents due to a change in surface elevation to obtain Δ STOR(X). For Lillinonah proper, and substituting $(x - 10.9)$ for X, we obtain:

$$FRCTXL = 0.2016 * (x - 10.9) ** 0.632 \text{ with } r^2 = 0.995$$

where x is downstream distance in river miles. At the end of the backwater, i.e., $x = 10.9$, FRCTXL = 0. At the dam, $x = 23.5$ and FRCTXL = 1.0. Thus, outflow at the dam equals inflow minus DVTDH * DH where DH is the change in height of the lake surface. For the Shepaug Arm

$$FRCTXS = (48.2 * x_s - 6.63 * x_s ** 2) / 86.62$$

where x_s is measured upstream from the confluence. At $x_s = 0$, FRCTXS = 0 and at $x_s = 3.25$ miles FRCTX = 1.0. Similarly, for Zoar:

$$FRCTXZ = 0.269 * (x - 23.5) ** 0.57$$

with $r^2 = 0.993$ where FRCTXZ equals 1.0 at $x = 33.5$ and equals zero at $x = 23.5$.
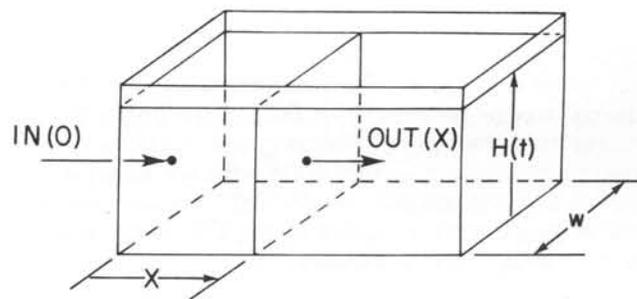
Fig. II-4. The notation used to calculate the change in water storage with distance along the river.

D. Summary

The mathematical functions for cross-sectional area and storage volume derived here, along with measurements of flow, allow us to determine the average rate of water movement from Bulls Bridge to Stevenson Dam. The sensitivity of the model to uncertainties in these functions will be tested in Section V. With this information we now proceed to the determination of a water balance.

III — WATER BALANCE

In this chapter we develop a budget of water entering and leaving the river through the boundaries shown in Fig. III-1. The amount or load of chemical substance passing any point in the river in unit time depends directly on the amount of water flowing in the river, since the load is simply the product of concentration and flow. There-

fore, the ability of our model to predict concentrations depends upon our ability to balance the water budget.

We require a balanced budget on an annual, on a monthly, on a weekly, and even on a daily basis. The balance for shorter periods is important for that section of our model (Section IV) which describes the downstream position of a parcel of water as a function of time. To make a water balance we use: 1) the daily average discharge in cfs gaged by the USGS (1968, 1969) at the eight stations identified on Figure III-1, 2) the daily flow of water pumped or used in generation at Rocky River by CL&P and 3) the water surface elevation each day at 2400 hours at the forebay of both Shepaug and Stevenson dams, again measured by CL&P.

These primary data allow us to calculate the net of inflow, outflow and change in storage and are sufficient to make a water balance. We can ignore, without serious error, storage as ice on the river and lakes, and bank storage. Direct evaporation from and rainfall on the lake is accounted for by daily measurements of water surface elevation. On the other hand, the possibility of bank overflow and temporary storage behind levees during floods could significantly affect a short-term water balance made with these data, and any serious imbalances are examined for this possibility. The assumptions of this paragraph are justified in Section III.E.

A. Ungaged Flow

Of the 551 miles^2 of watershed between the USGS gaging stations at Gaylordsville and Stevenson Dam, only 336.3 miles^2 are measured by gages on tributary streams, leaving the remaining 184.7 miles^2 ungaged. We assume that the runoff from ungaged portions of the watershed is similar to that from adjacent gaged areas with similar topography. Accordingly, we have defined four ungaged areas as shown in Fig. III-1 and assigned them to gaged areas as follows:

Acronym	Ungaged Area	Assigned Gage & Area
GU	30.7	Gaylordsville (GAY) 994
WAU	41.2	West Aspetuck (WA) 23.3
SU	66.3	Shepaug River (SHP) 133
PU	46.5	Pomperaug (PMP) 75.3

There are three other streams with USGS gaging stations. The Marshepaug River is gaged near Milton, CT, but it flows into the Shepaug and is gaged there. The Nonewaug River is gaged at Minortown, CT, but it is also subsequently measured by the Pomperaug gage. The other stream, Copper Mill Brook, has a drainage area of only 2.50 miles^2 and we have therefore included it in the larger ungaged area.

Daily ungaged flows were calculated as a proportion of the gaged flows. Since flows are proportional to drainage areas, the proportionality factor is simply the fraction of ungaged area divided by gaged area. For example, on any day, GU is 30.7/994 times the flow gaged at Gaylordsville.

B. Time of Travel for Flow Information

To construct a short-term balance of water when flows and sizable distances are involved, we require a knowl-
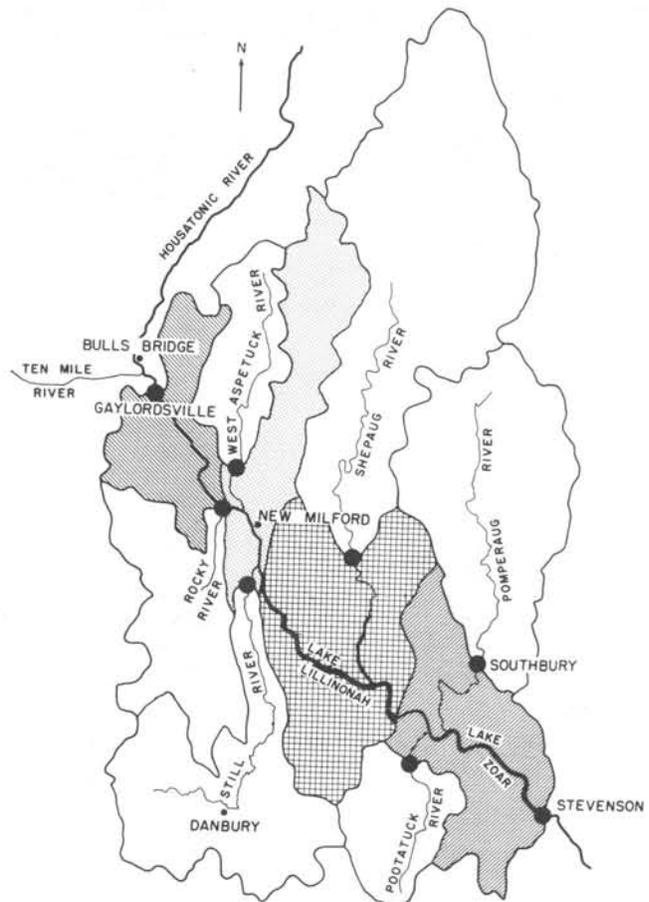

Fig. III-1. The watershed between the major USGS gages on the Housatonic River at Gaylordsville, CT and at Stevenson Dam, CT. The watersheds of the major tributaries are shown, and the location of their gages is indicated by solid black dots. The cross-hatched and stipled regions indicate ungaged areas, and the runoff from these areas was estimated and added to the water balance. From left to right they are GU, WAU, SU and PU.

edge of the time, t_h, for a hydraulic signal carrying information about a change in flow at one point in the river to another point down the river, e.g., between two tributaries. In other words, t_h is the time for the water level, or stage height (and thus the flow), to respond to a change in upstream flow.

There is an important distinction between the time required for a hydraulic signal to travel and the time for a volume of water to travel from Bulls Bridge to Stevenson Dam. The difference can be months. The information about increased flow is carried downstream by waves of two types: kinematic and dynamic. To estimate the time t_h, we must consider the propagation of a flood wave through three hydraulically different sections of the river. From Bulls Bridge at mile 0 to the backwater of Lake Lillinonah at mile 10.9 we assume that the flood wave travels with the characteristics of a kinematic wave (Lighthill and Whitham 1955, Eagleson 1970). The speed c_k (ft/sec) of such a kinematic wave is approximately:

$$c_k = m\overline{u} \tag{III-1}$$

where $\overline{u}$ is the mean speed (cross-sectional average) of water flow in the channel in ft/sec and where m is a

parameter that relates water volume flow RQ (cfs) to local stage height H by the equation

$$RQ = a H^m \qquad (III-2)$$

Taking the flow vs. stage height at Gaylordsville (Fig. III-2) as typical of this section of the river we find $m = 2$. This relatively high m reflects the steepness of this section of the river (cf. Fig. II-3) and the relatively wide, rectangular river channel. Our value of m lies between the value expected for laminar flow in a uniform wide channel, i.e., $m = 3$, and the value expected for fully turbulent flow, i.e., $m = 5/3 = 1.67$ (Eagleson 1970). By Eqn (III-1), then, a rising stage height travels downstream at a speed twice the mean flow. From our analysis of cross-sectional area A vs. flow given in Section II, we can estimate $\bar{u}$ versus RQ at Gaylordsville since $\bar{u} = RQ/A$. Combining these, we find that

$$\bar{u} \simeq 0.054 * RQ ** 0.48 \qquad (III-3)$$

and $\bar{u}$ ranges from about 1 ft/sec at a low flow of 500 cfs to about 4 ft/sec at a relatively high flow of 7000 cfs. Thus, the flood wave traverses the 10.9-mi.-long section of river (i.e. that portion where the water surface is approximately parallel with the channel bottom) in about 15 hrs at low flow and in only about 4.2 hrs at high flow.

Once the flood wave reaches the level surface of the lake, it travels at roughly the speed of a gravity wave c_g (ft/sec), i.e.,

$$c_g \simeq \sqrt{g d} \qquad (III-4)$$

where g is the gravitational constant and d is the depth of water. To calculate an average wave speed $\bar{c}_g$ we can use an average water depth in Eqn III-4 since depth is a nearly linear function of distance. From the backwater of Lillinonah to Lovers Leap, which is near the confluence of the
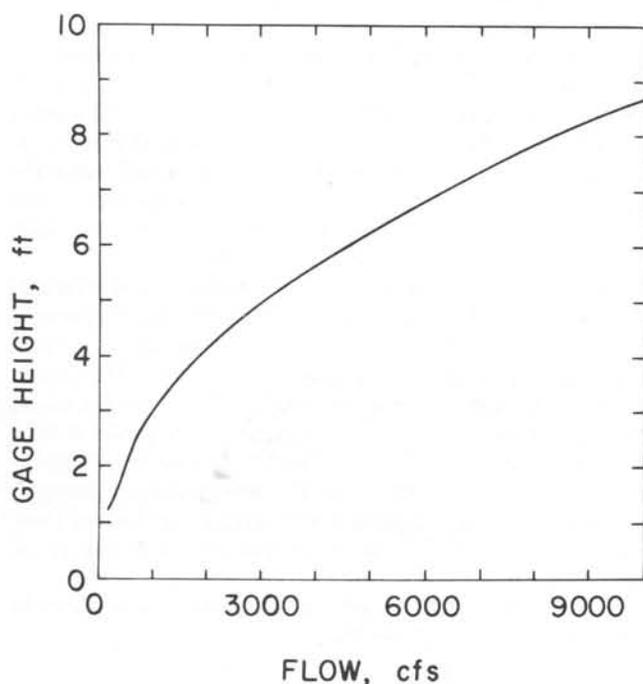
Housatonic and Still rivers at mile 12.5, the average depth is about 15 ft. Thus $\bar{c}_g$ in this section of the river is about 15 mph and the flood wave traverses this reach in about 0.1 hr.

The narrow gorge at Lover's Leap near Silex Mill controls flows coming from upstream and delays the communication of increased upstream flows to the lake below the gorge. From a comparison of measured water surface levels at Silex Mill and at the forebay of Shepaug dam during steady generation we estimate this delay to be about 1.5 to 2 hrs.

For the reach from mile 12.5 to 23.5, the average depth of water in Lake Lillinonah is about 65 ft. Therefore $\bar{c}_g \simeq 31$ mph, and the flood wave traverses this reach in about 0.35 hrs. A similar calculation for Lake Zoar, whose average depth is about 40 ft, gives a time of travel of about 0.4 hrs.

Finally, adding these times together we get a total time t_h for an increased stage height to be communicated the entire 33.5-mi. length of river under study. During relatively low flows t_h is about $(15 + 0.1 + 1.5 + 0.35 + 0.4) = 17.4$ hrs and during relatively high flows t_h is about $(4.2 + 0.1 + 1.5 + 0.35 + 0.4) = 5.8$ hrs. Similarly, we can calculate the delay for flow information at any other point on the river. For example, an increased flow on the Still river would be gaged at Shepaug dam in about 2.25 hr and an increased flow on the Pomperaug would be gaged at Stevenson dam in about 0.1 hr. Our estimate of the time of travel of a hydraulic signal agrees well with observations made by CL&P.

To make a daily water balance, it is necessary to take a weighted average of the present and of the previous day's flow. The contribution to the daily average flow at Stevenson on day I due to the flow at Bulls Bridge QBB for low flows can be estimated from our example by:

$$\frac{17.4\,QBB(I-1) + 6.6\,QBB(I)}{24}$$

and from

$$\frac{5.8\,QBB(I-1) + 18.2\,QBB(I)}{24}$$

for high flows. In this bulletin we are concerned with weekly averages and therefore, can usually neglect travel time of a hydraulic signal in our calculations of a water budget. For the water balance to be presented here we have added daily flows to compute weekly means. In doing so, we have corrected for the approximate travel time of the hydraulic signal although its effect is minor.

C. Change in Storage

We must also account for changes in storage at Candlewood, Lillinonah and Zoar. The electricity consumed or generated during pumping or generation at Rocky River reflects changes in Candlewood and is recorded hourly by CL&P. We converted these data to flow according to factors supplied by CL&P. For Lillinonah and Zoar, the elevation of the lake surface is recorded hourly by CL&P. Using the change in contents of these two impoundments with change in water elevation established in Section II, the change in gage height during the day was used to calculate the flow past the dam in cfs. An increase in the

Fig. III-2. Calibration of the USGS gage at Gaylordsville.

contents of Lillinonah and Zoar reduces the downstream flow. Low water in the Housatonic prevents pumping at Rocky River. When water is pumped, however, downstream flow is also reduced.

D. Calculated Water Balance and Estimated Accuracy

The difference between calculated and observed weekly mean flows, as a percentage of the observed flow, is shown in Fig. III-3 for weeks 1 – 52 with week 1 starting on August 1, 1968. Several large discrepancies are evident, especially between weeks 20 and 30. We will first estimate the error expected from uncertainties in measurements of flow and storage. Later, in this same section, we will examine in detail some possible causes of these errors.

We illustrate our procedure for estimating the expected uncertainty in the water balance by the calculations entered in Table III-1 for a typical low flow week in August. By adding all of the flows and the changes in storage along the river we calculate a flow given as SUM = 568 cfs. This is 61 cfs less than was observed at Stevenson Dam (STV) during the same period, for a negative relative error of –9.7%.

Table III-1. Water balance and error analysis for a low flow week in August.

Station	Flow, cfs	Daily Accuracy, %	Variance, (cfs)²
GAY	488	5	51.6
GU	15	5	3.2
CWPUMP	0	10	0
CWGEN	81	10	5.7
WA	4	10	0.01
WAU	7	10	0.1
STL	34	10	1.0
SHP	21	10	0.4
SU	10	10	0.8
DVLDH	-131	10	14.9
POOT	12	15	0.3
PMP	27	10	0.6
PU	18	10	1.1
DVZDH	-18	10	0.3
SUM	568	3.2	80.0
STV	629	5	85.7
Difference	-61	—	—
Percent Error	-9.7%	±4.1%	—

Determining the uncertainty in calculating the flow requires an analysis of the reported accuracy of each of the gaging stations and an analysis of the propagation of errors. The "accuracy" of the gaging stations is defined by the USGS as follows: "excellent" means that about 95% of the daily discharges are within 5%; "good" means within 10%; and "fair" within 15%. We assume that this means that 95% of all daily observations of flow would be expected to be within two standard deviations of the mean. The reported accuracy of the gaging stations is shown in Table III-1. The ungaged flows are calculated from the gaged flows and hence are given the same accuracy. The changes due to diversion at Candlewood are assumed to be known to 10%. The gage heights at Lillinonah and Zoar are read to the closest 0.1 ft, which on a daily basis amounts to 90 and 50 cfs respectively. Since the change in elevation during a day is generally of the
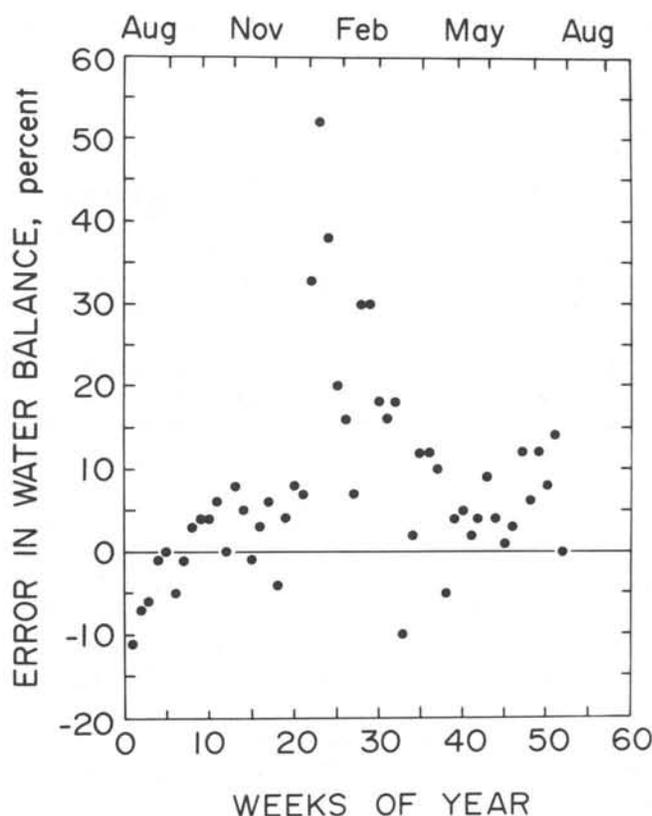

Fig. III-3. The percentage error of calculated weekly average flows at Stevenson Dam for August 1968 to September 1969. Note that the error is generally positive and the largest errors occur during the winter. Week 1 is August 1-7, 1968.

order of 1 ft, we will assign an "accuracy" of 10% to this measurement. In truth, we are dealing with both precision and accuracy and we shall try to make that distinction clear as we proceed.

In the present study we derive weekly mean flows by averaging successive daily flows at any given station. Hence, we require an estimate of the accuracy of weekly mean flows. There are two possibilities. If the errors in daily flows at any one gage are uncorrelated, then the coefficient of variation of the sum of flows for a week decreases and may be approximated by dividing the daily coefficient of variation by $\sqrt{N-1}$ (Ku 1966).

On the other hand, if the errors in daily measurements were correlated, uncertainty in the weekly average would not decrease nearly as much. We know that flows on successive days at any one gage are correlated. If the error in measurement was proportional to flow, then the errors would also be correlated. At present, we lack information on the correlation of errors. We know, however, that the correlation coefficient between successive daily flows for a week at any one station is about 0.5 regardless of flow. Hence, an upper limit on the correlation of errors would also be $r = 0.5$.

In this case, the variance of the mean s_m^2 is given by (Matalas and Langbein 1962):

$$s_m^2 = \frac{s^2}{N}\left[\frac{1+r}{1-r} - \frac{2}{N}\frac{r(1-r^N)}{(1-r)^2}\right]$$

If $r = 0.5$, this reduces to $s_m^2 = (2.43/N) s^2$. Thus, a gage with a daily "accuracy" of 5% would have a weekly "accuracy" of 2.94%.

To determine the uncertainty in the sum of all of the gaged flows, we require the formula for the variance s^2 of a sum of variates.

$$\text{If} \quad u_i = x_i + y_i + \ldots$$

$$\text{then} \quad s_u^2 = s_x^2 + s_y^2 + 2r_{xy} s_x s_y + \ldots$$

where r_{xy} is the correlation coefficient between the errors of measurement of the variates x_i and y_i. In the case of the gaged flows, we assume that an error in reading one gage does not affect the error of reading another gage. Since the ungaged flows are calculated from the gaged flows, however, these errors are perfectly correlated and thus $r_{xy} = 1$.

The variances shown in Table III-1 were calculated by the above rules. For example, at Gaylordsville (GAY) the variance is $(488 \times .0147)^2 = (7.2)^2 = 51.6 \, (cfs)^2$. Since Gaylordsville Ungaged (GU) is calculated as the flow at GAY times $30.7/994 = 15$, we must add a term due to the correlation of errors. Assuming $r_{xy} = 1$, the variance of the sum of these two flows is:

$$s^2 = (7.2)^2 + (.22)^2 + (2)(7.2)(.22)$$

$$\text{or} \quad s^2 = 51.6 + .05 + 3.17$$

In Table III-1 we combine the correlation error into the error for GU; thus we enter the variance $.05 + 3.17 = 3.2 \, (cfs)^2$. This example also shows how large errors can be introduced when the errors of observation are correlated. The remaining values in Table III-1 were calculated in a similar fashion, and the variance of the sum is 80. Converting this to USGS "accuracy", we have $((80)^{1/2} \cdot 2)/568 = 3.2\%$. The variance of the difference, i.e., CALC − OBS at STV, is also given by the sum of the variances, or $80 + 86 = 166$. Again, converting this to USGS "accuracy", we find that $((166)^{1/2} \cdot 2)/629 = 4.1\%$. Thus, we would expect that 95% of the time the error in the calculated weekly flow would be within ±4% and that 99% of the time it would be within ±6%. Similar calculations for a high flow week in April yield similar calculated "accuracy" for the sum of the flows.

Thus, the "accuracy" of a weekly balance apparently can vary from 2.9% if the errors are uncorrelated, to 4% if the errors are correlated with $r = 0.5$. Many of the errors are greater than ±4% of the mean (Fig. III-3), and a few larger and persistent positive errors occur in January and February. Therefore, we next examine other possible sources of error.

E. Sources of Error

Since the gage just below the Stevenson Dam is the reference for our water balance, we examined it carefully and discovered some large discrepancies between the observed and calculated flows, particularly in April 1969. As an independent check we examined CL&P power generation data at Stevenson Dam where the turbines are calibrated in cfs per kilowatt-hour. For January 1969 we converted power generation to cfs and, using correlation analysis, compared this with flow measured by USGS.
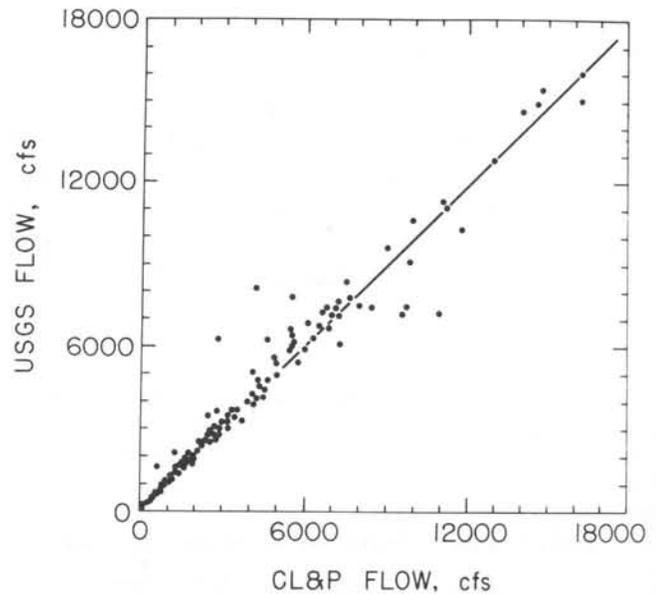
Fig. III-4. Correlation between the daily flows measured by the USGS and by CL&P at Stevenson Dam. The r^2 is 0.969 and the slope of the line is 0.975.

The result was:

$$\text{USGS Flow} = 54 + 0.902 * \text{CL\&P Flow}$$

with $r^2 = 0.996$. Therefore, the USGS gage agrees well with flows calculated from power generation, although there is a difference in calibration of about 10% at high flows. Checking USGS flows in April against the CL&P gage we identified the same discrepancies discovered by our model. We requested verification of the published record for April 1969 from the Water Resources Division, USGS, Hartford. Their reply is quoted below:

"Analysis of the April 1969 digital recorder tape showed clock stoppage occurred intermittently during the month, resulting in three random days of missing record. We used the daily stream flow hydrograph for our station at Gaylordsville to adjust the Stevenson record laterally along the time ordinate, and to estimate stream flows on the missing days."

(Personal communication, M.A. Cervione, Jr., March 15, 1973).

The original and revised flows are shown in Table III-2.

Table III-2. Original and revised flow data (USGS) at Stevenson Dam.

April 1969	Published flow, cfs	Revised flow, cfs
1	6,070	6,400
2	3,970	5,800
3	3,510	6,100
4	9,840	6,200
5	12,900	8,100
14	4,680	8,400
15	4,730	6,600
18	5,870	6,000
19	10,300	6,200
20	14,300	6,800
21	11,000	6,600
22	10,000	7,800
23	13,000	15,000
24	15,000	16,000

We cite this as an example of the large errors that can occur in data said to be accurate to within 5% for 95% of the time. For the USGS, we must say maintaining continuous records at many sites is harder than maintaining a

few days of observation at one site. These revisions considerably improved our predicted water balance in April 1969. We did not discover other large discrepancies between these two gages for the rest of the water year.

Using these corrected April flows, correlation analysis of 365 daily flows gave:

USGS Flow = 197.83 + 0.9748 * CL&P Flow

with $r^2 = .969$. Thus, the discrepancy seems largely a matter of calibration, rather than inability to measure the regulated flow at Stevenson. (See Fig. III-4). There are a few large discrepancies, but we cannot determine which gage is correct.

Having satisfied ourselves that the USGS reference gage at Stevenson dam was free of further large errors, we turned our attention to other possible sources of error, either in our model or in the data used in its verification.

1. Temporary Storage Due to Ice on the Lake Surface:

Since our largest apparent loss of water from the system occurs in the cold winter months, we estimated the equivalent flow of temporary storage in ice on the reservoirs. Although we did not measure the ice thickness in Winter 1969, 2 ft seems a generous estimate. Except for a small amount of ice near the banks, about 0.1 · 2 ft or 0.2 ft would float above water and be ungaged. From the functions derived in Section II for change in storage with change in height of reservoir, i.e., DVLDH and DVZDH, and assuming that this layer of ice forms uniformly over a month, it would reduce flow about 6 cfs in Lillinonah and about 3.4 cfs in Zoar. The reduction of about 10 cfs does not account for the large errors observed.

2. Bank Storage:

Finally, storage in the soil of the banks seems possible. Most of the bank in the section of the river under study is along the two lakes, which are maintained at a reasonably constant level throughout the year. We plotted the percent error versus the weekly change in height of Lillinonah (Fig. III-5). We could have included the change in height of Zoar, but since the two are correlated, omitting Zoar should not affect the outcome. Because the largest errors occurred with no change in height of the reservoir, changes in bank storage are not likely responsible for these errors. Since the largest errors occurred during 6 weeks in January and February (Fig. III-3), we examined these more carefully by checking each weekly error against the sum of the *absolute* value of daily height changes for each week. This analysis also showed that errors were greatest when the level of the lake changed
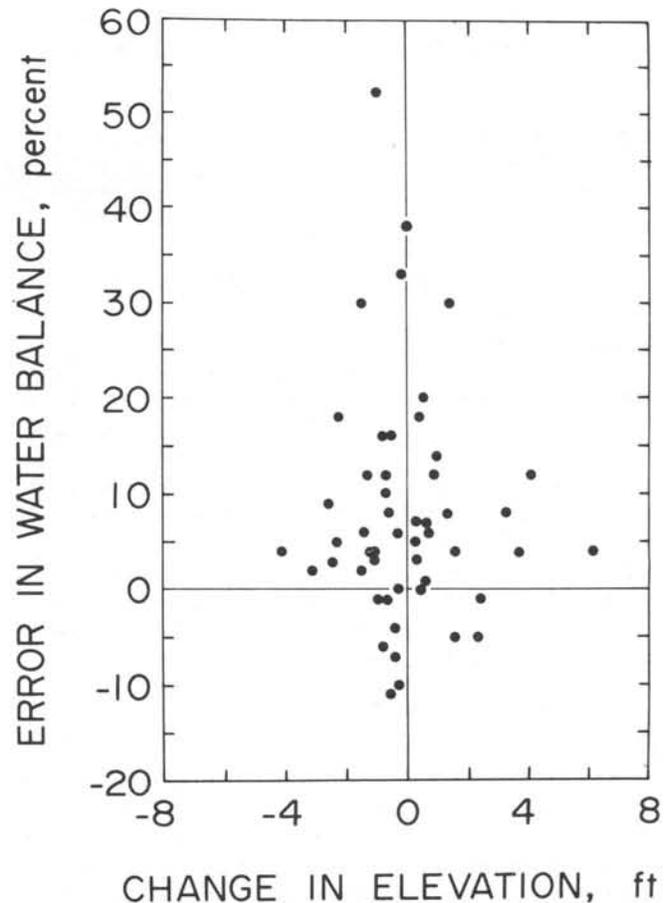
Fig. III-5. Percentage error in the weekly water balance vs. the weekly change in elevation of Lake Lillinonah, showing the largest error during little or no change in elevation.

least. Therefore, bank storage is not the main source of error.

3. Overflow of Banks and Temporary Storage behind Levees:

A large storm could cause overflow of the river bank, particularly between Gaylordsville and Silex Mill, and water could be temporarily impounded behind natural levees and not be immediately gaged downstream. We have examined this possibility carefully for the period January-February 1969. Daily flows never exceeded 2500 cfs at Gaylordsville during this period and pond heights at Lillinonah did not exceed 200 ft. According to CL&P, these conditions have never caused flooding.

Table III-3.   The water balance for 6 weeks using flows at Gaylordsville (GAY) all other gaged and ungaged flows (OTHER), corrections for changes in reservoir storage (DIVERSION) and flow at Stevenson (STV). Percent error is (GAY + OTHER + DIVERSION – STV)/STV.

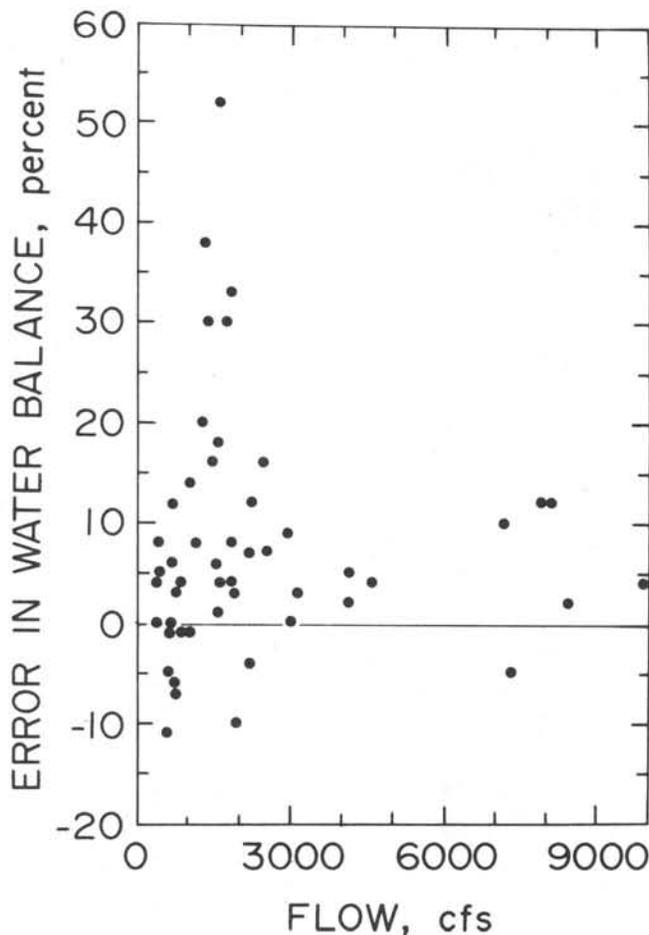| Week Ending	GAY, cfs	OTHER, cfs	DIVERSION, cfs	STV, cfs	% Error
Jan. 1	1703	672	+55	1821	33.4
Jan. 8	1693	659	+78	1604	51.5
Jan. 15	1321	483	+23	1323	38.1
Jan. 22	1201	420	-76	1283	20.4
Feb. 12	1321	574	-158	1365	27.2
Feb. 19	1476	614	+148	1724	29.8

Fig. III-6. Percentage error in the weekly balance vs. flow at Stevenson Dam, showing the largest errors during low flow.
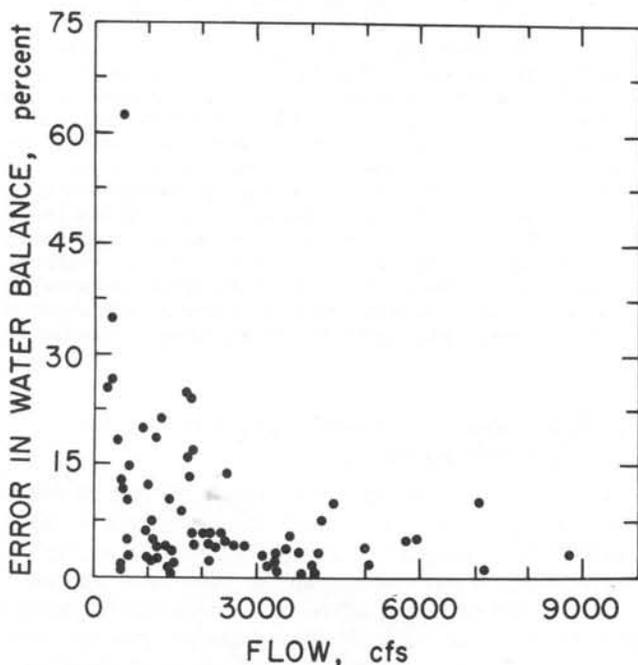

Fig. III-7. Percentage error in monthly water balance for the six water years 1966-1971, showing the largest errors during low flow.

4. Other Possible Sources of Error:

We return to the data itself. The percent error was greater when flow at Stevenson Dam was less. Closer examination of Fig. III-6 shows that the largest errors occur between 1000 and 2000 cfs. Specifically, during 6 weeks in 1969, the error exceeded 20% as shown in Table III-3.

Not only do the errors appear at low flow, but they also appear during the cold months of January and February as noted earlier. To examine this further, we calculated a monthly water balance for the six water years 1966-1971. The relative monthly errors are shown in Fig. III-7 as a function of flow at Stevenson. The USGS calibration, which implies a constant relative error, is apparently incorrect at low flows. The largest error of 62.4% occurred in October 1970 when the mean monthly flow was only 492 cfs. This error of 307 cfs may be an error in the record. If this error is omitted from the data set, we have the following means for the 6-year period:

Table III-4. Monthly water balance for the 6 water years 1966-1971.

Month	STV, cfs	% Error	Absolute error, cfs
Oct★	722	10.9	78.7
Nov	1699	4.1	69.6
Dec	2278	4.6	104.8
Jan	1582	18.2	288.0
Feb	2674	8.4	224.6
March	4372	−1.0	43.7
April	5607	2.3	129.0
May	3086	2.6	80.2
June	2058	3.2	65.8
July	1047	11.1	116.2
Aug	1250	9.5	118.8
Sept	958	−1.5	14.4

★The Oct. 1970 flow of 492 cfs giving a % error of 62.4% was omitted.

Having concluded that the Stevenson gage was free of large errors, we examined the gage at Gaylordsville, Figure III-2 shows the USGS calibration curve for this gaging station. It is interesting that the sensitivity of this gage changes dramatically at about 1000 cfs and is not a smooth function between 1000 and 2000 cfs. This suggested to us the possibility that ice in the flow control section for the Gaylordsville gage and perhaps other gages in the watershed could lead to high apparent flows in January and in February. However, we have no way of confirming this.

The remaining large errors at low flow are attributed to the fact that the USGS gaging stations do not have a constant relative error. Finally, it is apparent that even on a monthly basis the errors of measurement are larger than would be expected from the accuracy attributed to these gages. For purposes of conservation of mass of chemical substances, we shall have to force a water balance. We examine various alternatives for doing this in Section V.

F. Summary

This study highlights the need for better data before water and solute balances can be used with confidence. It

appears to us that a few gages, strategically located, carefully calibrated, and frequently inspected, might be preferable to a large number of less reliable gages maintained at various locations for relatively short periods of time.

IV — MODEL

In this section we use mathematics, physics, and observations at fixed locations to predict changes in the chemical quality of water that traverses the Housatonic River from Bulls Bridge to Stevenson Dam, CT. Although water quality is usually measured at fixed locations, the quality of different volumes of water at different locations in the river at the same time bear no direct relation to one another. Thus, we learn little from fixed locations about changes in the quality of a volume of water as it moves through the river and reservoirs. If, on the other hand, we could track a particular volume of water from its entrance into Lake Zoar to its exit from Lake Zoar, we could tell whether *that* volume of water lost or gained a net amount of chemical. This change in quality of water, which reveals mechanisms, is our subject. Such a description of the motion of a fluid volume from known initial coordinates is known as a Lagrangian description of fluid motion.

We shall specify the concentration of solute (P, Cl, N, etc.) in the volume of water V_N identified by initial coordinates x_o, t_o as it moves down the river. In doing so we will make a distinction between conservative elements such as chloride and non-conservative elements such as phosphorus. A conservative element is one whose total amount in solution undergoes little or no chemical change.

For conservative elements we must account for three processes that change their concentration in the volume of water:

1) advection, or horizontal transport, of the center of the volume;
2) augmentation or dilution of the element when the volume passes the confluence of a tributary;
3) change in concentration within the volume because of dispersion by turbulent mixing in the channel and exchange with regions of relatively still water.

For non-conservative elements we must also account for:

4) the decay or augmentation of the element by biological or physical processes. For example, phosphorus might be removed from solution by algae in summer, fall to the sediments in autumn and be released to the water in winter or early spring.

Our calculation of concentrations will explicitly include changes due to transport, inflows, and dispersion but will not explicitly include changes due to biological or chemical reaction. We will calibrate our model against measurements of chloride, a conservative element, for which chemical change is not important. We will then use this calibrated model to determine losses or gains of non-conservative elements, like phosphorus (P) or nitrogen (N), by comparing predictions of the concentrations of P or N at specific locations with actual measurements.
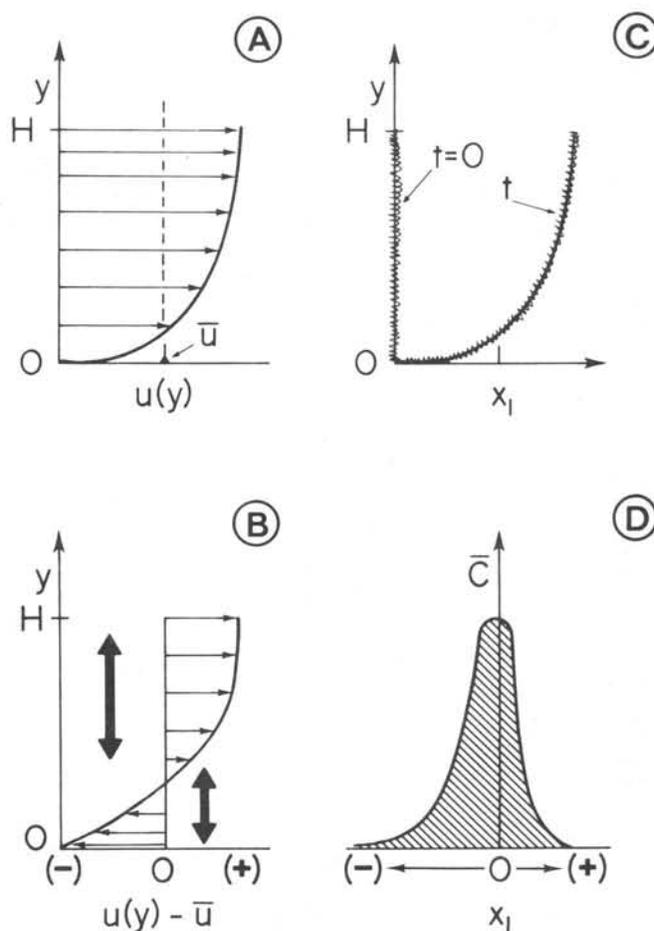
Fig. IV-1.   Schematic of dispersion. We have illustrated dispersion using the vertical variation in flow speed although cross-stream variation in flow usually dominates dispersion in natural streams. Part A shows the relative variation of speed at different heights where longer arrows represent higher speeds. The dashed vertical line locates the vertically-averaged speed $\bar{u}$. Part B shows the variation of the difference in speed $u - \bar{u}$ at different heights, where arrows pointing to the right are positive differences and arrows pointing to the left are negative differences. The heavy vertical arrows represent turbulent exchange, which tends to mix the slower with the faster moving water. Part C shows the downstream distortion at time t of an initially ($t = 0$) concentrated amount of a substance represented by the squiggly line. Part D shows the relative distribution of mean concentration $\bar{C}$ about point x_1 located at a distance $\bar{u} \cdot t$. The distribution of $\bar{C}$ results from the distorted profile shown in Part C being uniformly mixed in the vertical direction by turbulence.

A. Transport and Dilution of Conservative Chemicals by Rivers

Chemicals transported by rivers are diluted by longitudinal dispersion as an initially concentrated amount of chemical is stretched in the direction of flow and mixes with cleaner water surrounding it. Physically, this dilution is a combination of advective transport and turbulent mixing and is more effective than turbulent mixing alone.

In fluid flows of uniform speed, dilution of a substance and the decrease in its concentration C depends entirely upon turbulent mixing by the irregular, three-dimensional movements of parcels of fluid known as

eddies. When the speed changes markedly, as from the bank toward the middle or from the top to the bottom of a stream (cf. Fig. IV-1), dilution is greatly enhanced. This nonuniform speed spreads the substance in the direction of flow because parcels of water at the center or at the top move faster and thus race ahead of those nearer the bottom or the bank. Were it not for vigorous turbulent mixing vertically and across the stream, this advection in the nonuniform velocity field would eventually stretch a concentrated amount of an element into a fine filament containing little of the substance per unit of stream length. Remarkably, this indefinite spreading out is checked by rapid cross-stream and vertical turbulence, which keeps the material somewhat concentrated within the volume V_N moving downstream at the average speed of the water in the cross-section of the stream. To introduce the discussion to come later, we pause to consider some mathematical details of the longitudinal dispersion process.

B. The Dispersion Equation

The concentration $C(\text{lb}/\text{ft}^3)$ of a conservative solute like chloride in a flowing fluid is described by a three-dimensional equation based on the conservation of mass in an infinitesimal volume dV:

$$\frac{\partial C}{\partial t} + u_i \frac{\partial C}{\partial x_i} = \frac{\partial}{\partial x_i}\left(D_i \frac{\partial C}{\partial x_i}\right) \quad (IV\text{-}1)$$

where u_i is the local fluid velocity in the ith direction, x_i is the ith direction, say x, y, z for the downstream, vertical and cross-stream directions, and $D_i(\text{ft}^2/\text{sec})$ is the turbulent transfer coefficient of C in the ith direction. The subscript i when repeated in a term stands for summation over all i, thus if the D_i's are constant, Eqn (IV-1) actually has seven individual terms, four on the left side and three on the right side. The first term on the left side of Eqn (IV-1) is the change in concentration in dV with time, and the other three terms are changes due to the transport of material into or out of dV by the local instantaneous velocities u_i. The terms on the right side are due to turbulent diffusion.

In rivers, the local velocity in the direction of flow is usually much greater than perpendicular to the flow, while turbulent diffusion is usually much larger across the stream and vertically than downstream. If $u(y, z)$ is the velocity in the direction of flow, i.e., the x-direction, Eqn (IV-1) can be simplified to:

$$\frac{\partial C}{\partial t} + u(y,z)\frac{\partial C}{\partial x} = \frac{\partial}{\partial y}\left(D_y \frac{\partial C}{\partial y}\right) + \frac{\partial}{\partial z}\left(D_z \frac{\partial C}{\partial z}\right) \quad (IV\text{-}1a)$$

In general, Eqn (IV-1a) must be solved together with the equations of conservation of momentum and conservation of mass of water and with appropriate boundary and initial conditions. This formidable set of equations has not yet been solved in their entirety for a river such as ours.

Fortunately, G.I. Taylor (1953, 1954) obtained an approximate one-dimensional form of Eqn (IV-1a) for flow in a pipe, which helps analyze dispersion in streams

(Fischer 1966, 1967, 1968). Taylor's approximate equation qualitatively describes the dispersion in streams that we have discussed in the preceding section.

Taylor's one-dimensional dispersion equation replaces Eqn (IV-1a) (Taylor 1953; Fischer 1968) by

$$\frac{\partial \overline{C}}{\partial t} + \overline{u}\frac{\partial \overline{C}}{\partial x} = K\frac{\partial^2 \overline{C}}{\partial x^2} \quad (IV\text{-}2)$$

where $\overline{C}$ is the cross-stream average concentration, $\overline{u}$ is the cross-stream average flow velocity, t is time and x is the downstream direction, which is in the direction of $\overline{u}$. Finally, K is a dispersion coefficient describing the "effective" diffusion in the x direction. The dispersion coefficient K combines the action of vertical and cross-stream variation in velocity, which was $u(y,z)$ in Eqn (IV-1a) and the vertical and cross-stream turbulent mixing, which were D_y and D_z. Equation IV-2 yields the remarkable and useful result that an initially concentrated slug of solute spreads out about its center, which moves downstream with the speed of the spatial average flow $\overline{u}$ (cf. Fig. IV-1).

The basic conditions governing Eqn (IV-2) are: 1) the velocity $\overline{u}$ and the dispersion coefficient K are constants, and 2) mixing vertically and across the stream is so rapid that $\overline{C}$ depends only on x and t. Since the channel shape and the flow speed vary considerably with distance x in the Housatonic River (cf. Fig. II-3), condition 1 is not true. In addition, the second condition will not hold immediately downstream of a tributary where solute is not thoroughly mixed across the stream (Fischer 1966). Despite these difficulties, Eqn (IV-2) suggests an attack for our more complex problem, and we shall adopt the main simplification underlying Eqn (IV-2), i.e., one-dimensional analysis.

To summarize, in adopting one-dimensional analysis we assume that the essential features of dispersion can be described by mean quantities (i.e., cross-sectional averages) that vary only in the direction of flow. Further, we assume that the flow is governed entirely by the geometry of the reservoirs. Since this last assumption would not be valid if there were substantial thermal stratification of the reservoirs, we will examine this possibility in the next section.

1. Thermal Stratification and Flow in the Reservoirs:

Gravity makes patterns of flow in stratified bodies of water dramatically different from flow in a homogeneous body of water subject to the same boundary conditions. In particular, certain portions of a stratified fluid can be blocked from flowing by interposing an obstacle, or thin layers of such fluids can be withdrawn without disturbing the remaining fluid layers (Yih 1965). This latter phenomena, known as selective withdrawal, can be important for impoundments on rivers (Brooks and Koh 1969, Imberger and Fischer 1970, Wunderlich and Elder 1973), and is currently an active area of research (Kao et al. 1974, Pao and Kao 1974, Imberger et al. 1976, Kao 1976).

Since a separation of the water into flowing and stagnant layers in either Lake Lillinonah or Zoar, would require modification of our model, we examined the extent of thermal stratification in these lakes and its effect
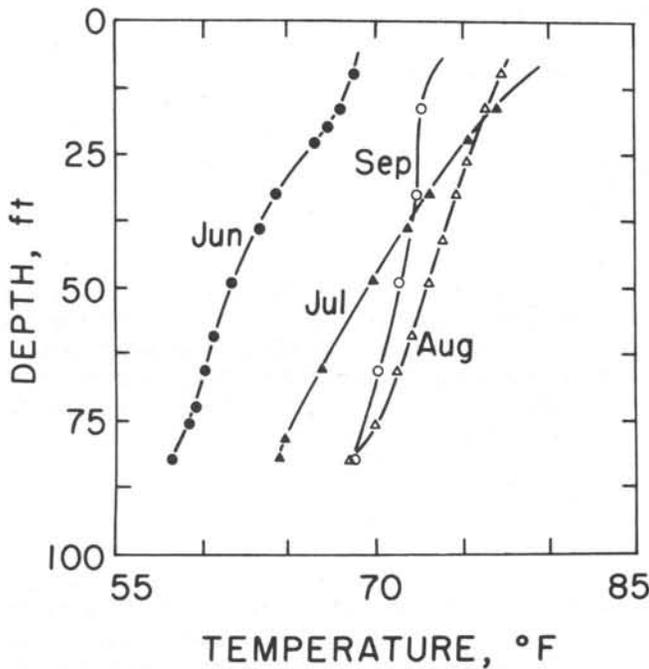
Fig. IV-2. Water temperature vs. depth in Lake Lillinonah in various months. At any time, the temperature at a given depth is essentially the same anywhere in the lake.

on the patterns of fluid flow. During summer, the time of greatest stable stratification, we examined patterns of flow indirectly by comparing tail race or outlet temperatures for diverse flows with the temperature profiles in the lakes and directly by the simultaneous movement of several drag plates drifting with the current at various depths in the lakes.
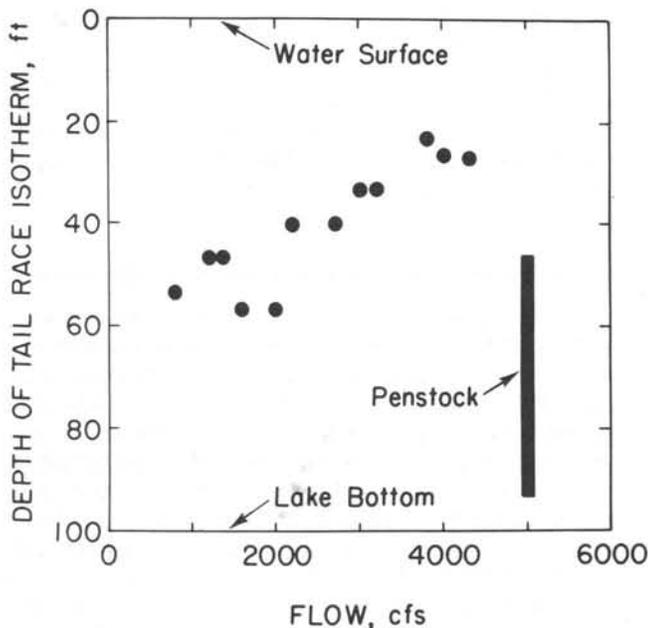
Fig. IV-3. Depth of the tail race isotherm TR versus outflow at Shepaug Dam. The internal Froude number F is $5 \times 10^{-6}Q$; thus for $Q = 2000$ cfs, $F = 0.01$. The solid bar shows the approximate vertical location and extent of the penstock opening in the dam face.

Lillinonah and Zoar, because of their considerable flow, are not stratified by a thermocline into a distinct epilimnion and hypolimnion as are many Connecticut lakes (Norvell and Frink 1975). They do, nevertheless, have a noticeable temperature variation with depth during the summer (Fig. IV-2). In both lakes the temperature gradient ranges between 0.14 and 0.19° F/ft in July and August. During 7 years of observation (1971-1977) the maximum gradient was 0.27° F/ft in July 1971. From early October until early or mid-May, temperature varies little with depth in these lakes.

Theoretical and laboratory investigations of withdrawal of stratified fluids from a container usually consider flow into an opening of small vertical but large horizontal dimension, which is called a line sink. These studies show that layered or separated flow develops when the flow is slow and the vertical gradient of fluid density is large, not when the flow is fast or the density stratification is small. While such studies have qualitative importance, they cannot be readily applied here because the penstock openings in Shepaug and Stevenson dams have a large vertical extent (cf. Fig. II-3). Therefore, we tried to determine withdrawal flow directly.

From the location of the openings in the dam face, withdrawal of water from Zoar tends to be from near the top while withdrawal from Lillinonah tends to be from near the bottom. Since temperature varies at different depths, the temperature of the outlet, or tail race, water should roughly indicate the vertical location and extent of water withdrawal. The temperature at the tail race TR is related to the temperature profile in the reservoir by:

$$TR = \frac{1}{Q} \int_0^h u(y) \, w(y) \, T(y) \, dy$$

where Q is outflow, $u(y)$ is the vertical velocity profile, $w(y)$ is the width of the lake as a function of depth y at the dam, $T(y)$ is the water temperature at depth y, and h is the thickness of the withdrawal layer. While this one equation cannot define both $u(y)$ and h uniquely, it can be a reasonable check on one of these, given the other.

The qualitative effect on h of increasing Q is shown for Lake Lillinonah in Fig. IV-3, where we have plotted the depth d_{TR} of the isotherm with temperature TR vs. outflow Q. These data, selected from periods when the generation was reasonably steady for about 2 hours, show a clear tendency for d_{TR} to become shallower and thus for the withdrawal layer to become thicker as Q is increased. Typically, outflows are greater than 2000 cfs. Similar observations of outlet temperature at Stevenson show no relation between d_{TR} and Q.

Withdrawal layers, which take considerable time to develop, form ideally when Q is low and steady for many hours (Imberger and Fischer 1970, Koh 1976). While average daily summer flows in the Housatonic River are low, instantaneous flows are neither low nor steady. An example of a generation pattern typical of the operation of either Shepaug or Stevenson Dam is shown in Fig. IV-4 where the peak flow is in excess of 5000 cfs while the average flow for the day is only about 1420 cfs. Withdrawal layers do not readily develop in such unsteady flows.

From what has been said so far we would not expect well-defined layered flows to develop in either Lillinonah or Zoar, and this expectation was substantially confirmed by mapping vertical velocity profiles observed with drag plates (Fig. IV-5). Although patterns of flow measured by drag plates at Shepaug Dam are far from regular, they indicate that flow is more or less uniform over the entire depth of the lake. Therefore, thermal stratification does not seem to invalidate one-dimensional analysis.
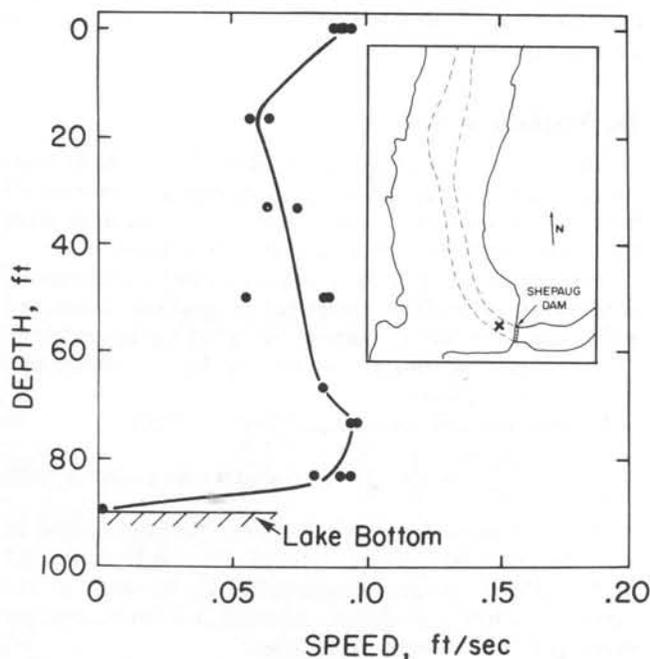
Fig. IV-5. **Speed of the flow vs. depth in Lake Lillinonah near Shepaug Dam on July 6, 1978. The inset shows a plan view of the river at the dam with X marking the location of the measurement.**

C. Equations for Conservation of Mass

The major features of the transport of chemicals in the Housatonic River can be incorporated into two equations: one equation for the conservation of water and another equation for the conservation of a chemical substance. In these equations, the cross-sectional area A and the mean velocity $\bar{u}$ are not constants as in Eqn (IV-2); rather, they are functions of both distance and time. Furthermore, water and substance enter into the system from tributaries at several locations x_j downstream.

1. The Conservation of Water:

The conservation of water can be described by:

$$\rho \frac{\partial A}{\partial t} + \rho \frac{\partial A\bar{u}}{\partial x} = \sum_{j=1}^{n(x)} q_j \delta (x - x_j) \qquad (IV\text{-}3)$$

which can be integrated to give

$$\bar{u}A = \sum_{j=1}^{n(x)} Q_j(t) \, \mu (x - x_j) - \int_{x_0}^{x} \frac{\partial A}{\partial t} dx \qquad (IV\text{-}4)$$

where

$$\mu (x - x_j) = \begin{cases} 0 \text{ for } x < x_j \\ 1 \text{ for } x \geq x_j \end{cases}$$

and

$$\delta (x - x_j) = \begin{array}{l} 1 \text{ for } x = x_j \\ 0 \text{ otherwise} \end{array}$$

Here q_j is a concentrated inflow of water per unit length of stream (lb/ft sec), Q_j is the inflow (ft^3/sec) of the jth stream where $j = 1$ is the Housatonic at Bulls Bridge and $j = 2, 3,...$ are tributaries, ρ is the density of water (lb/ft^3), and the summation Σ extends over all rivers upstream of location x. The unit step function μ and the unit impulse function δ mathematically represent an inflow of water entering at location x_j. Thus, our model assumes that all water is added at major tributaries as discussed in Section III. The change in area with time on the right side of (Eqn IV-4) mainly represents the change in surface elevation of the lakes due to power generation. As discussed in Section III, the surface elevation adjusts to changes in storage within about half an hour. To be consistent with our use of average weekly concentrations and flows, we also use the average change in reservoir contents for an entire week and write

$$\frac{\partial A}{\partial t} = \frac{\Delta A}{\Delta t}$$

$$\frac{1}{\Delta t} \int_{x_0}^{x} \Delta A(x) dx = \frac{1}{\Delta t} \Delta v(x)$$

where $\Delta v(x)$ is that portion of the change in storage occurring between positions x_0 and x. Finally, we can
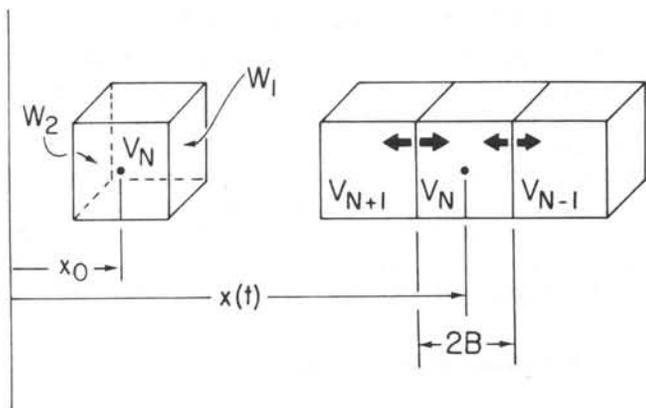
Fig. IV-6. The control volume V_N, initially located at position x_0, arrives at position $x_0 + \int \overline{u}\, dt$ at some later time t. While V_N is being transported downstream, water and chemicals are exchanged with neighboring volumes, V_{N-1} and V_{N+1}. These exchanges, shown schematically by the arrows, occur through the adjacent faces such as W_1 and W_2.

determine the average speed $\overline{u}(x,t)$ from

$$\overline{u}(x,t) = RQ(x,t)/A(x,t)$$

$$\text{(IV-5)}$$

where

$$RQ(x,t) = \sum_{j=1}^{n(x)} Q_j(t)\mu(x - x_j) - \frac{\Delta v(x)}{\Delta t}$$

In our model we consider the movement of a volume V_N defined by the channel boundaries, the free water surface and two imaginary end walls W_1 and W_2. Material is free to move through W_1 and W_2 but not through the other surfaces. This volume V_N, shown in Fig. IV-6, moves downstream at speed $\overline{u}$, and in a time t its center moves from x_0 to $x(t)$. Substance is exchanged with neighboring volumes through the imaginary end walls W_1 and W_2 by longitudinal dispersion already described (cf. Fig. IV-1).

Since we know $\overline{u}(x,t)$ from Eqn (IV-5), we can now determine the downstream position of volume V_N as a function of time by:

$$x(t) = x_0(t_0) + \int_{t_0}^{t} \overline{u}(x,t)\, dt$$

$$\text{(IV-6)}$$

where x_0 and t_0 are the initial coordinates of V_N, say Bulls Bridge on August 1, 1968. Notice that the variable x, which we wish to determine, appears both inside and outside of the integral sign requiring that Eqn (IV-6) be solved by successive approximation. This is accomplished by recasting Eqn (IV-6) as an ordinary differential equation:

$$dx/dt = \overline{u}(x,t)$$

$$\text{(IV-7)}$$

with initial condition $x(t_0) = x_0$, and then solving Eqn (IV-7) numerically using a third order Runge-Kutta technique (Scarborough, 1955).

2. The Conservation of Substance

The conservation of a substance having concentration

$\overline{C}(\text{lb}/\text{ft}^3)$ can be expressed as

$$\frac{\partial \overline{C}A}{\partial t} + \frac{\partial\, A\overline{u}\overline{C}}{\partial x} = \frac{\partial}{\partial x}\left(KA\,\frac{\partial \overline{C}}{\partial x}\right) \qquad \text{(IV-8)}$$

$$+ \frac{1}{\rho}\sum_{j=1}^{n(x)} q_j(t) \cdot C_j(t) \cdot \delta(x - x_j)$$

where $\overline{u}(x,t)$ is given by Eqn (IV-5). To solve this equation we must know the $q_j(t)$, the $C_j(t)$ as well as the initial distribution of $\overline{C}$ all along the river, and either the concentration or the spatial derivative of the concentration at both the upstream and downstream ends of our section of the river. Equation (IV-8) cannot be solved directly but, in general, must be solved approximately by numerical methods (for example, see Bella and Dobbins 1968). Unfortunately, such numerical methods require more information than we have. For example, a different value of K must be specified for transport across the ends of each finite elemental volume. Because K cannot be specified exactly from first principles, this procedure will introduce many individual values of K, which must be adjusted by matching the solution of the finite difference form of Eqn (IV-8) to observations of chemical concentrations. In the present case, we do not have detailed concentration data at several locations along the Housatonic, but rather we only have input and the output concentrations (cf. Appendix A). Furthermore, artificial or numerical dispersion inherent to finite difference approximations can greatly increase the difficulty of determining a physical dispersion coefficient (Roache 1972). This difficulty is especially bothersome in a system such as ours where the average speed $\overline{u}$ depends strongly on both location and time. Because of these difficulties we chose to forego a numerical solution of Eqn (IV-8) and instead developed an approximate solution commensurate with the data we have available.

D. Method of Solution

Although the seasonal variation of flow in the Housatonic is large (cf. Fig. II-2), during much of the year travel time is relatively short and advection is more important than dispersion in determining concentrations.

Our approach, then, is to obtain an initial estimate of concentration by first neglecting dispersion. Later, we will correct this initial estimate by including the exchange of mass with neighboring volumes, using the concept of a dispersion coefficient.

Neglecting dispersion, Eqn (IV-8) becomes

$$\frac{\partial \overline{C}A}{\partial t} + \frac{\partial A\overline{u}\overline{C}}{\partial x} = \frac{1}{\rho}\sum_{j=1}^{n(x)} C_j(t) \cdot q_j(t) \cdot \delta(x - x_j) \quad \text{(IV-9)}$$

where the right side represents chemical loads added at discrete points by tributary inflows. The load on the right of Eqn (IV-9) changes discontinuously by steps at the various x_j's but is constant, although a different one for every volume, between tributaries.

Between tributaries no water is added, and the flow, $A\overline{u}$, is constant. Furthermore, since the depth of the lakes varies relatively little with time, we consider the area A not to be a function of time in Eqn (IV-9). Therefore, for a
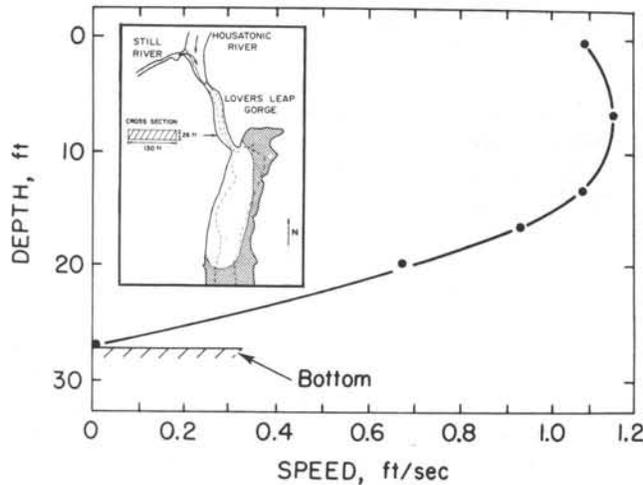
Fig. IV-7. Flow speed vs. depth in Lovers Leap gorge. The inset shows a cross-section and a plan view of the river near the gorge. Mixing was revealed by water color (stipled area) and distribution of algae (cross-hatched area) on September 22, 1977.

section of river between tributaries, Eqn (IV-9) can be written as:

$$\frac{\partial \overline{C}}{\partial t} + \overline{u}(x,t) \frac{\partial \overline{C}}{\partial x} = 0. \qquad (IV-9a)$$

Of course, the initial concentration at the beginning of the section must be specified. The solution of Eqn (IV-9a), obtained by the method of characteristics (Hilde-brand 1949) is of the form:

$$\overline{C}(x,t) = C_0\, g\left[x - \int_0^t \overline{u}(x,t)dt\right] \qquad (IV\text{-}10)$$

where $C_0 g(x)$ is the initial distribution of the substance. Equation (IV-10) simply states that a pulse of substance described by the function $g(x)$ with initial coordinates x_0, t_0 will be transported downstream *unchanged* to a new position $x = x_0 + \int_0^t \overline{u}(x,t)dt$ at some later time t. When the pulse of substance reaches a tributary, however, its concentration will be changed.

An important thing to notice about Eqn (IV-9) is that the concentrations and the flows on the right side contain the time t explicitly. Therefore, no matter when our control volume V_N started its journey, the load added by a tributary is given by the flow and concentration in the tributary at the time that V_N arrives at the confluence with that tributary. Equation (IV-6) for $x(t)$ allows us to calculate this time-of-arrival for each confluence. At each confluence, then, the concentration is increased according to the rule

$$\widetilde{C}(x,t) = \frac{\displaystyle\sum_{j=1}^{n(x)} \mu\,(x - x_j)\cdot C_j\,(t)\cdot Q_j\,(t)}{R Q(x,t)} \qquad (IV\text{-}11)$$

The upper limit n in the summation refers to the number of tributaries that the volume V_N has passed on its journey, which depends explicitly upon downstream location x. Again, $j = 1$ corresponds to the Housatonic River itself at Bulls Bridge and $j = 2, 3, ... n$ correspond to the other

tributaries. We have used the tilde ($\sim$) to denote the concentration calculated by neglecting dispersive mixing.

In deriving Eqn (IV-11) we have assumed that any introduced substance is rapidly and thoroughly mixed across the stream so that we need only describe its distribution in the direction along the river. This assumption is examined in the next section.

1. Mixing at Confluences:

In general, complete mixing is not accomplished immediately at a confluence, but occurs gradually over some distance downstream. The distance L over which thorough cross-stream mixing occurs is approximately (Fischer 1966):

$$L \simeq 1.8 \ \frac{l^2 \ \overline{u}}{R \ u_*}$$

where l is the half width of the stream, R is the hydraulic radius (Chow 1959), u_* is the friction velocity which is a measure of turbulent mixing by the flow, and $\overline{u}$ is the cross-stream average flow velocity. For a rectangular channel:

$$L \simeq 1.8 \frac{l^2}{d}\left(1 + \frac{d}{l}\right)\frac{\overline{u}}{u_*} \qquad (IV\text{-}12)$$

where d is the depth of the water. The friction velocity u_*, which depends on the average flow speed $\overline{u}$ and on channel roughness, can be estimated from measured velocity profiles. Generally, u_* is $0.1\,\overline{u} < u_* < 0.2\,\overline{u}$. For wide channels (i.e., $2l/d > 10$) like the Housatonic we find $L \simeq 45\, d\overline{u}/u_*$, and with $u_* = 0.2\overline{u}$, the length of the mixing region is $L \simeq 225\ d$.

All of the tributaries north of the Still River enter the Housatonic River where it has an average depth of less than 10 ft, and where its bed roughness is great. For those tributaries we estimate that complete mixing occurs within about ½ mile of entering the main stream.

a. The Still River

The Still River, entering the Housatonic at Silex Mill, is a major contributor of chemical loads and how well it mixes with the main stream is important to our model. Fortunately, the Still River enters just upstream of Lovers Leap Gorge, where rapid and thorough mixing occurs. The gorge at Lovers Leap serves as a hydraulic control section for flows upstream of it and water movement through the gorge is relatively turbulent and swift. An example of a vertical profile of average velocity measured in the lower part of the gorge with drag plates on September 22, 1977 is shown in Fig. IV-7 where speed in ft/sec is plotted at several depths. The slightly faster flow just beneath the surface is typical of open channel flows (Chow 1959).

The friction velocity u_* is estimated from the flow profile (Fig. IV-7) to be about 0.14 ft/sec. Also, since $l/d \simeq 2.5$ for the gorge, Eqn (12) gives $L \simeq 116d$. Since this L is less than the length of the gorge, we expect mixing to be complete. Another measure of this mixing and of the
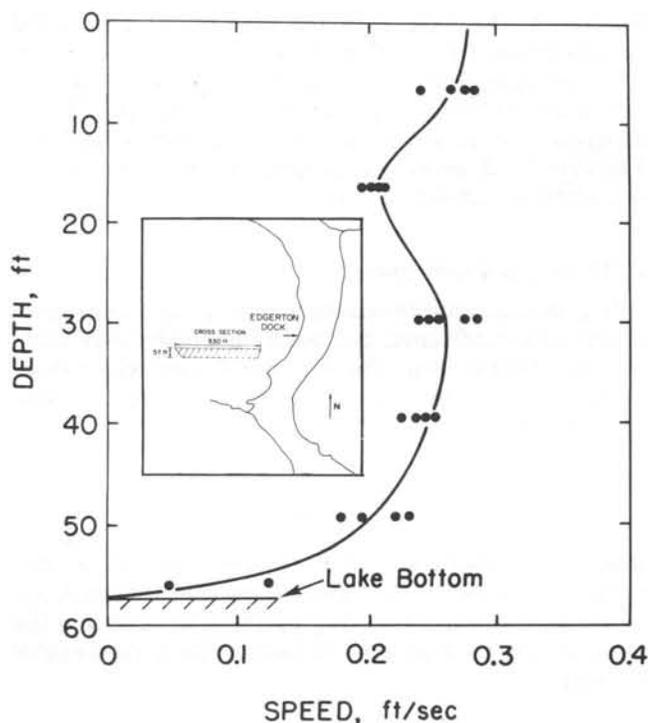
Fig. IV-8. Flow speed vs. depth in Lake Zoar at a location about 2.7 miles upstream of Stevenson Dam. The inset shows a cross-section and a plan view of the river at this location.

flow near the gorge is shown in the inset of Fig. IV-7 where the shade of the surface water outlines the flow. On September 22, 1977 the distinctive color of the water entering from the Still River followed the west bank near Silex Mill upon first entering the Housatonic. However, it became indistinguishable from the main flow a short distance inside the gorge. Below the gorge, patterns of surface algae separated regions of flow from more nearly quiescent regions. The quiescent water region to the northeast of the gorge outlet shown in the figure is an example of a region that contributes to the dispersion of solute by delaying its downstream progress.

b. The Shepaug River

Observations of natural alkalinity in the Housatonic River allowed us to assess qualitatively the mixing of water from the Shepaug River with that in the main river. Since the soils in northwestern Connecticut, where much of the Housatonic River water originates, are more calcareous than those in the Shepaug's watershed, the alkalinity of this water is about three times greater than in the Shepaug River. Water from the Housatonic encroaches up the Shepaug Arm of Lake Lillinonah (cf. Section III) due to 1) differences in the relative flows of the two streams, 2) differences in the relative volume of water storage in the Shepaug Arm and the remainder of Lake Lillinonah and 3) fluctuating levels caused by the periodic drawdown and recovery of the lake during power generation. Vertical profiles of alkalinity observed in summer within the Shepaug Arm indicate that mixing is effectively completed before the waters enter the main stream.

2. Dilution of Substance by Dispersion:

When travel time is short, dilution by mixing along the length of the channel can be neglected, and downstream concentrations can be determined from upstream information alone as was described earlier. However, when the water is flowing at a slower pace, such as in late summer and early fall, the exchange of material with neighboring volumes (both upstream and downstream) must be accounted for. In this case, we must also have some knowledge of the amount of substance downstream of our volume V_N.

How do we obtain this downstream information? The most straightforward way is to start a volume at a time just before the one we are considering and calculate its concentration as we have described for volume V_N. Of course, this other volume is also exchanging material with the volume element downstream of it and so on. Fortunately, the length of upstream transfer is relatively small, and we can often obtain a reasonable approximation by considering only one or two downstream segments. The concentration C in volume V_N is then just a weighted average of the amount of substance in V_N and in its nearest upstream and downstream neighbors.

The solution of the advection equation Eqn (IV-9) in the previous section gives the information necessary to estimate the change in concentration in V_N by mixing with neighboring volumes. This diffusion of substance between neighboring volumes can be described by a weighted average of concentrations in succeeding volumes:

$$\hat{C}_N = a_1 \widetilde{C}_{N+2} + b_1 \widetilde{C}_{N+1} + e\widetilde{C}_N + b_2 \widetilde{C}_{N-1} + a_2 \widetilde{C}_{N-2} \quad \text{(IV-13)}$$

Here, $\hat{C}_N$ is the concentration at a particular location at time t, allowing for diffusion; the $\widetilde{C}$'s on the right side are those obtained at that location from the advective model alone Eqn (IV-11) for volumes N, $N+1$, $N-1$, etc., and the a_1, b_1, e are weighting factors. Since the $\widetilde{C}$'s are already known from Eqn (IV-11) we need only determine the weighting factors.

We require a method for determining these weighting factors a_1, b_1, etc. from measurements of concentration at Stevenson Dam. To do so, we first examine the simple case of constant geometry and constant flow, for which the weighting factors can be determined exactly, and then introduce by analogy an approximate solution for the actual river with its varying geometry and flow.

To estimate the weighting factors in Eqn (IV-13) we first discuss the solution of Eqn (IV-2)

$$\frac{\partial \overline{C}}{\partial t} + \overline{u}\frac{\partial \overline{C}}{\partial x} = K\frac{\partial^2 \overline{C}}{\partial x^2} \quad \text{(IV-2)}$$

subject to the initial condition that at $t = 0$, $\overline{C} = C_0$ for $0 \leqslant x \leqslant 2B$ while $\overline{C} = 0$ outside of this region, and the boundary conditions $\overline{C}(x,t) = 0$ for $x \rightarrow \pm\infty$. These conditions represent a uniform slug of substance of finite length $2B$. Following Taylor (1953), we let $x_1 = x - \overline{u}t$ and obtain:

$$\frac{\partial \overline{C}}{\partial t} = K\frac{\partial^2 \overline{C}}{\partial x_1^2} \quad \text{(IV-14)}$$

where it is understood that $\overline{C}$ represents the cross-stream

average concentration and $x_1 = 0$ is the coordinate of a plane that moves downstream with constant speed $\bar{u}$. In other words, Eqn (IV-14) represents the diffusion, or mixing of material due to motions relative to the plane identified by $x_1(t)$ as shown earlier in Fig. IV-1D.

The solution of Eqn (IV-14) for a finite slug is (Csanady 1973):

$$\overline{C}(x_1,t) = \frac{C_0}{2}\left\{ \text{erf}\left(\frac{B+x_1}{\sqrt{4Kt}}\right) + \text{erf}\left(\frac{B-x_1}{\sqrt{4Kt}}\right)\right\} \quad \text{(IV-15)}$$

At the center of volume V_N, $x_1 = 0$, and therefore:

$$C_N(0,t) = C_{N_0}\,\text{erf}\left(\frac{B}{\sqrt{4Kt}}\right) \quad \text{(IV-15a)}$$

where C_{N_0} is the initial concentration in V_N, K is the effective longitudinal dispersion coefficient as before, t is time after introduction of the slug of substance into the river, B is the half length of the slug of substance, and erf is the error function.

In our river, the concentration of substance is not initially zero outside of the region $0 \leqslant x \leqslant 2B$ containing the initial slug. Instead, it is variable and non-zero all along the river, and the total concentration at the center of volume V_N located by $x_1 = 0$ is given by:

$$\hat{C}_N(0,t) = C_N(0,t) + C_{N-1}(2B,t) + C_{N+1}(-2B,t) + C_{N-2}(4B,t) + C_{N+2}(-4B,t) + \ldots\ldots$$

where $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (IV-16)

$$C_{N-1}(2B,t) = \widetilde{C}_{(N-1)_0}\frac{1}{2}\left[\text{erf}\left(\frac{B+2B}{\sqrt{4Kt}}\right) + \text{erf}\left(\frac{B-2B}{\sqrt{4Kt}}\right)\right]$$

with analogous expressions for the other terms. For the simple case of Eqn (IV-2) the weighting factors a_1, b_1, etc. of eqn (IV-13) are obtained by equating Eqns (IV-13) and (IV-16).

Depending on the value of the arguments of the error function, one, three, five, or more terms are required to account for diffusive mixing. We can estimate the number of terms required by calculating the solution of the following problem. Suppose that the concentration is initially equal to 1 everywhere in an infinitely long river,

i.e., $C(x,0) = 1$ for $-\infty < x < \infty$,

then, $\hat{C}_N(x_1 = 0) = 1$ for all time.

We can vary the value of the argument of the error function and see how many terms are required in Eqn (IV-16) for the right side to add to 1 which means that the total mass of substance is conserved. The results of such a calculation are shown in Fig. IV-10. For short times, the value of the argument is relatively large and only one or at most three terms are required to effectively conserve mass within 2%; thus, e is the dominant coefficient in Eqn (IV-13) This agrees with what we have said earlier: for short travel times the most important mechanism is advection and the mixing with neighboring volumes is secondary. For longer travel times, on the other hand, more terms

must be retained in Eqn (IV-16) and the relative magnitude of a_1, b_1, etc., become larger relative to the value of e. In any case, to conserve mass, we must always have that

$$\Sigma a_i + \Sigma b_i + e \equiv 1.$$

Unfortunately, Eqn (IV-8) describing conservation of the substance in the Housatonic River cannot be reduced to the form of Eqn (IV-2) and therefore the relatively simple solution given by Eqn (IV-15a) does not apply. Nevertheless, we take a bold step and assume that an equation of the form of eqn (IV-15a) holds for volume V_N approximately in some *average* sense, and we write by analogy to Eqn (IV-15a)

$$C_N = C_{N0}\,\text{erf}(A1/\sqrt{\tau}) \quad \text{(IV-17)}$$

where $A1$ is an adjustable parameter to be determined by comparing our predictions of concentration to observed concentrations and where τ is the time-of-travel.

Note especially that we have replaced the variable t with the parameter τ which is a fixed number for any V_N, i.e., τ is the time required for volume V_N to traverse the section of river under study. Of course, travel time is determined uniquely by Eqn (IV-6).

The geometry of the river allows us to make a further simplification. The form of the cross-sectional area function in Section II makes it clear that water takes relatively little time to travel from Bulls Bridge to the Still River: essentially all of the time is required to travel the rest of the way to Stevenson Dam. Furthermore, anticipating some of the results, we know that most of the chemical load has already entered the river by the time water has passed the confluence with the Still River. These two facts serve as our justification for using Eqn (IV-17) in Eqn (IV-16) to estimate the effect of dispersion on concentrations at Stevenson Dam. Thus, we are ignoring the dispersion of the limited amounts of solute that enter the system downstream from the Still River. The justification of this
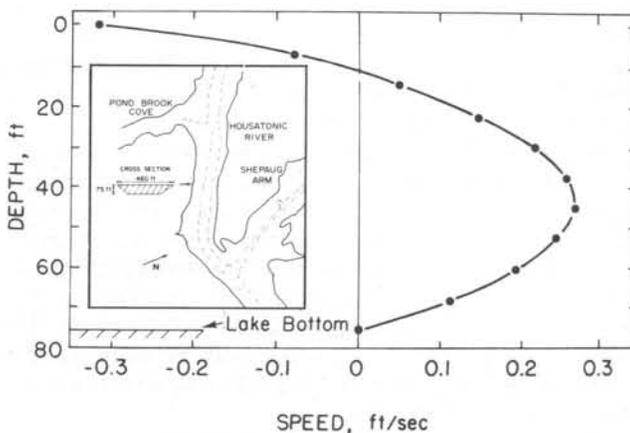
Fig. IV-9. Flow speed vs. depth in Lake Lillinonah due to the shear stress of the wind. At the surface, flow is upstream with a speed of about 0.3 ft/sec. The measurements of speed by the drag plates have been corrected to allow for the drag on the line that suspends them in the water. The inset shows a cross-section and a plan view of the river at this location just upstream of the Shepaug Arm.

approach will be tested in our subsequent comparisons of predicted and observed concentrations of Cl at Stevenson Dam.

a. Estimate of the Dispersion Parameter $A1$

To obtain a first estimate of $A1$ we take $A1 = B(x)/\sqrt{4\ K(x)}$ and let K be given by (Fischer 1967):

$$K \simeq 1.9\,u_* \frac{l(l+d)}{d} \qquad \text{(IV-18)}$$

where d is stream depth, l is half the stream width and u_* is friction velocity. Friction velocity can be estimated from vertical velocity profiles measured by drag plates. As an example, we discuss the vertical velocity profile shown in Fig. IV-8 that was observed in Lake Zoar on June 6, 1978 when the flow was about 7000 cfs through Stevenson Dam, 2.7 miles downstream.

Notice first that the flow is not separated into regions and is consistent with the assumption of one-dimensional analysis fundamental to our model. The friction velocity is obtained from the measured profile by (Sutton 1953):

$$u_* = k\,y\,\frac{du}{dy}$$

where y is the vertical direction and k is von Karman's constant equal to 0.4. From this equation and from Fig. IV-8 we find that $u_* \simeq 0.08$ ft/sec. Thus, since $d = 57.4$ ft and $l \simeq 220$ ft, K is about 162 ft^2/sec. This estimate of K was obtained during maximum power generation of approximately 7000 cfs. Profiles measured when flows were less than this gave correspondingly lower estimates of K. This relatively low value of K in Lake Zoar is in marked contrast to the much greater dispersion of freely flowing streams. For example, Eqn (IV-18) estimates K as large as 2000 to 4000 ft^2/sec in the Housatonic River between Bulls Bridge and the confluence with the West Aspetuck River.

Since we are concerned here with concentrations that change only from week to week, we take $B(x) = \frac{1}{2}\,\bar{u}(x)$ · 1 week, where $\bar{u}$ is the cross-sectional mean velocity as before. We evaluated $A1$ at several locations along the river and chose the average of these values as a first estimate in Eqn (IV-17). Between the Still River and Stevenson Dam $A1$ varies somewhat, but on average is about 2 when τ is expressed in terms of weeks.

In summer, generation patterns are irregular, and during many hours each day flow is nearly zero. During these quiescent periods, dispersion is mainly due to wind-driven currents. A good example of how an upstream wind can distort vertical flow patterns is shown in Fig. IV-9 for observations on October 27, 1977 at a location just upstream of the Shepaug Arm in Lake Lillinonah. The wind at 6.5 ft. above the water was blowing directly upstream at 5 to 8 mph and caused an upstream surface drift of about 0.33 ft/sec.

The dispersion coefficient K increases when the variance of the spatial deviation of the flow speed given by $\overline{u'^2}$ increases (Fischer 1967). At any particular river location x, $u' = u(y,z) - \bar{u}$. As is evident in Fig. IV-9, wind stress can substantially increase $\overline{u'^2}$ and thus increase K. Importantly, this example illustrates a difficulty of trying to specify values of K expected to be generally valid for all
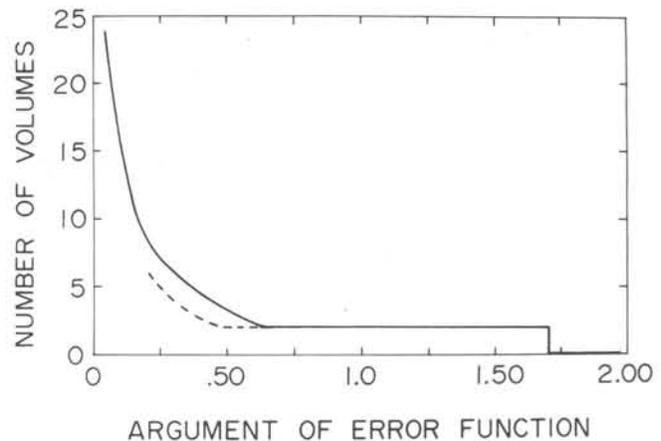
Fig. IV-10. The number of neighboring volumes required to effectively conserve the mass of substance is plotted against the argument of the error function. Mass is conserved to within 2% along the solid line and to within 5% along the dashed line.

distances and all times such as would be required by a finite difference model.

Anticipating the results of solving Eqn (6), we know that travel time varies seasonally from one to 11 weeks. With $A1 = 2$ and travel time $\tau = 1$ week, the argument of the error function is 2; also when $\tau = 11$ weeks, it is 0.6. From Fig. IV-10 we see that for the short travel time of 1 week the concentration in volume V_N is modified little by the surrounding volumes, while for the longest travel time of 11 weeks, the concentration in volume V_N is modified considerably by exchange of material from volumes that lead or lag, V_N by as much as 2 weeks. Thus, we have taken the weighted average of concentrations, including as many as 5 weeks for the longer travel times. Certainly the value of $A1 = 2$ is only a first guess, and we show in Section V.B that by adjusting the value of $A1$ somewhat the agreement of our predictions of Cl with observations of Cl at Stevenson Dam can be improved.

In summary, it does not seem feasible to describe the details of the dispersion coefficient at every river location for all times. Instead, we estimated an average K from average flows and average channel cross section to determine a dispersion parameter $A1$. We will adjust the final value of $A1$ by comparing model predictions with observations of chloride concentrations. Since $A1$ is not determined entirely from first principles, we cannot necessarily extrapolate it to other years. In spite of this, the value of $A1$ obtained in Section V.B will be used until a more detailed analysis yields a better value.

E. Computer Algorithm

Our computer model RVRFLO in Appendix C requires weekly mean flows and concentrations at the nine gaging stations shown in Fig. III-1 for all 52 weeks in a year. Using these data and Eqn (IV-6), the computer calculates the downstream location $x(t)$ of a volume of water V_N. A concentration C of a substance (e.g., Cl, P, N) corresponding to that at Bulls Bridge is initially assigned to V_N. Changes in concentration due to V_N passing the tributaries is calculated according to the rule given by Eqn (IV-11), where the load of the entering stream is

arithmetically averaged with the load in the Housatonic River at that point and time.

The new concentration downstream of the first tributary is computed:

$$C_{12} = \frac{C(1,t_0) * Q(1,t_1) + C(2,t_1) * Q(2,t_1)}{Q(1,t_1) + Q(2,t_1)}$$

where C_{12} is the concentration in V_N due to mixing the main stream (1) with the first tributary (2) as shown in Fig. IV-3. The * symbolizes multiplication. Thus, C_{12} is the sum of the loads divided by the combined flow. Time t_0 is the time when V_N started its journey from Bulls Bridge and $C(1,t_0)$ is the concentration in the Housatonic at Bulls Bridge at t_0. Time t_1 is the time that V_N reaches the confluence with stream 2, i.e., the first tributary which is the Ten Mile River. Note that the contribution to the load at the confluence from the upstream source contains the flow at the current time t_1. After passing the next confluence at time t_2,

$$C_{123} = \left\{ C_{12}(t_2) * \left[RQ - Q(3,t_2) \right] + C(3,t_2) * Q(3,t_2) \right\} / RQ$$

where

$$RQ = Q(1,t_2) + Q(2,t_2) + Q(3,t_2)$$

With the help of Fig. IV-11 we can see that the next concentration in the series C_{1234} is easily gotten by changing C_{12} to C_{123}, and by changing the indexes from 3 to 4 and from t_2 to t_3. RQ is also increased by the flow $Q(4,t_3)$. This pattern holds for all of the tributaries except the Shepaug River, which is a special case.

1. Confluence with the Shepaug River:

Unlike the other tributaries, the Shepaug does not have its flow and concentration measured at or near the confluence with the Housatonic River at mile 21.3, but rather, these parameters are effectively measured 3.25 miles away at the head of the Shepaug arm of Lake Lillinonah. This required a subroutine, SHPARM. To determine the concentration of a substance entering the Housatonic from the Shepaug, we must first calculate the time for water to travel the 3.25 miles to the Housatonic. That is, just as we say that the volume of water in the Housatonic River arriving from Bulls Bridge at mile 21.3 on week I actually left Bulls Bridge NW weeks earlier, we also say that the water arriving on week I at mile 21.3 from the Shepaug River actually left the mouth of the Shepaug River MW weeks earlier. We determine this number of weeks, MW, by conceptually running the flow of water in the Shepaug arm backwards. This computation is as already described for Eqn (IV-6) except that the specific geometry of the Shepaug arm (cf. Section II) is used, the positional parameter x_s is measured upstream; and time is decreased, rather than increased, by a small increment of time at each iteration.

The essential difference in calculating the load from the Shepaug River on week I then is that the concentration used is the one measured on week ($I-MW$). The flow used, however, is the one measured on week I because hydraulic signals travel quickly (cf. Section III B). Specifically, the algorithm for adding the load of the Shepaug River is:

$$C_{12\ldots7} = \left\{ C_{12\ldots6}(t_6) * \left[RQ - Q(7,t_6) \right] + C(7,NX) * Q(7,t_6) \right\} / RQ$$

with week t_6 determined by the MAIN program in the usual way and week $NX = (I-MW)$ determined by subroutine SHPARM as explained above.
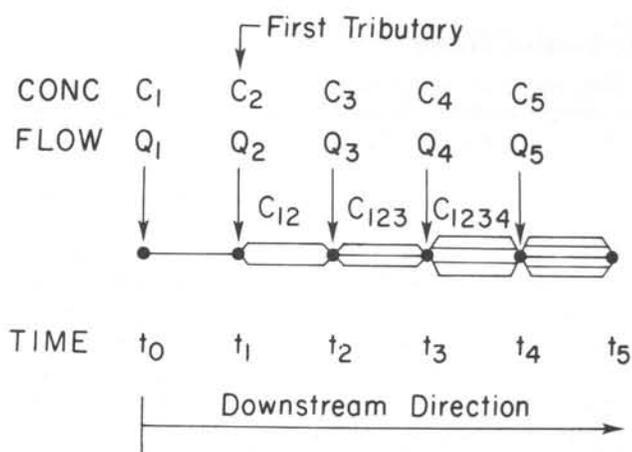
Fig. IV-11. A schematic of the way the computer adds chemical loads to volume V_N as it travels downstream and passes tributaries.

F. Summary

We have solved Eqn (IV-9) for the concentration C of a substance by a method of characteristics. The characteristics are along the line $x_1 = x - \int_0^t \overline{u} dt$, where $\overline{u}(x,t)$ is determined by Eqn (IV-5) for the conservation of water. We have incorporated this solution into a computer program, RVRFLO, that uses river channel geometry, measured flows, and chemical contents of tributaries to predict the concentration in a volume V_N travelling along the river and then the consequent concentration of chemical substance at a specific location, for example Stevenson Dam, for each of the 52 weeks for 1 year. We then make allowance for dispersion by averaging these concentrations. The weighting factors for the average depend on the time for a volume of water to travel the length of the reservoirs. These averaged concentrations are then compared with observations of concentration for the conservative element Cl. In the next section, the constant $A1$ describing dispersion is adjusted to allow the best agreement between predicted and observed Cl at Stevenson Dam. When this is done, we say that our model is calibrated, and we will use it to determine seasonal losses or gains of non-conservative chemicals, such as phosphorus, in Lakes Lillinonah and Zoar.

V — RESULTS

Using the methods described in Section IV, we can calculate directly the concentration of a conservative chemical, such as Cl, in a volume of water V_N arriving at Stevenson Dam at any time during the year, provided we know the inputs of water and Cl by tributaries and we know the geometry of the river channel. The concentration of Cl in V_N is comprised of contributions added at distinctly different times from several streams, each of whose concentration of Cl varies considerably with time. To properly account for these additions of Cl it is necessary to calculate the instantaneous position of volume V_N as it journeys downstream.

A. Time of Travel

The results of our calculation using RVRFLO for the position of a water volume V_N at any time $x(t)$ are shown in Fig. V-1 for water starting at Bulls Bridge on week 2.5 (August 1968) and week 35.5 (March 1969). Note that a volume of water leaving Bulls Bridge takes more than 10 times longer to arrive at Stevenson when travelling in late summer and early fall than in the spring. This behavior is a reflection of the difference in runoff during the two seasons. The abrupt changes in slope of these curves is due to changes in the river channel cross-sectional area with downstream distance as described in detail in Section II.

The amount of mixing of a chemical with neighboring volumes depends upon the time required for these volumes to traverse the length of the river. Travel time, calculated as arrival time at Stevenson Dam minus starting time at Bulls Bridge, is shown in Fig. V-2. The longer travel times for water flowing in late summer and early fall afford the greatest opportunity for longitudinal dispersion.
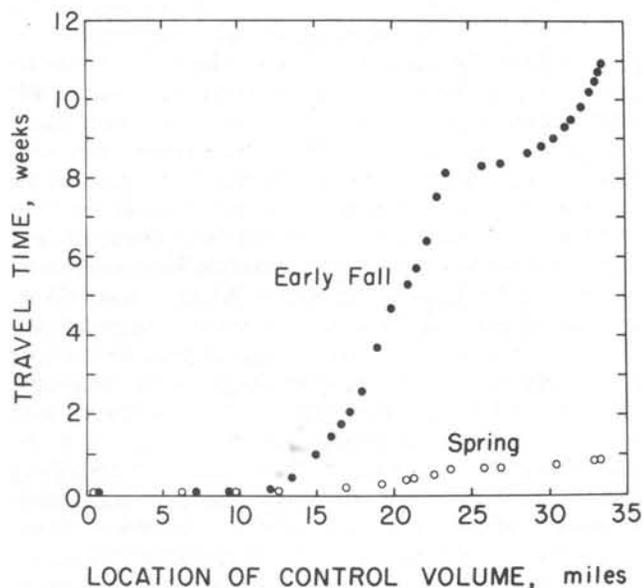
Fig. V-1. Instantaneous location of a volume of water V_N starting at Bulls Bridge in August 1968 (week 2) or in March 1969 (week 35). The volume traveling in late summer and early fall takes 10 times longer to arrive at Stevenson as the one traveling in the spring.
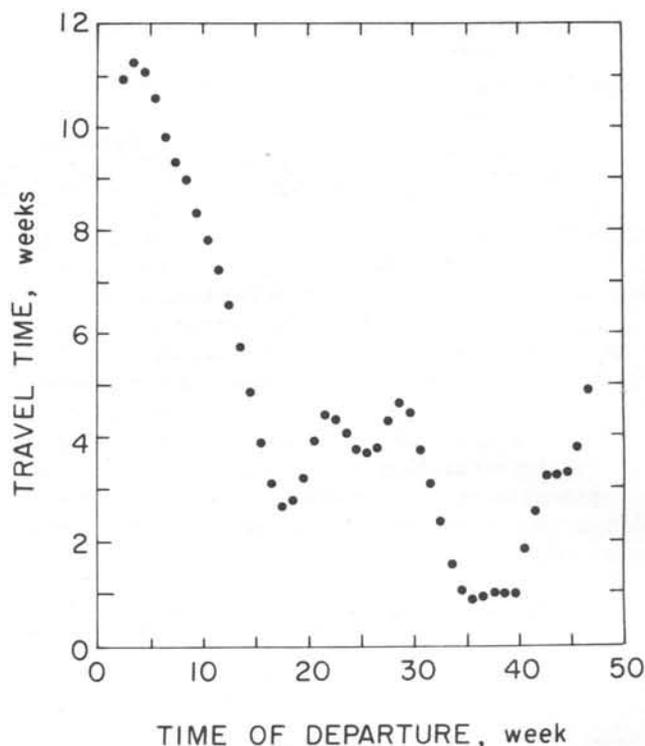
Fig. V-2. The time for a volume of water to travel from Bulls Bridge to Stevenson Dam. Travel time is, of course, directly related to flushing time. Week 1 is August 1-7.

B. Predicted vs. Observed Cl

Since Cl is a conservative element, we can test our model by comparing weekly predictions of Cl concentrations with those observed at Stevenson Dam. Details of the water quality measurements including frequency of collection and accuracy of analysis, as well as the data itself, are given in Appendix B. RVRFLO accounts for advection of Cl and additions of Cl by tributaries. We account for the effects of longitudinal dispersion by introducing the dispersion parameter $A1$ (cf. Section IV.D.2.a).

Since we must first calculate chemical loads to determine concentration, we also require a water balance to test our model. We examined various alternatives for correcting the errors in flow discussed in Section III by testing their effect on the agreement between predicted and observed Cl concentrations. The errors in flow are too large to be assigned entirely to the Gaylordsville gage. Moreover, assigning the errors uniformly to all gages except Stevenson has no effect on predictions of Cl concentration, as can be easily shown. Hence, we assumed that the two gages on the Housatonic River were essentially correct and assigned the percent error of the water imbalance to all gaged and ungaged flows between. This provided the necessary water balance and had remarkably little effect on the correlation between predicted and observed Cl concentrations.

The correlation between observed Cl and that predicted at Stevenson Dam using the corrected flows derived from the forced water balance is shown in Fig. V-3. This comparison is based on predictions of Cl obtained
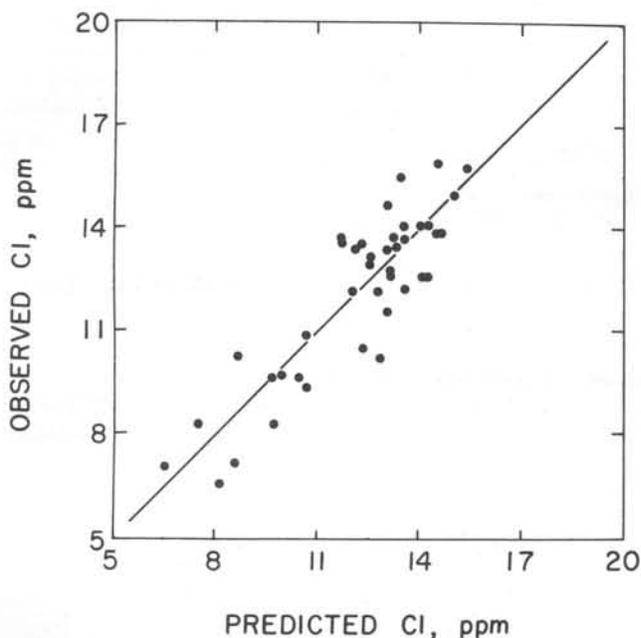
Fig. V-3. Correlation between chloride observed at Stevenson Dam and that predicted by the model ($r^2 = 0.774$).

by running RVRFLO each week, beginning with week 2 having a travel time of about 10.9 weeks, and ending with week 46 having a travel time of 4.9 weeks, for a total of 45 observations. The correction due to longitudinal mixing described in Section IV.D.2 utilizes information from two preceeding and two following weeks; thus, the number of observations are reduced to 41.

The best correlation between the two variates for these 41 observations is $r^2 = 0.774$ with intercept = 0.0003,
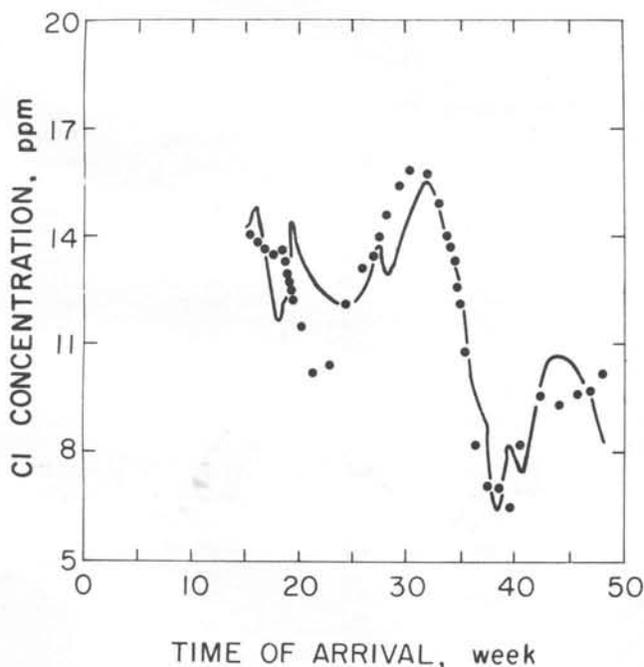
Fig. V-4. Predicted (line) and observed (solid circle) seasonal changes in chloride concentration at Stevenson Dam. Week 1 is August 1-7.
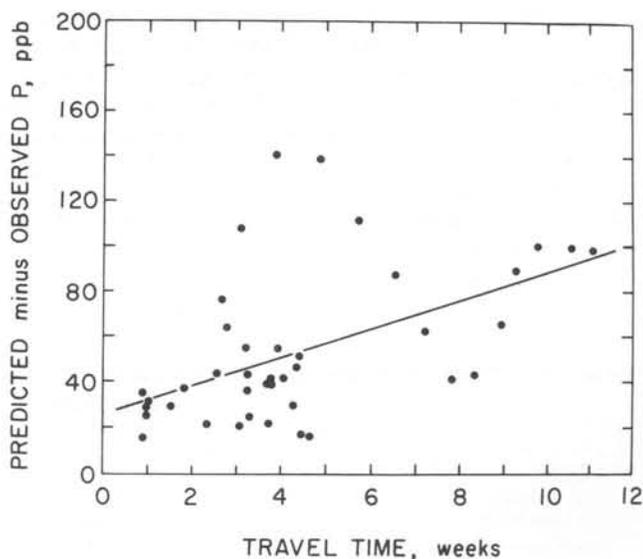
Fig. V-5. Retention of phosphorus in the river system vs. travel time from Bulls Bridge to Stevenson Dam.
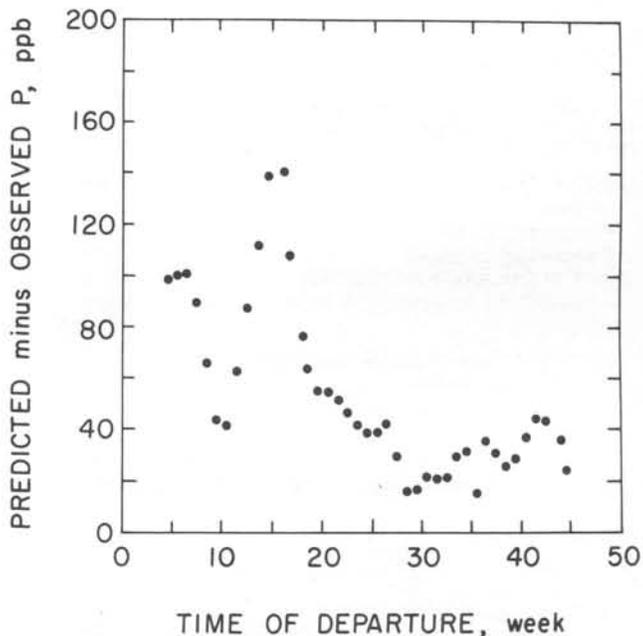
Fig. V-6. Retention of phosphorus in the system vs. week of departure from Bulls Bridge. Week 1 is August 1-7.

slope = 0.989, and standard error of the estimate (s.e.e.) = 1.17 ppm, obtained when the parameter $A1$ of Eqn(IV-17) is 1.33. The correction for dispersion improved these statistics, since for these same 41 observations without correction $r^2 = 0.734$, the intercept = 0.692, slope = 0.932 and s.e.e. = 1.27. Furthermore, an analysis of variance showed that the differences between weeks were highly significant but that the differences between predicted and observed Cl concentrations were not significant. Considering the substantial seasonal changes in Cl observed at Stevenson Dam and the reasonable agreement between our model and the observations shown in both Figs. V-3 and V-4, it seems that RVRFLO simulates the transport properties of the river well.

Since chemical concentrations were also observed on the Housatonic at Shepaug Dam (cf. Appendix B), we compared our model with observations made there as well. For the prediction of Cl concentration flowing into Lake Lillinonah as measured at Shepaug Dam, correlation analysis similar to Fig. V-3 gave the following: $r^2 = 0.750$, intercept = 0.861, slope = 0.959 and s.e.e. = 1.31. Although the correlation is not as good as for the entire river, the prediction is still within the experimental error of the analytical determination of Cl.

Using computed values of water flow at Shepaug Dam, we can predict Cl concentrations at Stevenson Dam based on observed Cl concentrations at Shepaug Dam for the same number of weeks. In this case, correlation analysis gave $r^2 = 0.891$, intercept = 1.234, slope = 0.873 and s.e.e. = 0.86. The uncertainty in prediction is less for this simple case, but there is some deviation from the 1:1 line.

Finally, we compare the predicted and observed mean annual load of Cl into Lakes Lillinonah and Zoar (Table V-1). Since this is an annual load, we have ignored

Table V-1.   **Mean Annual Chloride Budget, Bulls Bridge to Stevenson Dam, August 1968-July 1969**

	Mean Flow cfs	Mean Cl† ppm	Load Cl lbs/yr x 10⁶	Load Cl %	Watershed miles²	Cl Runoff lbs/acre
STATION						
Bulls Bridge	1390	10.6	28.9058	65.1	791	57.1
Ten Mile River	235	9.4	4.3445	9.8	203	33.4
Gaylordsville Ungaged	43	10.6	0.8928	2.0	30.7	45.4
Candlewood Out (Pump)	-35	10.9	-0.7523	-1.6	—	—
Candlewood In (Generate)	65	7.4	0.9509	2.1	40.7	36.5
West Aspetuck River	32	10.6	0.6662	1.5	23.3	44.7
West Aspetuck Ungaged	57	10.5	1.1780	2.6	41.2	44.8
Still River	92	23.7	4.2875	9.7	69.8	96.0
Shepaug River	171	7.5	2.5117	5.7	133	29.5
Shepaug Ungaged	85	7.5	1.2521	2.8	66.3	29.5
DVTDH	7	10.9	0.1504	0.3	—	—
SUM CALCULATED INTO LILLINONAH	2142	10.5	44.3876	100.0	1399	49.6
OBSERVED AT SHEPAUG DAM	—[1]	10.9	46.0166	103.7	1399	51.4
OBS/CALC, %	—	103.7%	103.7%	—	—	—
OBSERVED OUT LILLINONAH	2142	10.5	44.3876	94.6	1399	51.4
Pootatuck River	35	15.9	1.0919	2.3	24.2	70.5
Pomperaug River	94	11.6	2.1428	4.5	75.3	44.5
Pomperaug Ungaged	58	11.6	1.3233	2.8	46.5	44.5
SUM CALCULATED INTO ZOAR	2329	10.7	48.9456	104.2	1545	49.5
OBSERVED AT STEVENSON DAM	2331	10.2	46.8924	100.0	1545	47.4
OBS/CALC, %	100.1%	95.8%	95.8%	—	—	—

[1]Flow is not measured at Shepaug Dam.
†Flow weighted mean.

Table V-2.   **Mean Annual Phosphorus Budget, Bulls Bridge to Stevenson Dam, August 1968-July 1969**

	Mean Flow cfs	Mean P† ppb	Load P lbs/yr	Load P %	Watershed miles²	P Runoff lbs/acre
STATION						
Bulls Bridge	1390	70.8	193,681	54.4	791	0.38
Ten Mile River	235	104.9	48,492	13.6	203	0.37
Gaylordsville Ungaged	43	70.7	5,982	1.7	30.7	0.30
Candlewood Out (Pump)	-35	77.3	-5,323	-1.5	—	—
Candlewood In (Generate)	65	15.3	1,951	0.5	40.7	0.07
West Aspetuck River	32	30.3	1,909	0.5	23.3	0.13
West Aspetuck Ungaged	57	30.1	3,375	1.0	41.2	0.13
Still River	92	538.0	97,358	27.4	69.8	2.18
Shepaug River	171	14.6	4,906	1.4	133	0.06
Shepaug Ungaged	85	14.6	2,445	0.7	66.3	0.06
DVTDH	7	71.1	979	0.3	—	—
SUM CALCULATED INTO LILLINONAH	2142	84.4	355,755	100.0	1399	0.40
OBSERVED AT SHEPAUG DAM	—*	71.1	299,658	—	1399	0.33
OBS/CALC, %	—	84.2%	84.2%	—	—	—
OBSERVED OUT LILLINONAH	2142	84.4	299,658	94.3	1399	0.33
Pootatuck River	35	204.7	14,090	4.4	24.2	0.91
Pomperaug River	94	14.2	2,628	0.8	75.3	0.05
Pomperaug Ungaged	58	14.2	1,623	0.5	46.5	0.05
SUM CALCULATED INTO ZOAR	2329	69.4	317,999	100.0	1545	0.32
OBSERVED AT STEVENSON DAM	2331	42.2	193,476	—	1545	0.19
OBS/CALC, %	100.1%	60.8%	60.8%	—	—	—

*Flow is not measured at Shepaug Dam.
†Flow weighted mean.
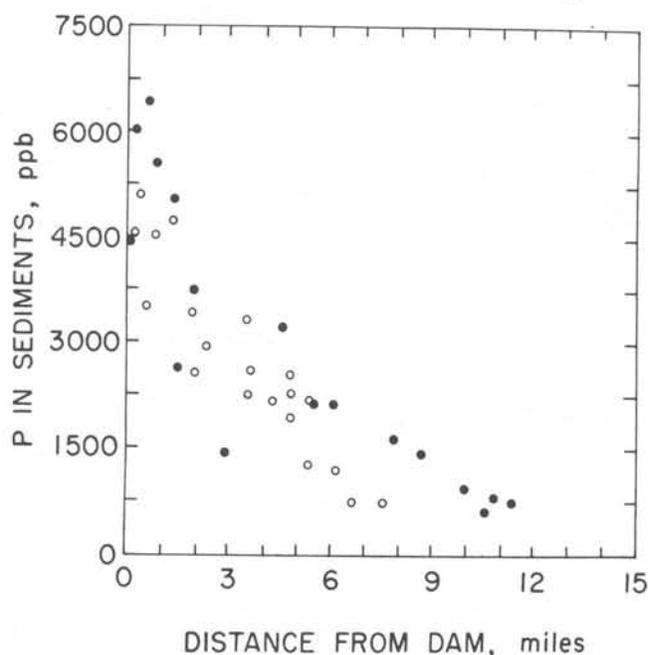
Fig. V-7. **Concentration of phosphorus in the bottom sediments of Lakes Lillinonah (●) and Zoar (o) collected at various distances from the dams.**

time of travel for these calculations. Also, since changes in concentration from year to year are small, we have ignored changes in storage of Cl within the reservoirs. As shown in Table V-1, the load measured at Shepaug Dam is 103.7% of the predicted load, while at Stevenson Dam the observed load is 95.8% of the predicted load. Given the uncertainties in the data and our model, we conclude that a reasonable mass balance of Cl has been obtained.

Table V-1 also includes estimates of Cl runoff in lbs Cl/acre of watershed per annum. These may be compared with observations in the Merrimack River with a watershed of about 4800 miles[2] in New Hampshire and Massachusetts. In this river, described as urbanized and polluted (Ceasar et al. 1976), the Cl runoff was as low as 39 lbs/acre in the upper reaches and increased to about 670 lbs/acre near the mouth. By contrast, the Housatonic receives relatively modest loadings of Cl.

In conclusion, our analysis shows that predicted and observed concentrations of Cl are not significantly different, and most of the seasonal variation is accounted for by our model. Therefore, we next discuss phosphorus concentrations and use our calibrated model to calculate retention of phosphorus in Lakes Lillinonah and Zoar.

C. Predicted vs. Observed P

Phosphorus is a non-conservative element, i.e., its concentration is attenuated due to retention of P in lakes as water moves through. This attenuation may be caused by settling of P adsorbed on suspended sediment, by uptake of P by phytoplankton and subsequent settling to the bottom, and by chemical precipitation of various phosphorus compounds. Our present model does not calculate this attenuation directly. Instead, we calculate the concentration of P accounting for the three processes des-

cribed for Cl: 1) advection, 2) addition by tributaries and 3) longitudinal dispersion. Since our calibrated model accounts for these processes quite well (cf. Figs. V-3 and V-4), we can then calculate the weekly attenuation of P concentration in the lakes by subtracting the concentration of P observed from that predicted at Stevenson Dam. Since this attenuation should be related to retention time (Vollenweider 1976; Norvell, Frink and Hill 1979), we examined this relationship first.

The relationship between phosphorus retention and travel time is shown in Fig. V-5 for the same 41 weeks shown for Cl in Fig. V-3. While there is much variability, ($r^2 = 0.267$) phosphorus retention tends to increase with travel time as expected. The slope of the regression line is 6.25 ppb/wk. The points above the curve for 3 to 7 weeks of travel time correspond to water that started from Bulls Bridge on weeks 12 through 16. During this period, the concentration of P in the Still River was unusually high. If these 5 weeks are omitted, the correlation between the variates increases to $r^2 = 0.548$ with little change in the slope of the line.

If attenuation of phosphorus was caused largely by biological processes, we would expect attenuation to be greatest during the summer months. Figure V-6 does show considerable attenuation for water travelling in late summer and early fall. The greatest attenuation, however, is for the 5 weeks discussed above when concentrations in the Still River were so high. According to stirred reactor theory for changes in lake phosphorus (Vollenweider 1976), we might expect

$$\Delta P \propto P_0^\alpha \tau^\beta$$

where ΔP is predicted minus observed P, P_0 is P observed at the outlet, and τ is travel time, with $\alpha \simeq 1$ and $\beta \simeq \frac{1}{2}$. For our data, with P concentration in ppb and τ in weeks, regression analysis gave

$$\Delta P = 116.2 \, P_0^{-0.42} \, \tau^{0.44}$$

but with a coefficient of determination r^2 of only 0.336. While the exponent of τ is nearly the 0.5 given by stirred reactor theory, the exponent of P_0 differs considerably from the expected value of 1. This is another indication of the concentration dependence of P attenuation in these impoundments. These results suggest that P retention by these reservoirs depends on P concentration as well as on travel time and season of the year, a result consistent with findings by Burns (1976) that the net sedimentation rate of P in Lake Erie increased with increasing P concentration. This would be expected if phosphorus was removed either by chemical precipitation (Norvell 1974) or by adsorption of P on particulates.

As with Cl, we have the opportunity to compare P concentrations predicted and observed at Shepaug Dam, and to run the model from Shepaug Dam to Stevenson Dam using observed rather than predicted P concentrations at Shepaug Dam. When these comparisons are made, the scatter in the data obscures any dependence on travel time. Of the approximate average annual attenuation of P in the system of 50 ppb (Fig. V-5) about 20 ppb is lost from Bulls Bridge to the Shepaug Dam and the other 30 ppb is lost from the Shepaug Dam to Stevenson Dam.

This is true despite an average travel time from Bulls Bridge to Shepaug Dam of 4.3 weeks and from Shepaug Dam to Stevenson of 1.2 weeks.

Our calculated mean annual phosphorus budget shown in Table V-2 confirms this greater retention in Lake Zoar. Of the observed load into Lake Lillinonah of about 356,000 lbs per year, some 84.2% leaves Lake Lillinonah and enters Lake Zoar. Thus, only 15.8% is retained, with nearly 300,000 lbs entering Lake Zoar. Of this load, 60.8% leaves Lake Zoar and continues downstream. The 39.9% retained in Zoar exceeds the absolute amount retained in Lake Lillinonah by a factor of about 2.2. The overall retention of P in the system from Bulls Bridge to Stevenson Dam is substantial, amounting to about half that entering the system. While this may seem high, as much as 72% of the total P was retained in Callahan Reservoir in Missouri. There, most of the incoming P was adsorbed on suspended sediment (Rausch and Schreiber 1977). Our calculation of the amount retained in the Housatonic River greatly exceeds that based on average annual water loads calculated by Norvell et al. (1979). Although not revealed in our measurements of flow, the smaller retention of P in Lake Lillinonah may be due to greater removal of bottom water where P concentrations are known to be higher.

Table V-2 also includes estimates of apparent annual P runoff in lbs of P/acre of watershed. These may be compared with P export coefficients derived by Norvell, Frink and Hill (1979) in a study of non-point sources of P to Connecticut lakes. For streams low in P, such as the Shepaug and the Pomperaug, the apparent P runoff in Table V-2 is less than that calculated from land use in the watershed by a factor of 3 to 6. This suggests significant P attenuation in streams as well as in impoundments. For streams high in P, such as the Still and the Pootatuck, the estimates from the non-point source model are low by a factor of 2 to 4 since these streams receive direct discharge of sewage effluent.
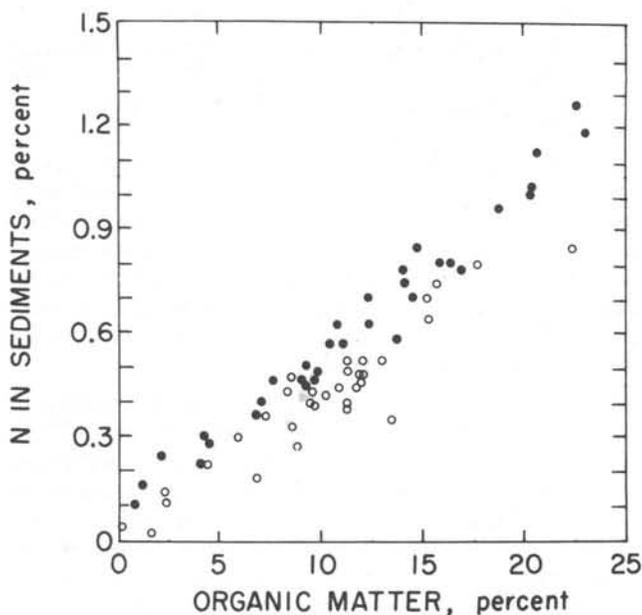
Fig. V-8. Concentration of nitrogen vs. organic matter in the bottom sediments of Lakes Lillinonah (●) and Zoar (o).

D. Predicted vs. Observed Nitrogen

Nitrogen is also a non-conservative element, and its concentration is attenuated largely by biological immobilization in weeds and algae. Thus, the accumulation of nitrogen in lake sediments is thought to be largely the result of organic detritus settling to the lake bottom.

A mean annual total nitrogen budget for the river system is shown in Table V-3. Like phosphorus, more nitrogen is retained in Lake Zoar than in Lake Lillinonah. This is largely the result of a low mean concentration of nitrate nitrogen of 58 ppb observed at Stevenson Dam as compared with a mean of 267 ppb observed at Shepaug Dam. Since no preservatives were added to the samples, biological changes could have occurred between collection and analysis. Hence, we combined the nitrate and Kjeldahl nitrogen fractions for the budget shown in Table V-3 and do not attach too much importance to the apparent distribution of nitrogen between the two fractions. However, the differences in retention are consistent with those observed for phosphorus and could be attributed to release of NH_4 from anoxic sediments in Lake Lillinonah and concomittant withdrawal of bottom water.

E. Sediment Analyses

The concentration of phosphorus in surficial sediments collected in the center of the former river channel in Lakes Lillinonah and Zoar is shown in Fig. V-7. Phosphorus in sediments is chemically associated with fine-grain material (Frink 1967, 1969), which tends to settle out where the water moves slowly as in deeper and wider areas near the dams. Thus, the P concentration in these sediments is highest near the dams and decreases in a fairly regular fashion with increasing distance from the dam. Although there is some scatter in the data shown in Fig. V-7, the P concentrations in the sediments of both lakes are similar at similar distances from the dam.

Our previous calculations showed that Lake Zoar retained about twice as much phosphorus as did Lake Lillinonah, yet Fig. V-7 shows that the concentrations of P are similar in the sediments of the two impoundments. This apparent discrepancy could be explained if the rate of accumulation of sediment in Lake Zoar was also greater; but, we have little evidence about sediment transport. Preliminary seismic work by the USGS has shown that the sediments in Lake Zoar are thicker and more dense than in Lake Lillinonah. However, Lake Zoar has been impounded since 1919, while Lake Lillinonah was impounded in 1955. Further work is underway in connection with studies of the transport and accumulation of PCBs which may help resolve these uncertainties.

Nitrogen, on the other hand, is generally associated with organic detritus and hence is strongly correlated with organic carbon in sediments. The relationship between these two variates for 67 sediment samples collected throughout Lakes Lillinonah and Zoar is shown in Fig. V-8 and is similar for the two lakes. In addition, this C/N ratio is nearly identical to that observed in a natural lake in Connecticut (Frink 1969) and in a variety of lakes in Wisconsin (Keeney et al. 1970; Konrad et al. 1970). Thus, nitrogen accumulation is apparently controlled by

biological processes that tend to maintain a constant C/N ratio in sediment organic matter.

F. Sensitivity

Clearly, our predictions depend on the cross-sectional area of the river channel because the speed that volume V_N travels is determined by the flow and the cross-sectional area. To test the sensitivity of the model to the choice of the functional representation of the cross-sectional area, we varied this function in three ways. First, since our estimated volumes for Lakes Lillinonah and Zoar using the equations in Section II were somewhat low, we increased the area functions of both impoundments by 5%. Moreover, the smooth exponential function we chose does not precisely reflect the changes in cross-sectional area with distance. Instead, we used three straight line segments to represent the cross-sectional area of Lake Lillinonah. Finally, since in summer the inflow from the Shepaug River does not immediately mix with the water in the Shepaug Arm but instead begins to flow as a layer while mixing gradually, we reduced the effective area for flow in the Shepaug Arm by reducing the area of the Shepaug Arm by 2/3.

None of these changes significantly changed the correlation between the predicted and observed chloride concentrations at Stevenson Dam. Therefore, we conclude that our model is relatively insensitive to modest errors in the estimates of the cross-sectional area of the Housatonic River.

As noted earlier, another potential source of error is the failure to achieve a water balance. While our method of forcing a water balance described in Section V.B did not materially alter the correlation between predicted and observed chloride concentrations, it does affect the mass balances shown for phosphorus, chloride, and nitrogen in Tables V-1, V-2 and V-3. At this point, however, we present these as our best estimates of the transport of these chemicals in the Housatonic River system.

G. Summary

Our calculations of the time of travel showed that a volume of water V_N leaving Bulls Bridge takes more than 10 times longer to arrive at Stevenson Dam when travelling in late summer and early fall than in the spring. Average annual travel times (or residence times) are often calculated for rivers and impoundments, but this obscures seasonal changes that may be important in biological reactions.

We tested our model by comparing Cl concentrations observed at Stevenson Dam with those predicted by RVRFLO. The correlation between these two variates for 45 weeks of observation was $r^2 = 0.774$ with intercept = 0.0003 and slope = 0.989. Since Cl is a conservative element, we require a mass balance as well. After corrections for water imbalance, the observed annual load of Cl at Stevenson Dam was 95.8% of the predicted load. Given the uncertainties in the data and our model, we consider that it is verified.

We calculated the attenuation of P in the system by accounting for advection, addition by tributaries, and longitudinal dispersion as we did for Cl and then subtracted the concentration of P observed at Stevenson Dam from that predicted using RVRFLO. Phosphorus attenuation increases with increasing travel time, and is greater during the summer. We also found that P attenuation increased with increasing P concentration. The overall retention of P in the system from Bulls Bridge to Stevenson Dam is substantial, amounting to about 50% of the total entering the system. This greatly exceeds the calculated retention based on average annual water loads

Table V-3.   Mean Annual Nitrogen Budget, Bulls Bridge to Stevenson Dam, August 1968-July 1969

	Mean Flow cfs	Mean N† ppm	Load N lbs/yr x 10⁶	Load N %	Watershed miles²	N Runoff lbs/acre
STATION						
Bulls Bridge	1390	0.555	1.517553	53.3	791	3.0
Ten Mile River	235	1.471	0.680184	23.9	203	5.2
Gaylordsville Ungaged	43	0.554	0.046870	1.6	30.7	2.4
Candlewood Out (Pump)	-35	0.701	-0.048279	-1.7	—	—
Candlewood In (Generate)	65	0.327	0.041800	1.5	40.7	1.6
West Aspetuck River	32	0.529	0.033283	1.2	23.3	2.2
West Aspetuck Ungaged	57	0.525	0.058852	2.1	41.2	2.2
Still River	92	1.673	0.302769	10.6	69.8	6.8
Shepaug River	171	0.409	0.137616	4.8	133	1.6
Shepaug Ungaged	85	0.410	0.068601	2.4	66.3	1.6
DVTDH	7	0.676	0.009500	0.3	—	—
SUM CALCULATED INTO LILLINONAH	2142	0.676	2.848750	100.0	1399	3.2
OBSERVED AT SHEPAUG DAM	—*	0.690	2.906917	102.0	1399	3.2
OBS/CALC, %	—	102.0%	102.0%	—	—	—
OBSERVED OUT LILLINONAH	2142	0.676	2.848750	92.3	1399	3.2
Pootatuck River	35	0.779	0.053628	1.7	24.2	3.5
Pomperaug River	94	0.622	0.114969	3.7	75.3	2.4
Pomperaug Ungaged	58	0.622	0.070997	2.3	46.5	2.4
SUM CALCULATED INTO ZOAR	2329	0.674	3.088343	100.0	1545	3.1
OBSERVED AT STEVENSON DAM	2331	0.483	2.213451	—	1545	2.2
OBS/CALC, %	100.1%	71.7%	71.7%	—	—	—

*Flow is not measured at Shepaug Dam.
†Flow weighted mean.

discussed by Norvell et al. (1979) and suggests that chemical precipitation is important at high P concentrations.

We also compared predicted and observed annual nitrogen loads. As with phosphorus, Lake Zoar retained more nitrogen than Lake Lillinonah. Analyses of surficial sediments in the two impoundments confirm the calculated enrichment of the sediments with P and N, but we lack sufficient information at present to make quantitative comparisons.

In conclusion, our model appears to provide reliable predictions of conservative elements such as Cl and to provide a means of calculating the attenuation of nonconservative elements such as P and N. In the final Section (VI), we discuss in greater detail some of the uses for a model such as RVRFLO.

VI — SUMMARY AND CONCLUSIONS

A. Model

The 33.5 mile stretch of the Housatonic River studied here contains two large impoundments and numerous tributaries, and, except for tides, it contains most of the complexities likely to be encountered in New England rivers. We calculated the transport of chemicals in the Housatonic using the principles of hydraulics, estimates of river channel cross-sectional area, and measurements of weekly mean flow and concentration of major tributaries.

Our model accounts directly for three major processes governing the change in concentration of conservative chemicals, like chloride, transported in rivers: 1) advection, or horizontal transport, 2) addition of chemicals by tributaries, and 3) dispersion of chemical. Our model does not account directly for removal of chemical from the water by biological or physical processes. Instead, we calculate the removal of a nonconservative chemical, like phosphorus, by subtracting observations of phosphorus from the predictions of our model.

In developing our model, we had to force a water balance since we found large errors in the flows recorded by USGS gaging stations. While forcing a water balance did not affect the correlation between predicted and observed chloride concentrations, future models would profit from improved measurements of flow. We used approximate mathematical functions to describe river channel cross-section; fortunately, our model is not particularly sensitive to small changes in the form of these functions. Lack of significant thermal stratification in the two impoundments allowed us to assume one-dimensional flow, which simplified our analysis greatly. We verified one-dimensional flow with detailed measurements of velocity at various depths using drag plates.

B. Water and Solute Movement

We tested our model by comparing the predicted and observed concentrations of Cl at several points along the river. For the 33.5 mile stretch from Bulls Bridge to Stevenson Dam, the correlation between these two variates for 45 weeks of observation was $r^2 = 0.774$ with intercept = 0.0003 and slope = 0.989. The annual Cl load observed at Stevenson Dam was 95.8% of that predicted by RVRFLO. Given the uncertainties in our model and in our data we consider this to be adequate verification.

Phosphorus attenuation, calculated as the difference between P predicted by RVRFLO and P observed at Stevenson Dam, was found to increase for longer travel times, and to increase with increasing incoming P concentrations. Attenuation was considerably greater than expected from calculations based on average annual water loads and suggests the need for additional investigations of mechanisms of P removal.

Nitrogen attenuation was calculated in a similar manner. No nitrogen appeared to be retained in Lake Lillinonah, but about 30% of the total load was retained in Lake Zoar. Since nitrogen is not considered an important pollutant in these lakes, we did not investigate this apparent difference in attenuation in greater detail.

C. Implications for Control Strategies

As stated in the Introduction, our ultimate goal is to provide information necessary to devise more effective management of water quality in the impoundments on the Housatonic River. Thus, we ask if our model suggests strategies for the control of pollutants such as P.

The data show that there are two major sources of phosphorus: one is the Housatonic River itself, with about 54.4% of the load, and the second is the Still River with 27.4%. A third and much smaller source, based on data obtained from New York State, is the Ten Mile River.

These data support present efforts to remove P in effluent from sewage treatment plants on the Still River and on the upper Housatonic in Massachusetts. Conversely, while the P concentration in the Pootatuck is high, its contribution to the load into Lillinonah is insignificant.

Tertiary treatment of effluent for P removal is expensive, raising the question as to whether year-round operation is necessary. Based on our model, we offer the following observations: Significant amounts of phosphorus are retained in the sediments of both impoundments. In Lake Lillinonah, it appears that little or no P from sediments returns to the surface water (Norvell and Frink 1975). Thus, limiting P inputs during the growing season (with due allowance for travel time) should reduce algal growth but removal of P during winter months is not expected to affect the growth of algae in the summer in Lake Lillinonah.

Despite our inability to detect selective removal of anoxic bottom waters from Lake Lillinonah, the release of both P and NH_4 to Lake Zoar is suggested by our mass balances. Thus, limiting P inputs into the Housatonic River during the growing season may have less impact on algal growth in Lake Zoar than it would in Lake Lillinonah. However, it is clear from our data and model that Lake Lillinonah intercepts substantial amounts of phosphorus that otherwise would reach Lake Zoar. It is tempt-

ing to speculate on how to enhance this P attenuation, but this is beyond the scope of our present study.

Finally, as efforts are increased to prevent P from entering these impoundments, the model suggests ways in which future monitoring programs could be designed to obtain maximum information at least cost. For example, four water quality stations located 1) at Gaylordsville, 2) on the Still River, 3) at Shepaug Dam and 4) at Stevenson Dam would now provide nearly as much information as that obtained with nine stations in the present study.

Considering next the problem of the accumulation of PCBs in the Housatonic River (Conn. Acad. Sci. Engineering 1978), the model can be extended with some modifications to include the headwaters near the presumed major source in Pittsfield. Since PCBs are nearly insoluble in water and are transported largely by sediment, their transport and accumulation is expected to resemble that of phosphorus. Sampling stations for suspended sediment will be installed at gaging stations on the river to measure the transport of particulates.

D. Challenges that Remain

The challenges that remain can be divided into two categories: theoretical and experimental.

The dispersion of solutes in Lakes Lillinonah and Zoar is a complex theoretical problem depending on channel geometry, wind driven currents, power generation patterns and, to a limited extent, thermal stratification. While longitudinal dispersion is only of secondary importance for much of the water year, it is important for determining concentrations of solutes during low flow periods in late summer and early fall. Our treatment of dispersion has relied on *ad hoc* assumptions that were guided by measurements of flow velocity near to and upstream from the dams. Improved estimates of dispersion await more detailed studies of the mixing of tracers at various locations in the river.

Our analysis of P retention suggests that removal of water near the lake bottom in Lillinonah might contribute more to dispersion than we have allowed. While we believe that the assumption of one-dimensional flow on which our model is based is workable, the inclusion of vertical flow structure in Lake Lillinonah near the Shepaug Dam might improve our estimate of dispersion. This may be particularly important in the transport of sediment from Lake Lillinonah to Lake Zoar. Before a more detailed analysis is undertaken, however, some measurements of actual sediment transport must be obtained.

Although the load entering the Housatonic from the Shepaug has little effect on chemical concentrations in the river, the encroachment of Housatonic River water up the Shepaug Arm during the summer has considerable effect on water quality in that portion of the lake. Measurements of alkalinity profiles indicate that a two-dimensional analysis might be required in this arm of the lake.

Under experimental challenges, the most obvious need is for better flow data to obtain a mass balance for water in the river. Indeed, our method of forcing a water balance increased the water imbalance reported earlier for Candlewood Lake (Frink 1971). We believe that our experience is not unique to the Housatonic River but rather reflects the fact that accurate daily or weekly flow balances have not been attempted on this or other major rivers.

The next challenge is to increase our understanding of the mechanisms of P attenuation in streams and reservoirs. Contemporary models assume that P attenuation in lakes increases with increasing retention time. Our analysis suggests that in these reservoirs the attenuation is also controlled by incoming P concentrations. Thus, as obvious point sources of P are eliminated, it appears that further reductions in lake phosphorus will occur more slowly.

Acknowledgements

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APPENDIX A — SYMBOLS

A	the cross-sectional area of the river channel (ft^2)
$A1$	a semi-empirical parameter to account for dispersion ($(\text{wk})^{1/2}$)
a_1, b_1, e, etc.	the weighting factors that account for the mixing of substance between neighboring volumes of water
B	the half length of a pulse of substance (ft)
C	concentration, or amount per unit volume of water, of substance (lb/ft^3)
$\overline{C}$	the cross-stream average concentration
$\tilde{C}$	the concentration of substance in volume V_N neglecting dispersive mixing
$\hat{C}_N$	the concentration of substance in volume V_N corrected for dispersive mixing
Cl	the chloride ion
c_g	the speed of a gravity wave (ft/sec)
c_k	the speed of a kinematic wave (ft/sec)
D	turbulent transfer coefficient (ft^2/sec)
DVLDH	the change in volume of water stored in that part of Lillinonah contained in the Housatonic River channel corresponding to a change in surface elevation of Lillinonah (ft^3)
DVSDH	the change in volume of water stored in the Shepaug arm due to a change in surface elevation of Lillinonah (ft^3)
DVTDH	the change in volume of water stored in the entire Lake Lillinonah corresponding to a change in surface elevation of Lillinonah (ft^3)
DVZDH	the change in volume of water stored in Lake Zoar due to a change in surface elevation of Zoar (ft^3)
d	depth of water (ft)
d_{TR}	depth of the tail race isotherm (ft)
erf	the error function
exp	the exponential function
F	Froude number
FRCTXL FRCTXS FRCTXZ	the fractional change in storage at location X in Lillinonah proper, in the Shepaug arm and in Zoar
$f(x), g(x)$	functions describing the spatial distribution of concentration
$f(\text{flow})$	an arbitrary function of flow used to estimate missing concentration data
g	the acceleration of gravity (ft/sec^2)
H	the height of the water surface above the lake or river bottom (ft)
HL, HZ	elevation (above mean sea level) of the surface of Lake Lillinonah and Lake Zoar (ft)
h	the thickness of the withdrawal layer of water (ft)
I	an index referring to a week of the year
K	the dispersion coefficient (ft^2/sec)
k	von Karman's constant
L	a mixing length (ft)
$l(x)$	the half-width of the river (ft)
ln	the natural logarithm
N	total nitrogen, and number of observations
$n(x)$	the number of tributaries to be summed depending on the location x of volume V_N
P	total phosphorus
$Q_j(t)$ or $Q(J,t)$	the volume inflow of water from the jth stream (ft^3/sec) at time t
q_i	the mass inflow of water per infinitesimal length of river from the ith tributary (lb/ft sec)
R	the hydraulic radius of the river channel (ft)
$RQ(x,t)$	the net downstream volume flow of water at position x and time t (ft^3/sec)
r	the correlation coefficient
s^2	the variance of a sum of variates
$T(y)$	water temperature, a function of depth y (°F)
TR	temperature of water exiting at the tail race of the dam (°F)
t	time (hr)
t_h	the time of travel of a hydraulic signal (hr)
t_0	initial time coordinate of volume V_N
u	flow velocity in the downstream direction (ft/sec)
u	the cross-stream average velocity
u_*	the friction velocity, a measure of turbulent mixing
u'	the deviation from the cross-sectional mean speed $\overline{u}$
V_N	an identifiable volume of water whose chemical content is monitored as it journeys downstream
W_1, W_2	the end walls of volume V_N
w	the width of the river (ft)
X	distance measured from the backwater of the lake (mi)
x	spatial coordinate in the downstream direction
x_0	initial space coordinate of volume V_N
x_1	a coordinate measured from the center of volume V_N
x_s	distance measured in the Shepaug arm of Lillinonah (mi)
$x(t)$	the instantaneous downstream location of the center of volume V_N (mi)
y	spatial coordinate in the vertical direction
z	spatial coordinate in the cross-stream direction

$\Delta STOR$	the change in downstream flow (ft^3/sec) due to a change in reservoir storage	
$\Delta v(x)$	the change in water content of the lakes due to changes in surface elevation (ft^3)	
δ	the unit impulse function	
μ	the unit step function	
ρ	the density of water (lb/ft^3)	
τ	travel time (weeks)	

APPENDIX B — WATER QUALITY DATA

Automatic samplers (Brailsford 1965) collected weekly composite water samples at nine stations along the river and its major tributaries for a period of 52 weeks beginning August 1, 1968. Samples were analyzed according to Standard Methods (APHA 1965) for nitrate nitrogen, organic nitrogen, chloride, total phosphorus and volatile, fixed and total solids[1]. Surficial sediments were collected from Lakes Lillinonah and Zoar with an Eckman dredge and analyzed as previously described (Frink 1969). Some conclusions from these data have been published elsewhere (Frink 1970, 1971).

Since our model can be no better than the data used in its calibration, we describe the collection and determination of the various water quality parameters in some detail. The station locations were given in Table II-1, with the exception of a temporary station at the railroad bridge in New Milford approximately at river mile 11.2.

The water samplers were operated periodically by a pump to sample water at a constant rate, collecting a few cm^3 at a time to fill a two-gallon polyethylene container in 1 week. They were battery operated, permitting their use in remote locations, but could not be operated without protection from freezing. Where samplers could be placed in USGS gaging stations, the tygon intake tubing was extended from the gage to the water inlet located generally in the center of the stream. The end was covered with a coarse screen. Where the samplers were located on bridges, the tubing simply dipped into the stream.

Battery failure, vandalism, freezing and other circumstances occasionally caused samples to be lost. Where this occurred, we have replaced the missing observations with either the mean or with values calculated from regression analysis of concentrations and flows, i.e., $C = f(flow)$. Since the original study was intended to examine nutrient inputs during the summer months, samples during the winter were collected weekly but composited biweekly prior to chemical analyses. This was true for all stations for weeks 10-35 (Week 1 = August 1, 1968).

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[1]The analyses were performed by Continental Testing Laboratories, now Enviro Tech Laboratory, Windsor, CT 06095.

Bulls Bridge River Mile 0.0

The composite sampler was located in the generating station owned by CL&P and arranged to periodically sample river water flowing through the generating station. This sampler operated well with few missing observations (MOBS). Missing data were replaced by:

Chemical	MOBS	Replaced by
P	2	Mean
Cl	2	f(flow)
NO_3	3	Mean
Org N	1	f(flow)

Ten Mile River, River Mile 0.9, End of Reach 1

The data are from the USGS Gaging Station #1-2000 located 1.7 miles upstream from the confluence with the Housatonic River. This station, located in New York State was not included in our original survey; hence water quality parameters were estimated from data collected by the New York State Department of Environmental Conservation. Analyses of 40 samples collected at approximately monthly intervals from October 11, 1965 to November 30, 1970 were used to determine water quality as described below:

Phosphorus:
For flows <700 cfs, $ln P = ln A + B\, ln$(flow) with $r^2 = 0.302$.
For flows >700 cfs, the mean = 127 ppb.

Chloride:
$ln Cl = ln A + B\, ln$(flow) with $r^2 = 0.391$.

Nitrogen:
Neither NO_3 nor Kjeldahl Nitrogen showed any dependence on flow, and hence the following means were used: NO_3, 944 ppb; Org N, 524 ppb; Total N = sum of the two.

Candlewood Pump, River Mile 8.85, End of Reach 2.

Candlewood Lake is operated as a pump-storage reservoir. The concentrations of solutes in the water pumped out of the Housatonic River were calculated by adding the load from Bulls Bridge and the Ten Mile River and dividing by the flow in the Housatonic River calculated at Candlewood Pump.

Candlewood Generate, River Mile 8.95, End of Reach 3

The composite sampler was installed in the CL&P station at Rocky River and was activated only when water entered the Housatonic during power generation. The separation of 0.1 mile between Candlewood Pump and Generate was arbitrarily chosen for purposes of the RVRFLO model; in fact, pumping and generation occur physically in the same pipe. The few missing observations were replaced by:

Chemical	MOBS	Replaced by
P	3	Mean
Cl	6	Mean
NO_3	5	f(flow)
org N	6	Mean

West Aspetuck River, River Mile 9.7, End of Reach 4

This station was not included in the original survey. Water quality parameters were estimated as follows:
Chloride:
> In the absence of any other information, chloride was assumed to be the same as in the Housatonic River at Bulls Bridge.

Phosphorus:
> Land use information has been shown to provide accurate predictions of phosphorus in lakes (Norvell, Frink and Hill 1979). Given the land use in the West Aspetuck watershed, and our observations of water quality in Lake Waramaug (Norvell and Frink 1975), we estimate the mean phosphorus concentration as 30 ppb.

New Milford, River Mile 11.2

This was a temporary station with the composite sampler suspended from the railroad bridge below New Milford. The original intention was to determine if municipal discharges at New Milford had a measurable impact on water quality in the Housatonic River. Vandalism and fluctuating water levels limited collection of data at this site to weeks 2-10 in the fall and week 28 in the spring. No statistically significant differences were observed between concentrations of P or Cl predicted by RVRFLO and those observed at this station. However, it should be noted that this was not a protracted sampling period.

Still River, River Mile 12.5, End of Reach 5

The composite sampler was located in the USGS Gaging Station #1-2015.1 at Cross Road in New Milford, 0.9 miles upstream from the confluence with the Housatonic River. Unfortunately, in order to avoid flooding, the sampler was located above the frost line and was inactive for weeks 20-35. Other difficulties at this Station resulted in substantial missing observations as shown below:

Chemical	MOBS	Replaced by
P	29	f(flow)
Cl	30	f(flow)
NO_3	30	Mean
Org N	29	Mean

Shepaug River, River Mile 21.3, End of Reach 6

The composite sampler was located in the USGS Gaging Station #1-2030, 7.3 miles upstream from the confluence with the Housatonic River. The few missing observations were replaced by:

Chemical	MOBS	Replaced by
P	1	f(flow)
Cl	1	f(flow)
NO_3	2	Mean
Org N	1	Mean

Shepaug Dam, River Mile 23.5

The composite sampler was located in the generating station owned by CL&P and periodically sampled river water flowing through the turbine. The few missing observations were replaced by:

Chemical	MOBS	Replaced by
P	4	Mean
Cl	5	Mean
NO_3	5	Mean
Org N	5	Mean

This station is not used in the RVRFLO model directly, but can be used to verify predicted concentrations at Shepaug Dam. It can also be used to calculate actual solute loading into Lake Zoar.

Pootatuck River, River Mile 25.8, End of Reach 7

The composite sampler was suspended from a bridge across the river at the Fabric Fire Hose Co. in Sandy Hook, 1.7 miles from the confluence with Lake Zoar. Consequently, it could not be operated in freezing weather and missing observations are numerous.

Chemical	MOBS	Replaced by
P	28	Mean
Cl	28	Mean
NO_3	29	Mean
Org N	28	Mean

Pomperaug River, River Mile 26.9, End of Reach 8

The composite sampler was located in the USGS Gaging Station No. 1-2040 in Southbury, 5.8 miles upstream from mouth. The few missing observations were replaced by:

Chemical	MOBS	Replaced by
P	10	Mean
Cl	11	Mean
NO_3	10	Mean
Org N	11	f(flow)

Stevenson Dam, River Mile 33.5, End of Reach 9

The composite sampler was located in the generating station owned by CL&P and periodically sampled river water flowing through the turbines. The few missing observations were replaced by:

Chemical	MOBS	Replaced by
P	4	Mean
Cl	2	Mean
NO_3	3	f(flow)
Org N	2	Mean

APPENDED DATA

The data for each station are shown in Tables B-1 through B-11. Two columns of flow data are presented. The first is the observed weekly mean flow for weeks 1-52 beginning on August 1, 1968. These data were used in the water balance discussed in Section III. The second column contains the flows adjusted for errors in the water balance as discussed in Section V, and adjusted for sampling periods which actually began August 13, 1968.

Since our model predicts weekly concentrations, we wished to smooth the step function created by the bi-weekly compositing of samples in winter. A three-point running average was used to smooth the data at all stations for weeks 10-35. Of course, such a running average cannot reconstruct the maxima and minima that may have occurred.

Table B-1. Flow and water quality at Bulls Bridge, River Mile 0.0.

Week	Flow cfs	Flow cfs	P ppb	Cl ppm	NO₃ ppb	Org. N ppb
1	424	400	72	13.4	192	438
2	410	433	66	13.4	192	544
3	313	326	39	14.3	60	643
4	311	302	48	15.1	218	551
5	235	268	52	14.9	158	648
6	356	297	33	14.6	72	600
7	584	591	60	13.0	68	401
8	299	311	88	11.4	69	646
9	313	336	42	13.2	68	354
10	359	344	64	13.3	50	338
11	392	406	64	13.3	50	338
12	383	356	94	12.1	56	346
13	333	355	94	12.1	56	346
14	317	307	131	14.4	47	449
15	618	564	131	14.4	47	449
16	861	743	122	13.0	159	359
17	1032	1112	122	13.0	159	359
18	1312	1272	56	12.5	177	351
19	2356	2356	56	12.5	177	351
20	1307	1288	66	10.6	405	265
21	1363	1388	66	10.6	405	265
22	1400	1380	121	14.8	276	791
23	1429	1473	121	14.8	276	791
24	1105	1099	86	11.8	418	308
25	1018	1014	86	11.8	418	308
26	1557	1541	102	17.6	290	496
27	1585	1530	102	17.6	290	496
28	1120	1218	85	15.4	387	219
29	1267	1263	85	15.4	387	219
30	1000	1006	71	13.0	370	328
31	930	934	71	13.0	370	328
32	848	872	82	15.6	232	585
33	815	770	82	15.6	232	585
34	3524	2771	101	10.1	335	328
35	4355	4836	101	10.1	335	328
36	5982	5392	93	8.5	297	310
37	5125	5662	71	5.3	215	249
38	4149	3558	44	7.2	138	190
39	5707	6345	47	6.5	78	307
40	2054	2263	10	9.5	187	136
41	1897	1879	38	11.5	85	224
42	1481	1418	39	10.1	51	215
43	1640	1744	41	10.0	59	264
44	1320	1360	34	7.6	31	504
45	1035	1069	43	9.2	36	232
46	1541	1383	40	11.0	127	233
47	1269	1397	56	8.2	71	631
48	785	854	100	10.3	229	755
49	547	567	60	12.4	32	398
50	653	622	72	13.1	192	429
51	624	649	49	12.2	69	324
52	1264	675	56	12.5	30	164
MEAN	1402	1390	72	12.2	181	398

Table B-2. Flow and water quality in the Ten Mile River, River Mile 0.9, End of Reach 1.

Week	Flow cfs	Flow cfs	P ppb	Cl ppm	NO$_3$ ppb	Org. N ppb
1	52	58	134	13.3	944	524
2	49	72	136	13.4	944	524
3	40	59	145	13.9	944	524
4	36	41	150	14.1	944	524
5	30	23	158	14.5	944	524
6	49	29	136	13.4	944	524
7	70	103	123	12.7	944	524
8	30	32	158	14.5	944	524
9	29	27	160	14.6	944	524
10	39	33	146	13.9	944	524
11	40	36	145	13.9	944	524
12	44	44	141	13.6	944	524
13	47	37	138	13.5	944	524
14	37	32	148	14.0	944	524
15	115	96	106	11.7	944	524
16	159	139	96	11.1	944	524
17	141	136	100	11.3	944	524
18	146	125	98	11.2	944	524
19	390	377	74	9.6	944	524
20	261	222	83	10.2	944	524
21	263	227	83	10.2	944	524
22	302	124	79	10.0	944	524
23	264	16	83	10.2	944	524
24	217	70	88	10.6	944	524
25	183	82	92	10.8	944	524
26	316	205	78	9.9	944	524
27	309	269	79	10.0	944	524
28	201	89	90	10.7	944	524
29	209	69	88	10.6	944	524
30	183	126	92	10.8	944	524
31	163	117	95	11.1	944	524
32	153	88	97	11.2	944	524
33	224	200	87	10.5	944	524
34	1710	1144	127	7.6	944	524
35	1394	1578	127	7.8	944	524
36	1095	735	127	8.1	944	524
37	687	535	62	8.8	944	524
38	857	708	127	8.5	944	524
39	1090	1239	127	8.1	944	524
40	460	481	70	9.4	944	524
41	470	445	70	9.3	944	524
42	359	303	75	9.7	944	524
43	317	316	78	9.9	944	524
44	205	192	89	10.7	944	524
45	182	192	92	12.4	944	524
46	336	276	77	9.8	944	524
47	244	225	84	10.4	944	524
48	133	135	101	11.4	944	524
49	90	71	114	12.2	944	524
50	108	81	108	11.8	944	524
51	121	86	104	11.6	944	524
52	264	104	83	10.2	944	524
MEAN	286	235	106	11.2	944	524

Table B-3. Flow and water quality at Candlewood Pump, River Mile 8.85, End of Reach 2.

Week	Flow cfs	Flow cfs	P ppb	Cl ppm	NO₃ ppb	Org. N ppb
1	0	0	79	13.3	274	447
2	0	0	73	13.4	272	541
3	0	0	51	14.2	160	629
4	0	0	59	14.9	293	548
5	0	0	64	14.8	246	633
6	0	0	45	14.4	177	590
7	-11	-15	67	12.9	161	414
8	0	0	94	11.6	148	634
9	0	0	52	13.3	142	368
10	0	0	72	13.3	137	356
11	0	0	72	13.3	132	355
12	0	0	99	12.2	147	364
13	0	0	99	12.2	165	368
14	0	0	133	14.3	140	456
15	-80	-57	127	13.9	187	460
16	-73	-72	118	12.7	281	384
17	-32	-32	119	12.7	253	378
18	-169	-168	60	12.3	253	368
19	-123	-122	59	12.0	285	375
20	-37	-33	69	10.5	494	308
21	-84	-53	69	10.5	492	306
22	-104	-38	114	13.9	394	743
23	0	-2	115	14.0	380	749
24	0	0	86	11.6	504	343
25	-98	-36	87	11.6	498	340
26	-20	-26	98	16.3	400	500
27	-41	-35	98	16.3	396	500
28	0	0	86	14.6	471	265
29	0	0	85	14.7	465	262
30	-34	-18	74	12.6	458	358
31	-11	-13	75	12.7	455	357
32	0	0	84	14.9	340	575
33	0	0	83	14.5	385	571
34	-80	-48	109	9.2	533	392
35	-218	-192	107	9.5	482	375
36	-218	-149	98	8.4	397	343
37	-106	-82	70	5.7	301	281
38	-75	-94	58	7.4	275	247
39	0	-10	60	6.7	216	341
40	-19	-18	21	9.4	325	206
41	-84	-57	44	11.0	255	283
42	-127	-129	46	10.0	225	275
43	-76	-58	47	9.9	202	306
44	-57	-56	41	8.0	153	506
45	-63	-61	50	9.6	171	275
46	-69	-58	47	10.7	273	285
47	-40	-51	61	8.5	211	613
48	-20	-19	100	10.4	332	721
49	-26	-14	68	12.3	160	415
50	-38	-36	77	12.9	298	442
51	-11	-8	58	12.1	211	356
52	-19	0	61	12.1	187	226
MEAN	-43	-35	76	12.0	292	418

Table B-4. Flow and water quality in Candlewood Generate, River Mile 8.95, End of Reach 3.

Week	Flow cfs	Flow cfs	P ppb	Cl ppm	NO_3 ppb	Org. N ppb
1	82	97	20	7.6	43	323
2	23	41	3	5.7	57	200
3	35	26	9	5.3	18	634
4	63	86	3	6.7	48	123
5	28	13	11	6.2	41	124
6	65	55	5	6.0	27	226
7	49	57	3	6.2	27	238
8	69	46	4	5.8	36	323
9	146	155	3	6.8	37	169
10	252	245	7	6.1	18	295
11	100	85	7	6.1	18	295
12	60	67	9	6.2	34	222
13	54	46	9	6.2	34	222
14	19	12	32	9.0	40	269
15	61	64	32	9.0	40	269
16	94	74	34	10.3	44	414
17	18	25	34	10.3	44	414
18	12	7	49	9.4	71	533
19	32	41	49	9.4	71	533
20	41	36	45	8.5	101	584
21	7	2	45	8.5	101	584
22	32	12	12	8.2	116	319
23	47	2	12	8.2	116	319
24	21	12	23	8.8	32	437
25	90	29	23	8.8	32	437
26	128	94	23	7.4	25	461
27	43	51	23	7.4	25	461
28	28	12	18	8.4	34	433
29	24	7	18	8.4	34	433
30	144	95	16	7.8	35	385
31	84	55	16	7.8	35	385
32	81	53	9	7.2	24	164
33	92	96	9	7.2	24	164
34	26	32	41	8.1	137	432
35	3	3	41	8.1	137	432
36	37	12	88	7.6	55	323
37	155	90	25	5.2	10	327
38	90	157	20	6.5	22	291
39	201	119	21	7.6	15	210
40	253	285	5	7.0	51	380
41	79	104	3	9.5	42	73
42	137	120	32	7.6	37	323
43	19	22	21	7.6	58	323
44	104	89	17	7.8	48	299
45	69	45	15	8.8	16	374
46	142	155	8	7.3	5	228
47	76	56	8	6.6	36	267
48	93	93	20	9.3	44	160
49	18	11	25	7.4	27	200
50	135	84	12	9.8	19	227
51	81	81	11	7.2	38	195
52	69	41	20	7.6	46	323
MEAN	75	65	20	7.6	44	322

Table B-5. Flow and water quality in the West Aspetuck River, River Mile 9.7, End of Reach 4.

Week	Flow cfs	Flow cfs	P ppb	Cl ppm	NO₃ ppb	Org. N ppb
1	4	4	30	13.4	192	438
2	3	4	30	13.4	192	544
3	2	3	30	14.3	60	643
4	3	3	30	15.1	218	551
5	3	2	30	14.9	158	648
6	9	2	30	14.6	72	600
7	12	24	30	13.0	68	401
8	3	3	30	11.4	69	646
9	3	3	30	13.2	68	354
10	7	6	30	13.3	50	338
11	4	4	30	13.3	50	338
12	6	6	30	12.1	56	346
13	6	5	30	12.1	56	346
14	5	4	30	14.4	47	449
15	31	26	30	14.4	47	449
16	32	29	30	13.0	159	359
17	20	20	30	13.0	159	359
18	25	20	30	12.5	177	351
19	70	70	30	12.5	177	351
20	39	35	30	10.6	405	265
21	38	31	30	10.6	405	265
22	29	12	30	14.8	276	791
23	22	1	30	14.8	276	791
24	19	6	30	11.8	418	308
25	20	8	30	11.8	418	308
26	39	26	30	17.6	290	496
27	39	33	30	17.6	290	496
28	26	12	30	15.4	387	219
29	24	8	30	15.4	387	219
30	23	15	30	13.0	370	328
31	23	17	30	13.0	370	328
32	22	13	30	15.6	232	585
33	32	32	30	15.6	232	585
34	193	145	30	10.1	335	328
35	125	135	30	10.1	335	328
36	145	98	30	8.5	297	310
37	67	52	30	5.3	215	249
38	148	130	30	7.2	138	190
39	145	167	30	6.5	78	307
40	50	52	30	9.5	187	136
41	73	69	30	11.5	85	224
42	54	41	30	10.1	51	215
43	40	44	30	10.0	59	264
44	24	23	30	7.6	31	504
45	16	16	30	9.2	36	232
46	93	79	30	11.0	127	233
47	45	40	30	8.2	71	631
48	22	23	30	10.3	229	755
49	13	11	30	12.4	32	398
50	40	29	30	13.1	192	429
51	31	23	30	12.2	69	324
52	47	18	30	12.5	30	164
MEAN	38	32	30	12.2	181	398

Table B-6. Flow and water quality in the Still River, River Mile 12.5, End of Reach 5.

Week	Flow cfs	Flow cfs	P ppb	Cl ppm	NO₃ ppb	Org. N ppb
1	33	36	847	28.2	742	933
2	96	135	691	20.4	742	732
3	43	66	365	24.2	359	849
4	34	39	1238	26.3	498	412
5	30	23	1309	30.5	914	646
6	65	34	775	27.2	742	933
7	94	147	710	26.3	742	933
8	34	37	845	28.2	742	933
9	30	29	854	28.4	742	933
10	36	33	840	28.2	742	933
11	32	27	849	28.3	742	933
12	37	39	1205	30.0	388	1430
13	35	27	1205	30.0	388	1430
14	31	26	1117	30.6	1160	1420
15	128	101	1117	30.6	1160	1420
16	172	155	567	24.8	1480	1220
17	112	118	567	24.8	1480	1220
18	111	91	371	21.0	630	862
19	328	334	371	21.0	630	862
20	131	116	627	25.1	742	933
21	132	104	625	25.1	742	933
22	111	48	672	25.8	742	933
23	81	5	739	26.7	742	933
24	71	23	762	27.0	742	933
25	91	40	717	26.4	742	933
26	120	82	652	25.5	742	933
27	114	97	665	25.6	742	933
28	67	29	771	27.2	742	933
29	78	26	746	26.8	742	933
30	77	52	748	26.8	742	933
31	67	49	771	27.2	742	933
32	53	32	802	27.6	742	933
33	102	89	692	26.0	742	933
34	400	275	23	16.4	742	933
35	299	350	250	19.7	742	933
36	224	153	418	22.1	742	933
37	139	106	296	22.4	88	766
38	247	207	433	21.8	40	570
39	323	359	287	20.0	583	755
40	136	140	616	24.9	742	933
41	184	173	508	23.4	742	933
42	144	109	598	24.7	742	933
43	126	134	329	19.6	751	1130
44	78	73	485	23.6	2470	574
45	69	72	678	25.2	1950	466
46	131	116	245	21.8	1070	1540
47	69	62	766	27.1	742	933
48	46	45	818	27.8	742	933
49	34	27	257	28.2	161	1780
50	30	24	886	36.0	40	353
51	35	25	870	31.0	43	526
52	127	47	691	30.3	45	507
MEAN	108	92	678	25.8	742	933

Table B-7. Flow and water quality in the Shepaug River, River Mile 21.3, End of Reach 6.

Week	Flow cfs	Flow cfs	P ppb	Cl ppm	NO₃ ppb	Org. N ppb
1	21	24	9	9.4	131	270
2	20	30	16	8.4	131	317
3	17	25	7	8.2	68	446
4	15	18	3	8.2	45	121
5	14	11	3	9.3	47	241
6	66	40	3	8.8	51	320
7	34	58	3	8.4	67	381
8	19	20	15	7.9	56	452
9	15	15	13	8.4	82	289
10	33	30	7	8.6	39	240
11	20	18	7	8.6	39	240
12	24	25	17	9.5	41	161
13	25	18	17	9.5	41	161
14	18	18	3	10.7	35	244
15	78	64	3	10.7	35	244
16	128	113	10	10.0	111	353
17	97	94	10	10.0	111	353
18	180	111	11	8.0	146	331
19	357	406	11	8.0	146	331
20	154	127	5	8.1	236	291
21	163	139	5	8.1	236	291
22	190	76	9	9.0	306	316
23	167	10	9	9.0	306	316
24	115	39	6	9.4	283	186
25	72	31	6	9.4	283	186
26	151	99	14	10.3	267	383
27	170	145	14	10.3	267	383
28	148	60	6	12.2	291	238
29	178	60	6	12.2	291	238
30	147	102	23	12.6	349	206
31	114	85	23	12.6	349	206
32	90	53	19	13.0	172	125
33	103	86	19	13.0	172	125
34	846	605	14	7.1	309	230
35	883	889	14	7.1	309	230
36	963	643	21	6.4	148	307
37	515	410	34	5.4	54	331
38	697	551	36	6.0	144	340
39	832	975	10	6.0	46	302
40	318	339	10	7.0	52	166
41	358	336	9	8.0	46	46
42	297	235	4	6.5	42	226
43	253	268	21	5.2	60	221
44	140	131	10	5.5	34	346
45	90	98	12	6.0	19	253
46	444	355	9	6.3	29	326
47	286	270	5	5.2	74	346
48	123	129	13	7.8	66	310
49	60	49	9	9.2	51	224
50	91	66	11	9.8	49	233
51	149	97	7	7.6	51	256
52	509	224	10	8.6	103	337
MEAN	211	171	11	8.6	133	269

Table B-8. Flow and water quality at Shepaug Dam, River Mile 23.5.

Week	Flow cfs	Flow cfs	P ppb	Cl ppm	NO$_3$ ppb	Org. N ppb
1	529	763	68	12.1	268	388
2	681	750	68	12.1	268	388
3	572	594	22	12.1	268	388
4	608	646	68	12.1	268	388
5	314	320	44	13.8	254	329
6	474	267	59	14.3	67	344
7	979	1166	48	14.1	149	366
8	779	716	58	13.4	82	474
9	369	419	48	13.2	62	351
10	898	896	37	12.2	68	372
11	656	628	37	12.2	68	372
12	625	618	67	13.2	36	530
13	379	373	67	13.2	36	530
14	420	376	67	14.5	59	394
15	758	680	67	14.5	59	394
16	1661	1336	152	14.0	45	594
17	1420	1526	152	14.0	45	594
18	1845	1776	56	13.8	217	585
19	4292	4348	56	13.8	217	585
20	1697	1292	71	10.8	359	480
21	2007	2155	71	10.8	359	480
22	2184	1765	62	9.6	447	490
23	2308	1594	62	9.6	447	490
24	1678	1276	89	13.8	455	336
25	1423	1129	89	13.8	455	336
26	2590	2300	104	14.2	559	443
27	2394	2193	104	14.2	559	443
28	1580	1471	99	18.2	575	340
29	2143	1651	99	18.2	575	340
30	1641	1425	91	15.2	608	297
31	1563	1295	91	15.2	608	297
32	1635	1665	123	16.4	602	293
33	1541	1338	123	16.4	602	293
34	7875	5851	101	11.8	560	269
35	7833	8518	101	11.8	560	269
36	8725	7218	97	12.1	493	360
37	7224	7459	71	5.5	346	379
38	6583	5356	58	7.9	273	314
39	9333	10510	68	7.5	240	304
40	3841	3895	19	8.0	242	380
41	3522	3595	14	9.5	155	79
42	1945	1669	29	10.6	334	431
43	2887	2759	17	9.3	262	276
44	1531	1802	39	9.0	99	728
45	1434	1417	29	10.2	33	254
46	3021	2690	28	9.6	40	206
47	2307	2247	48	7.0	60	291
48	1485	1465	48	8.0	69	359
49	700	820	57	11.3	42	183
50	1230	936	61	8.2	62	254
51	1059	912	68	12.1	268	388
52	2839	1593	78	11.3	41	738
MEAN	2308	2143	68	12.1	267	388

Table B-9. Flow and water quality in the Pootatuck River, River Mile 25.8, End of Reach 7.

Week	Flow cfs	Flow cfs	P ppb	Cl ppm	NO₃ ppb	Org. N ppb
1	12	13	218	16.0	263	506
2	8	13	209	13.0	263	618
3	5	7	218	16.0	263	506
4	4	6	218	16.0	263	506
5	3	2	218	16.0	263	506
6	28	17	270	15.7	281	375
7	9	19	175	11.4	144	520
8	5	5	384	16.1	89	1030
9	5	5	388	16.5	200	594
10	9	8	391	16.0	229	581
11	5	5	391	16.0	229	581
12	9	8	312	16.0	264	400
13	9	6	312	16.0	264	400
14	10	8	339	16.8	475	582
15	52	42	339	16.8	475	582
16	60	56	110	13.1	473	582
17	40	40	110	13.1	473	582
18	61	41	218	16.0	263	506
19	99	115	218	16.0	263	506
20	44	37	218	16.0	263	506
21	68	53	218	16.0	263	506
22	43	20	218	16.0	263	506
23	31	2	218	16.0	263	506
24	27	8	218	16.0	263	506
25	35	16	218	16.0	263	506
26	55	38	218	16.0	263	506
27	49	40	218	16.0	263	506
28	25	12	218	16.0	263	506
29	27	9	218	16.0	263	506
30	27	18	218	16.0	263	506
31	27	19	218	16.0	263	506
32	27	16	218	16.0	263	506
33	56	50	218	16.0	263	506
34	273	234	218	16.0	263	506
35	98	103	218	16.0	263	506
36	83	59	218	16.0	263	506
37	53	39	218	16.0	263	506
38	174	164	218	16.0	263	506
39	101	122	218	16.0	263	506
40	47	50	218	16.0	263	506
41	62	57	33	13.0	162	252
42	66	57	218	16.0	263	506
43	41	41	18	13.1	293	397
44	25	23	61	13.5	120	413
45	16	18	88	13.5	32	678
46	28	19	138	16.8	135	353
47	19	17	142	17.8	844	451
48	12	13	139	18.0	116	555
49	9	7	218	21.2	96	243
50	12	9	250	22.0	216	525
51	13	9	217	20.2	151	386
52	90	40	187	17.2	293	458
MEAN	42	35	217	15.9	263	505

Table B-10. Flow and water quality in the Pomperaug River, River Mile 26.9, End of Reach 8.

Week	Flow cfs	Flow cfs	P ppb	Cl ppm	NO₃ ppb	Org. N ppb
1	27	29	14	12.2	228	316
2	26	41	14	12.2	228	316
3	16	23	14	12.2	228	312
4	14	17	14	12.2	228	311
5	11	9	4	9.6	91	209
6	93	57	8	11.2	99	197
7	33	61	34	13.2	43	468
8	15	17	7	10.0	141	311
9	12	11	4	12.8	58	486
10	18	16	20	9.8	70	248
11	15	13	20	9.8	70	248
12	19	19	36	11.5	103	493
13	26	20	36	11.5	103	493
14	21	19	11	12.0	113	391
15	97	75	11	12.0	113	391
16	156	142	9	11.2	230	579
17	107	105	9	11.2	230	579
18	128	91	8	11.6	288	290
19	256	277	8	11.6	288	290
20	137	121	7	13.2	311	275
21	140	112	7	13.2	311	275
22	107	46	17	23.0	644	346
23	84	5	17	23.0	644	346
24	61	20	8	17.2	396	203
25	63	27	8	17.2	396	203
26	137	91	11	14.3	228	415
27	135	116	11	14.3	228	415
28	73	33	6	11.5	233	351
29	66	22	6	11.5	233	351
30	70	46	12	13.8	162	227
31	73	52	12	13.8	162	227
32	70	40	10	13.2	291	233
33	137	116	10	13.2	291	233
34	600	482	18	7.4	396	614
35	327	357	18	7.4	396	614
36	289	199	14	12.2	228	432
37	172	130	32	8.5	158	549
38	375	292	19	9.3	68	302
39	362	447	7	13.0	27	317
40	148	156	11	10.5	43	334
41	181	170	14	12.2	228	384
42	159	114	14	12.2	228	375
43	131	147	14	12.2	228	362
44	77	74	6	9.1	40	780
45	49	53	8	8.8	31	282
46	141	119	14	12.2	228	367
47	79	71	20	8.2	369	264
48	40	42	11	11.2	389	212
49	24	19	16	12.2	384	160
50	32	24	22	14.5	332	161
51	44	31	14	12.2	228	324
52	271	115	54	10.0	375	597
MEAN	114	94	14	12.1	228	354

Table B-11. Flow and water quality at Stevenson Dam, River Mile 33.5, End of Reach 9.

Week	Flow cfs	Flow cfs	P ppb	Cl ppm	NO₃ ppb	Org. N ppb
1	811	853	37	11.0	44	457
2	762	786	46	7.8	43	452
3	732	647	23	7.6	49	480
4	625	731	36	8.7	38	745
5	375	307	37	11.0	40	457
6	645	396	26	10.0	78	428
7	1048	1248	20	8.6	77	480
8	757	739	18	10.7	76	680
9	403	472	34	9.6	68	572
10	878	905	29	10.0	81	515
11	659	616	29	10.0	81	515
12	658	670	27	10.5	53	423
13	419	418	27	10.5	53	423
14	439	456	17	14.2	66	545
15	907	832	17	14.2	66	545
16	1897	1625	37	13.5	72	1170
17	1553	1738	37	13.5	72	1170
18	2216	1999	33	13.6	61	662
19	4599	4826	33	13.6	61	662
20	1823	1576	27	9.7	59	613
21	2171	2407	27	9.7	59	613
22	1821	1884	29	10.5	49	397
23	1604	1602	29	10.5	49	397
24	1323	1298	45	13.0	53	532
25	1283	1217	45	13.0	53	532
26	2451	2439	63	13.3	34	364
27	2530	2466	63	13.3	34	364
28	1365	1490	60	15.5	30	415
29	1724	1704	60	15.5	30	415
30	1539	1546	64	16.0	47	336
31	1438	1382	64	16.0	47	336
32	1541	1717	86	15.4	41	357
33	1962	1573	86	15.4	41	357
34	8448	6766	49	12.4	141	493
35	7878	9249	49	12.4	141	493
36	8084	7522	80	7.8	122	506
37	7147	7745	52	7.0	41	297
38	7308	5958	38	7.0	103	160
39	9929	11440	36	6.5	47	353
40	4127	4299	32	8.3	52	281
41	4097	4207	10	9.5	30	92
42	1825	1588	5	9.6	28	218
43	2937	3016	10	9.2	23	250
44	1630	1962	9	9.4	24	431
45	1576	1559	18	9.7	16	429
46	3134	2861	18	9.4	79	259
47	2219	2417	22	10.8	81	233
48	1530	1560	49	8.8	86	379
49	689	832	65	9.1	113	607
50	1166	997	42	9.5	41	316
51	1020	990	27	9.8	35	334
52	3017	1697	33	10.4	24	221
MEAN	2359	2331	37	11.0	58	456

```
C       THIS PROGRAM COMPUTES THE TIME, DISTANCE AND CONCENTRATION
C       COORDINATES OF A PARCEL OF WATER (NEUTRAL DENSITY) IN THE
C       HOUSATONIC RIVER FROM BULLS BRIDGE TO STEVENSON DAM.  IT USES
C       ONE-DIMENSIONAL ANALYSIS AND TAKES THE LAGRANGIAN POINT OF VIEW.
C       INITIAL COORDINATES ARE READ IN AS PARAMETERS.
        COMMON Q(10,52),HL(53),HZ(53),CP(10,52)
        DIMENSION RLR(9),JD(9)
C       THE RLR ARE DISTANCES IN RIVER MILES TO THE MAJOR TRIBUTARIES.
        DATA RLR/0.9,8.85,8.95,9.7,12.5,21.3,25.8,26.9,33.5/
C       READ FLOW DATA MATRIX Q(J,I) IN CFS, WHERE J=RIVER SEGMENT,
C       I=WEEK, J=1 CORRESPONDS TO SECTION FROM BULLS BRIDGE TO MOUTH OF
C       TEN MILE RIVER, ETC., WHILE I=1 CORRESPONDS TO WEEK BEGINNING
C       AUGUST 1, 1968.
      1 READ (5,500,END=5)((Q(J,I),I=1,52),J=1,10)
        READ(5,500)((CP(J,I),I=1,52),J=1,10)
    500 FORMAT(5(5X,10F7.0/),5X,2F7.0,61X)
C       READ GAGE HEIGHTS FOR LILLINONAH AND ZOAR
      2 READ(5,502) (HL(I),I=1,53),(HZ(I),I=1,53)
    502 FORMAT(5(10F6.1/),3F6.1,62X)
C       READ INITIAL COORDS, JDO=INITIAL WEEK, JDO MUST BE .GE. 2, JDEND=
C       FINAL WEEK, JDEND MUST BE .LE. 51
      3 READ(5,503)JDO,JDEND
    503 FORMAT(2I4)
      5 WRITE(6,6000)
   6000 FORMAT(1H ,'NEED MORE DATA')
C       INITIALIZE TIME, SECTION, DISTANCE, INTEGRATION INTERVAL, N, N2
C       AND JD(L). THE USE OF N AND JD(L) IS DESCRIBED BELOW AND THAT OF
C       N2 IS DESCRIBED IN SUBROUTINE CONC.
    504 DO 111 L=1,8
        JD(L)=0
    111 CONTINUE
        X=0.0
        XOLD=0.
        T=0.5
        J=1
        I=JDO
        N=0
        N2=0
        IF(X.LT.0.0.OR.X.GT.33.5)STOP
        WRITE(6,603)
    603 FORMAT(1H ,5X,'RVR SEG      WK     FRAC-WK    MILE     AREA      WKLY
       CMN FLOW       PL           TP'//)
C       THE FOLLOWING DO LOOP DETERMINES THE VALUE OF NK TO BE USED BY
C       SUBROUTINE FLOW.  NK TELLS HOW MANY TRIBUTARY FLOWS MUST BE SUMMED
C       IN RQ.
      9 DO 52 NN=1,9
        IF(J-NN)51,51,52
     51 NK=NN
        GO TO 59
     52 CONTINUE
C       THE NONLINEAR INTEGRAL EQN X(T)=XO+INTGL:V(X,T)*DT IS RECAST
C       AS AN ORD. DIFF. EQN. AND SOLVED BY RUNGE-KUTTA TECHNIQUE.
C       THE FACTOR 7*24*(3600/5280)=114.545454 GIVES DISTANCE IN MILES
C       IF DT IS FRACWK.
```

```
     59 DT=0.1
     60 CALL FLOW (NK,I,X,RQ,T)
     70 CALL AREA (RQ,X,I,A)
     72 A1=(114.545454)*(RQ/A)*DT
        X1=X+A1/2.
        T1=T+DT/2.
        CALL FLOW(NK,I,X1,RQ,T1)
        CALL AREA(RQ,X1,I,A)
        A2=(114.545454)*(RQ/A)*DT
        X2=X+2.*A2-A1
        T2=T+DT
        CALL FLOW(NK,I,X2,RQ,T2)
        CALL AREA(RQ,X2,I,A)
        A3=(114.545454)*(RQ/A)*DT
        DX=(1./6.)*(A1+4.*A2+A3)
        X=X+DX
        T=T+DT
        IF(N.GT.0) GO TO 700
C        THE PURPOSE OF THE FOLLOWING STATEMENT IS TO AVOID PRINTOUT WHEN
C        LITTLE DOWNSTREAM PROGRESS HAS BEEN MADE.
     73 IF((X-XOLD).LT..09) GO TO 60
C        THIS APPROACH, ALTHOUGH THRIFTY OF PRINTING TIME, CAN GIVE TROUBLE
C        WHEN A CONFLUENCE WITH A TRIBUTARY IS BEING APPROACHED, I.E.
C        STMNT NO. 80 ASKS RLR(J)-X TO BE .LT..05  THEREFORE, IF X HAD
C        OVERSHOT RLR(J) (SEE STMNT NO. 227) WE MUST REDUCE DX AND ITERATE
C        TO APPROACH THE CONFLUENCE FROM UPSTREAM.  IN THIS CASE WE MUST
C        AVOID STMNT NO. 73.  SETTING N=1 IN STMNT 227 ALLOWS 73 TO BE
C        AVOIDED.
    700 CALL CONC (RQ,PL,TP,NP,NQ,NR,NS,NT,NU,NX,NV,NW,J,N2,JDO,X,T,I)
        IF(X/RLR(J).LE.1.) GO TO 80
        IF(N.GT.0.AND.ABS(X-RLR(J)).LE..01) GO TO 80
    227 N=1
        X=X-DX
        T=T-DT
        DT=DT/2.
        GO TO 60
C        WHEN X GETS CLOSE TO, BUT LESS THAN RLR(J)--DEPENDING ON STMT. 80--
C        X IS SET EQUAL TO RLR(J) AND  J IS INCREMENTED BY 1.  THEREFORE THE
C        NEXT LINE PRINTED GIVES THE NEW J AND X=RLR(J) BUT AREA, FLOW, LOAD
C        AND CONCENTRATION ALL RETAIN THEIR VALUES JUST UPSTREAM OF THE
C        CONFLUENCE.  THESE VALUES ARE CALCULATED AND PRINTED FOR THE NEW
C        J ON THE NEXT LINE.
     80 IF((RLR(J)-X).GT..05) GO TO 100
C        THE FOLLOWING JD(J) VECTOR KEEPS TRACK OF WHEN THE PARCEL OF
C        WATER WE ARE FOLLOWING ARRIVES AT EACH CONFLUENCE.
        JD(J)=I
        NP=JD(1)
        NQ=JD(2)
        NR=JD(3)
        NS=JD(4)
        NT=JD(5)
        NU=JD(6)
C        WHEN X(T) GETS TO MILE 21.3 ON WK NU, THE SHPARM IS RUN
C        BACKWARDS TO DETERMINE THE WK NX WHEN THAT WATER ENTERING THE
```

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C       MAIN STREAM ACTUALLY LEFT THE MOUTH OF THE SHEPAUG RIVER
C       ACCORDING TO ONE-DIM ANALYSIS. THE SHPARM IS 3.25 MI. LONG.
C       THE LOAD ENTERING HOUSATONIC IS CONC ON WK NX TIMES FLOW ON
C       WEEK NU.
        IF(J .NE. 6) GO TO 85
        CALL SHPARM(NU,T,T1,NXX)
        NX=NXX
     85 NV=JD(7)
        NW=JD(8)
        X=RLR(J)
        J=J+1
        N=0
        N2=0
        IF(J.GT.9) GO TO 1900
    100 IF((1.-T).GT..05)GO TO 110
        IOLD=I
        WRITE (6,604)J,IOLD,T,X,A,RQ,PL,TP
        I=I+1
C       INITIALIZE TIME WEEKLY
        T=0.
        IF(I.GT.JDEND)GO TO 2000
C       OCCASIONALLY PROGRAM WILL NOT PRINT A RIVER SEGMENT DEPENDING ON
C       THE VALUE OF DT CHOSEN.  THIS HAS NO EFFECT, REPEAT NO EFFECT ON
C       CALCULATIONS.
    110 WRITE(6,604)J,I,T,X,A,RQ,PL,TP
    604 FORMAT(8X,I2,7X,I4,3X,F6.2,4X,F5.2,3X,F7.0,5X,F7.0,5X,F8.0,5X,F7.1
       A)
        XOLD=X
        GO TO 9
   1900 JKJ=J-1
        WRITE(6,604)JKJ,I,T,X,A,RQ,PL,TP
        WRITE(6,1901)
   1901 FORMAT(1H ,'STOP J EXCEEDED 9'/)
        RQ10=RQ
C       THE C'S AVERAGE THE FLOW AND THE CONCENTRATION ACCORDING TO
C       FRACTION OF WEEK WHEN PARCEL ARRIVES AT STEVENSON.
        IF(T.GE.0.5) GO TO 27
        C1=0.5-T
        C2=T+0.5
        C3=0.0
        GO TO 29
     27 C1=0.0
        C2=1.5-T
        C3=T-0.5
     29 CP10=C1*CP(10,I-1)+C2*CP(10,I)+C3*CP(10,I+1)
        Q10=C1*Q(10,I-1)+C2*Q(10,I)+C3*Q(10,I+1)
        WRITE(6,1902) RQ10, Q10, CP10
   1902 FORMAT(1H ,'***RQ10 EQUALS  ',F7.0,10X,'***Q10 EQUALS ',F7.0,10X,
       A'***TP10 EQUALS ',F7.1)
        GO TO 3000
   2000 WRITE(6,2001)
   2001 FORMAT(1H ,'STOP I EXCEEDED  JDEND'/)
        GO TO 3001
   3000 JDO=JDO+1
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         GO TO 504
    3001 STOP
         END
C $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
         SUBROUTINE FLOW(NK,I,X,RQ,T)
         COMMON Q(10,52),HL(53),HZ(53),CP(10,52)
C        THIS SUBPROGRAM CALCULATES THE TOTAL FLOW FOR WK I AT RIVER
C        DISTANCE X.  RESERVOIR STORAGE AND UNGAGED RUNOFF ARE ACCOUNTED
C        FOR.  THE YEAR IS DIVIDED INTO 13 28-DAY (OR 4-WK) MONTHS.
C        RQ IS THE TOTAL CORRECTED FLOW AND IS THE INFO RETURNED TO  MAIN.
C        FACTORS C1, C2, AND C3 ARE TIME WEIGHTING FACTORS TO SMOOTH THE
C        STEPWISE FLOW.
         IF(T .GE. 0.5) GO TO 2
         C1 = 0.5 - T
         C2 = T + .5
         C3 = 0.
         GO TO 5
       2 C1 = 0.
         C2 = 1.5 - T
         C3 = T - .5
       5 RQ = 0.0
         DO 10 JJ=1,NK
         RQ=RQ+(C1*Q(JJ,I-1) + C2*Q(JJ,I) + C3*Q(JJ,I+1))
      10 CONTINUE
C        THE UNGAGED AREA TOTALS 185 OF THE 551 SQ. MI. OR 34% OF THE
C        WATERSHED BETWEEN GAYLORDSVILLE AND STEVENSON.  UNGAGED AREA
C        IS APPORTIONED TO FOUR SUBWATERSHEDS ACCORDING TO GAY=30.7 SQ.
C        MI., WA=41.2 SQ. MI., SHP=66.3 SQ. MI., PMP=46.5 SQ. MI.
C        THIS SECTION ADDS THE UNGAGED FLOW DEPENDING ON THE CURRENT RIVER
C        SECTION JJ
         IF (NK.LT.2) GO TO 20
         GU=(C1*Q(1,I-1) + C2*Q(1,I) + C3*Q(1,I+1)) * (30.7/994.)
         RQ=RQ + GU
         IF (NK.LT.5) GO TO 20
         WAU=(C1*Q(5,I-1) + C2*Q(5,I) + C3*Q(5,I+1)) * (41.2/23.3)
         RQ=RQ + WAU
         IF(NK.LT.7) GO TO 20
         SU=(C1*Q(7,I-1) + C2*Q(7,I) + C3*Q(7,I+1)) * (66.3/133.)
         RQ=RQ + SU
         IF (NK.LT.9) GO TO 20
         PU=(C1*Q(9,I-1) + C2*Q(9,I) + C3*Q(9,I+1)) * (46.5/75.3)
         RQ=RQ + PU
      20 IF(X.LE.10.9) GO TO 25
C        ADD CHANGE IN LILLINONAH STORAGE TO GIVE CORRECTED FLOW
C        HL(I) CORRESPONDS TO HEIGHT AT 0100 OF  WK I THUS CHANGE IN
C        STORAGE IS HL(I+1)-HL(I)
C        THE AMOUNT OF CHANGE IN STORAGE ADDED TO THE FLOW VARIES
C        WITH UPSTREAM DISTANCE BY THE FRACTION FRCTXL WHICH =1
C        AT THE DAM AND = 0 AT LIMIT OF BACKWATER.
C        DVDH FOR ENTIRE RESERVOIR IS 0.781E+08 (FT)3/FT. OF THIS,
C        0.658E+08 IS IN THE MAIN ARM AND 0.123E+08 IS IN SHPARM.
         IF(X.GT.23.5)GO TO 24
      22 FRCTXL=0.2016*(X-10.9)**0.632
      23 RQ=RQ-(0.658E+08*(HL(I+1)-HL(I))/(24.*3600.*7.))*FRCTXL
```

```
          GO TO 25
      24  F=24.*3600.*7.
          FRCTXZ=0.269*(X-23.5)**0.57
          RQ=RQ-(0.658E+08*(HL(:+1)-HL(I))/F)
         C-(0.442857E+08*(HZ(I+1)-HZ(I))/F)*FRCTXZ
      25  RETURN
          END
C $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
          SUBROUTINE AREA(RQ,X,I,A)
          COMMON Q(10,52),HL(53),HZ(53),CP(10,52)
C         THIS SUBPROGRAM CALCULATES THE CROSS-SECTIONAL AREA FOR FLOW IN
C         THE RIVER AND RESERVOIR AS A FUNCTION OF X AND T.
C         THREE SEPARATE AREA FUNCTIONS ARE USED.  IF X IS LESS THAN 10.9
C         THE AREA AS A FUNCTION OF FLOW IS DETERMINED BY EXPONENTIAL
C         REGRESSION ANALYSIS OF GAYLORDSVILLE GAGE HGT VS FLOW AND AREA
C         VS GAGE HGT.  IF X .GT. 10.9 BUT .LT. 23.5 USE AREA OF LILLINONAH.
C         IF X .GT. 23.5 USE AREA OF ZOAR.  THESE AREAS ARE OBTAINED FROM
C         CONTOUR SHEETS WITH DEPTHS TAKEN AT 10 FOOT INTERVALS STARTING AT
C         FULL POND.  AREA VS RIVER MILES IS THEN FITTED WITH AN EXPONENTIAL
C         REGRESSION AS A FUNCTION OF CL&P RESERVOIR HEIGHTS.
          IF(X.GE.10.9) GO TO 10
          A=18.4 * RQ ** 0.52
          GO TO 40
      10  IF(X.GT.23.5) GO TO 20
C         C L AND P DATUM WAS USED FOR LILLINONAH
          XLIL=HL(I)
          IF(XLIL.GE.170.) GO TO 11
          A2=EXP(-11.8936 + 0.06268 * XLIL)
          B2=0.5438 - 0.001878 * XLIL
          GO TO 12
      11  A2=EXP(-12.1621 +0.06473*XLIL)
          B2=0.5438-0.001878*XLIL
      12  A=A2*EXP(B2*X)
          A=1000.*A
          GO TO 40
C         C L AND P DATUM WAS USED FOR ZOAR
      20  XZR=HZ(I)
          A3=EXP(-67.7226+0.5976*XZR)
          B3=2.0447-0.01684*XZR
          A=A3*EXP(B3*X)
          A=1000.*A
      40  RETURN
          END
C $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
          SUBROUTINE CONC(RQ,PL,TP,NP,NQ,NR,NS,NT,NU,NX,NV,NW,J,N2,JDO,X,T,I
         C)
          COMMON Q(10,52),HL(53),HZ(53),CP(10,52)
C         FOR THE CASE OF PURE CONVECTION, THERE IS NO MECHANISM TO CHANGE
C         THE CONCENTRATION OF A SOLUTE BETWEEN CONFLUENCES.  AT A
C         CONFLUENCE THE CONCENTRATION CHANGES DISCONTINUOUSLY.  AS USED
C         HERE, PL IS THE LOAD OF SOLUTE P WITH CONCENTRATION CP AND TP IS
C         THE TOTAL CONC OF SOLUTE P SUMMED OVER ALL SOURCES.
C         THE LOAD FROM UNGAGED AREA IS ACCOUNTED FOR BY INCREASING FLOW
C         IN PROPORTION TO UNGAGED AREA.  FOR EXAMPLE Q(5,NS) BECOMES
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C         Q(5,NS) * (1. + 41.2 / 23.3)  FOR WEST ASPETUCK.
C         FACTORS C1, C2, AND C3 ARE TIME WEIGHTING FACTORS TO SMOOTH THE
C         STEPWISE CONCENTRATION.
          IF(T .GE. 0.5) GO TO 2
          C1 = 0.5 - T
          C2 = T + .5
          C3 = 0.
          GO TO 5
        2 C1 = 0.
          C2 = 1.5 - T
          C3 = T - .5
        5 GO TO (701,702,703,704,705,706,707,708,709),J
      701 TP1 = C1*CP(1,JDO-1) + C2*CP(1,JDO) + C3*CP(1,JDO+1)
          TP = TP1
          PL=TP*RQ
          GO TO 722
C         WHEN X PASSES A CONFLUENCE N2 IS SET=0 IN MAIN AND A NEW CONC IS
C         CALCULATED BY STMNTS AFTER 702. ONCE THIS IS DONE N2 IS SET=1
C         AND TP CHANGES ONLY AS A FCTN. OF X AND T (TO BE DETERMINED LATER
C         BY MULTIPLYING THE R.H.S. OF 712,713,ETC. BY F(X,T) STILL TO BE
C         SPECIFIED).
      702 IF(N2.GT.0) GO TO 712
          Q2 = C1*Q(2,NP-1) + C2*Q(2,NP) + C3*Q(2,NP+1)
          CP2 = C1*CP(2,NP-1) + C2*CP(2,NP) + C3*CP(2,NP+1)
          TP = (TP*(RQ-Q2) + CP2*Q2) / RQ
          GU = (C1*Q(1,NP-1) + C2*Q(1,NP) + C3*Q(1,NP+1)) * (30.7/994.)
          TP = (TP*(RQ-GU) + TP1*GU) / RQ
          N2=1
      712 TP=TP
C         WHEN A CONFLUENCE IS PASSED  THE UPSTREAM LOAD AND THE LOAD OF THE
C         TRIBUTARY ARE ADDED AND THE SUM IS DIVIDED BY RQ.  BUT TO OBTAIN
C         THE UPSTREAM LOAD THE FLOW OF THE TRIBUTARY MUST FIRST BE
C         SUBTRACTED FROM RQ.
          PL=TP*RQ
          GO TO 722
      703 IF(N2.GT.0) GO TO 713
          Q3 = C1*Q(3,NQ-1) + C2*Q(3,NQ) + C3*Q(3,NQ+1)
          CP3 = C1*CP(3,NQ-1) + C2*CP(3,NQ) + C3*CP(3,NQ+1)
          TP = (TP * (RQ-Q3) + CP3*Q3) / RQ
          N2=1
      713 TP=TP
          PL=TP*RQ
          GO TO 722
      704 IF(N2.GT.0) GO TO 714
          Q4 = C1*Q(4,NR-1) + C2*Q(4,NR) + C3*Q(4,NR+1)
          CP4 = C1*CP(4,NR-1) + C2*CP(4,NR) + C3*CP(4,NR+1)
          TP = (TP*(RQ-Q4) + CP4*Q4) / RQ
          N2=1
      714 TP=TP
          PL=TP*RQ
          GO TO 722
      705 IF(N2.GT.0) GO TO 715
          Q5 = (C1*Q(5,NS-1) + C2*Q(5,NS) + C3*Q(5,NS+1)) * (1.+41.2/23.3)
          CP5 = C1*CP(5,NS-1) + C2*CP(5,NS) + C3*CP(5,NS+1)
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        TP = (TP*(RQ-Q5)+CP5*Q5)/RQ
        N2=1
  715 TP=TP
        PL=TP*RQ
        GO TO 722
  706 IF(N2.GT.0) GO TO 716
        Q6 = C1*Q(6,NT-1) + C2*Q(6,NT) + C3*Q(6,NT+1)
        CP6 = C1*CP(6,NT-1) + C2*CP(6,NT) + C3*CP(6,NT+1)
        TP = (TP*(RQ-Q6) + CP6*Q6) / RQ
        N2=1
  716 TP=TP
        PL=TP*RQ
        GO TO 722
  707 IF(N2.GT.0) GO TO 717
C       THE WEEK NX IS DETERMINED IN SUBPROG. SHPARM BY RUNNING FLOW
C       IN THAT ARM BACKWARDS.   NOTE THE ASYMMETERY IN SUBSCRIPTS IN
C       THE NEXT STMNT. COMPARED WITH ALL THE OTHERS FOR TP.
        Q7=(C1*Q(7,NU-1) + C2*Q(7,NU) + C3*Q(7,NU+1)) * (1. + 66.3/133.)
        TP=(TP*(RQ-Q7)+CP(7,NX)*Q7)/RQ
        N2=1
  717 TP=TP
        PL=TP*RQ
        GO TO 722
  708 IF(N2.GT.0) GO TO 718
        Q8 = C1*Q(8,NV-1) + C2*Q(8,NV) + C3*Q(8,NV+1)
        CP8 = C1*CP(8,NV-1) + C2*CP(8,NV) + C3*CP(8,NV+1)
        TP = (TP*(RQ-Q8) + CP8*Q8) / RQ
        N2=1
  718 TP=TP
        PL=TP*RQ
        GO TO 722
  709 IF(N2.GT.0) GO TO 719
        Q9 = (C1*Q(9,NW-1) + C2*Q(9,NW) + C3*Q(9,NW+1)) * (1.+46.5/75.3)
        CP9 = C1*CP(9,NW-1) + C2*CP(9,NW) + C3*CP(9,NW+1)
        TP = (TP*(RQ-Q9) + CP9*Q9) / RQ
        N2=1
  719 TP=TP
        PL=TP*RQ
  722 RETURN
        END
C $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
        SUBROUTINE SHPARM(NU,T,T1,NXX)
        COMMON Q(10,52),HL(53),HZ(53),CP(10,52)
C       THIS SUBROUTINE IS CALLED ONLY ONCE FOR EACH RUN OF THE RIVER.
C       IT CALCULATES THE WEEK(NXX) THAT A PARCEL OF WATER, WHICH REACHES
C       THE CONFLUENCE OF THE SHPARM WITH THE MAIN CHANNEL ON WEEK NU,
C       LEFT THE MOUTH OF THE SHEPAUG RIVER.   NXX GETS PASSED TO
C       SUBROUTINE CONC AND IS USED THERE IN STATEMENT 707 PLUS 1.
C       THE SHEPAUG ARM IS 3.25 MILES LONG AND FRCTXS IS A FRACTION OF
C       THE CHANGE IN STORAGE OF THIS ARM AS A FUNCTION OF LINEAL
C       DISTANCE: FRCTXS EQUALS 0 AT X1 = 0 AND 1.0 AT X1 = 3.25.
C
C       INITIALIZE WEEK AND DISTANCE IN THE SHEPAUG ARM.
        II = NU
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        X1 = 0.
        DT = 0.1
        T1=T
        F = 24.*3600.*7.
        DVSDH=0.123E+08
   10 FRCTXS = (48.2 * X1 - 6.63 * X1 ** 2) / 86.62
C       SUNG = UNGAGED FLOW CALC AS 66.3/133 SQ MILE * FLOW OF SHEPAUG
        SUNG=0.498*Q(7,II)
        QSHP=Q(7,II) +SUNG
C
        RQ = QSHP    - (DVSDH * (HL(II + 1) - HL(II)) / F) * FRCTXS
C       DETERMINE DISTANCE IN MILES USING A 3RD ORDER RUNGE-KUTTA SCHEME.
C       X1 = 0 AT THE CONFLUENCE AND X1 = 3.25 MILES AT THE MOUTH OF THE
C       SHEPAUG RIVER.
        CALL ASHP (II,X1,AS)
        A1=(114.545454)*(RQ/AS)*DT
        X11 = X1 + A1/2.
        CALL ASHP (II,X11,AS)
        A2 = (114.545454) * (RQ/AS) * DT
        X12=X1+2.*A2-A1
        CALL ASHP (II,X12,AS)
        A3 = (114.545454) * (RQ/AS) * DT
        DX = (1./6.) * (A1 + 4. * A2 + A3)
        X1 = X1 + DX
        T1 = T1 + DT
        IF (X1.GE.3.25) GO TO 20
        IF (T1.LT.1.)  GO TO 10
C       WHEN II = 1, I.E., AUG (1ST WEEK) 1968, FOLD YEAR ON ITSELF AND
C       GO TO WEEK 52 WHICH IS JULY (4TH WEEK) 1969.
C       *** ATTN. *** JULY 1969 WAS WET COMPARED TO JULY 1968. THE
C       HIGH FLOWS ALLOW MILE 3.25 TO BE REACHED SOONER THAN IT WOULD
C       WERE JULY,JUNE,ETC. 1968 DATA USED.
        IF (II.EQ.1) II = 53
        II = II - 1
        T1 = 0.
        GO TO 10
   20 NXX = II
        RETURN
        END
C $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
        SUBROUTINE ASHP (II,X1,AS)
C       THIS SUBPROGRAM CALCULATES THE CROSS SECTIONAL AREA OF THE
C       SHEPAUG ARM OF LAKE LILLINONAH.
        COMMON Q(10,52), HL(53), HZ(53), CP(10,52)
        XLIL=HL(II)
        A4=-195.1+1.235*XLIL
        B4=-50.8+0.325*XLIL
        AS=A4-B4*X1
        AS=1000.*AS
        RETURN
        END
```