CONNECTICUT VOLUME TABLES FOR PLANTATION-GROWN WHITE PINE, *Pinus strobus* L.

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Northern white pine, *Pinus strobus* L., has been used extensively in the establishment of forest plantations throughout the East during the past 40 years. In Connecticut 10,000 or more acres have been planted to this species, either pure or in mixture, and many of the plantations are now in need of thinning in order to maintain optimum growth. In some stands merchantable products have already been cut and marketed and their amount will increase greatly in the future.

Tables of volume for plantation-grown white pine, two in cubic feet and one in board feet were constructed by the alinement chart method in 1939 (1). Recently, additional measurements were secured on trees in the larger diameter and upper height classes to supplement the basic data used in constructing the 1939 tables. With this strengthened basic data, new tables were constructed by methods which, it is believed, permit extension of the tables beyond the limits of the basic data with greater safety than was afforded by the alinement chart method used in 1939.

DESCRIPTION OF PLANTATIONS

Data for the volume tables were secured from the Rainbow Forest in Windsor and East Granby, formerly owned by the Connecticut Agricultural Experiment Station, and from the Eli Whitney Forest in the vicinity of New Haven, owned by the New Haven Water Company. The stands represent a wide variety of planting sites and planting conditions.

The Rainbow plantations are situated on a site of very low productive capacity. The tract is part of an extensive glacial outwash plain about 160 to 180 feet above sea level. The soil is a coarse sand to a depth of many feet. Natural forest stands of the region are composed chiefly of pitch pine and inferior hardwoods such as gray birch, red maple and black and scarlet oak. Prior to the establishment of the plantations, the land had been abandoned for ordinary agricultural crops for a number of years and was reverting to forest growth. The oldest trees measured were planted in

[1] The authors wish to express their thanks to Professor Walter H. Meyer, School of Forestry, Yale University, for advice in preparing the volume tables and for reviewing the manuscript.

1902 and the youngest in 1907. Measured trees varied in breast-high diameters from 2.4 to 12.0 inches and in height from 23 to 49 feet. Growing space at the time of planting ranged from 20 to 50 square feet per tree. Thinnings were made in 1932 and 1936.

In the Eli Whitney Forest, plantations were established on soils of superior quality, representing the best sites for white pine in the region. Originally, these areas were farmed with fair success. The soil is, in the main, a good quality loam, somewhat better than the usual run of forest soils. Organic content, texture and moisture conditions are excellent. Growth has been rapid and well sustained with some of the better stands supporting 8,000 to 10,000 board feet of low grade material per acre at 30 years. In all cases plantings were made on recently cultivated fields or on pasture or mowing fields that were relatively free of shrub or tree growth. Spacing was 6 x 6 feet. The youngest stand sampled was 17 and the oldest, 35 years old at the time of measurement. Breast-high diameters ranged from 1.7 to 15.0 inches and heights from 16 to 57 feet.

BASIC DATA

Origin

Data obtained from the measurements on 1,031 trees served as a basis for three cubic-foot tables. Of this number, 847 were incorporated into a board-foot table. Five hundred and eleven trees were measured on the Rainbow Forest and 520 on the Eli Whitney Forest.

Field Measurements

The method of making field measurements on the felled trees varied somewhat. For the 921 trees used in the preparation of Bulletin 427, the following measurements were made on each tree:

a. Total height to the nearest tenth of a foot.
b. Diameter outside bark and bark thickness to the nearest tenth of an inch at:
    1. Stump height, taken at six inches above ground.
    2. Breast height.
    3. Intervals of one—, two—, three—, and four-fifths of the length of stem above breast height plus an additional measurement at mid-point for form quotient.

Measurements of the 110 trees felled in 1946 were made after the trees had been bucked into logs and were as follows:

a. Total height to the nearest tenth of a foot.
b. Stump height, taken at six inches above ground.
c. Distance between stump height and a top diameter of 2.0 inches, outside bark, to the nearest tenth of a foot.
d. Distance between stump height and a top diameter of 5.0 inches, inside bark, to the nearest tenth of a foot.
e. Length of each cut log.
f. Diameter outside bark to the nearest tenth of an inch and bark thickness to the nearest twentieth of an inch at:
    1. Stump height.
    2. Breast height.
    3. At the small end of each cut log.

PREPARATION OF THE TABLES

Volume Determination

The field measurements of the 1,031 trees were recorded on U. S. Forest Service Form 558a. All volume determinations were measured directly from the plotted taper measurements. Cubic contents were obtained by the conversion of planimetered areas, board-foot volumes (International Rule, ¼ inch saw kerf) by graduated transparent overlays or by computation. In the latter case, each tree was scaled as a composite of 8-foot logs with allowance for trimming plus a top log of shorter length. Trees with top diameters less than 5.0 inches, inside bark, at 8.65 feet above ground were considered to have no merchantable board-foot content. The top log when less than eight feet in length, was scaled by a separate calculation in units of two, four or six feet of length. Thus, a top log with a small-end diameter of 5.0 inches, inside bark, and a length of 7.5 feet was treated as a six-foot log with a top diameter, inside bark, somewhat larger than 5.0 inches. Since this top diameter rarely exceeded 5.5 inches, the possible combinations of diameter and length were so few that it was a simple matter to make up a table giving the volume of any combination.

The average form quotient of the 1,031 trees that served as a basis for the cubic-foot tables was .681, that of the 847 trees used in preparation of the board-foot table was .676.

Cubic-Foot Tables

Two volume tables in cubic feet were constructed directly from the basic data. These tables are presented in alinement chart and tabular form. The charts are somewhat easier to use for fractional diameters and odd heights because interpolation is unnecessary. Where diameters are grouped by whole inch classes and 5-foot or 10-foot height intervals, the tables are preferable.

1. Total Volume—Table 1 and Alinement Chart I give the volume in cubic feet of the entire stem, less bark, including a stump 0.5 foot high measured as a cylinder.

2. Merchantable Volume—Table 2 and Alinement Chart II give the volume in cubic feet of the stem, with bark, between a stump height of 0.5 foot and a top diameter of 2.0 inches, outside bark.

Tables 1 and 2 were read from Alinement Charts I and II, respectively, which were constructed as described below.

3. Merchantable Volume—Table 3, which was derived from Table 2, gives the volume in cubic feet of the stem, with bark, between a stump height of 0.5 foot and a top diameter of 5.0 inches, outside bark.

Construction

The values in Tables 1 and 2 were derived from the measured volumes by means of the equation $V = aD^bH^c$ with subsequent adjustments. In this equation V represents the volume in cubic feet, D the diameter-breast-high in inches, H the total height in feet and a, b and c, the constants derived from the basic data. This equation when expressed in the logarithmic form

$$\text{Log } V = \log a + b \log D + c \log H$$

can be solved by the method of least squares (2).

In order to facilitate the work resulting from the large number of trees involved, the data were grouped into 0.5-inch diameter classes and five-foot height classes which resulted in a total of 85 groups containing varying numbers of trees.

The average breast-high diameter, total height and measured volume were obtained for each group and these values, expressed as logarithms and weighted according to the number of trees in the group, were used in solving the equations.

The equations resulting from solution by the least squares method were:

1. Total Volume of Peeled Stem.

$$V = .0044781\ D^{1.88531}\ H^{1.04500}$$

or expressed in logarithmic form

$$\text{Log } V = -2.55219 + 1.88531 \log D + 1.04500 \log H$$

2. Merchantable Volume to 2.0 Inch Top Diameter.

$$V = .0045605\ D^{1.92075}\ H^{1.03718}$$

or expressed in logarithmic form

$$\text{Log } V = -2.54395 + 1.92075 \log D + 1.03718 \log H$$

Since the sum of the volumes calculated by means of the equations was found to be slightly different from the sum of the measured volumes, a correction was made for the systematic error occurring in the use of the logarithmic volume equation (3). The correction applied corresponded to that for the calculated standard error of estimate and was the same for both total and merchantable volumes, 1.0096.

Alinement Charts I and II were then constructed, an operation that included a regraduation of the volume axis (4) to fit the equations as subsequently corrected. Tables 1 and 2 were read from Charts I and II, respectively.

The values in Table 3 were obtained graphically by plotting estimated volumes to a top diameter of 2.0 inches, outside bark, over measured volumes to a top diameter of 5.0 inches, outside bark, obtained from the taper measurements on 20 per cent of the trees in each diameter and height class (177 trees in all). Estimated and measured volumes were from the same trees. For convenience the data were grouped into volume classes before plotting. Trees with a diameter-breast-high, outside bark, of less than 5.0 inches were excluded.

This method is similar to the one used in the first correction of an alinement chart (4). It employs the influence of the 1,031 trees used in the construction of Table 2, which was well within the allowable tolerance, to construct a table to another top diameter limit from a relatively few trees.

Two measures of accuracy were applied to the completed alinement charts and tables, the aggregate difference in per cent and the average percentage deviation. Approximately 20 per cent of the trees, or each fifth tree in the assembled data, were used as the basis for these tests. The

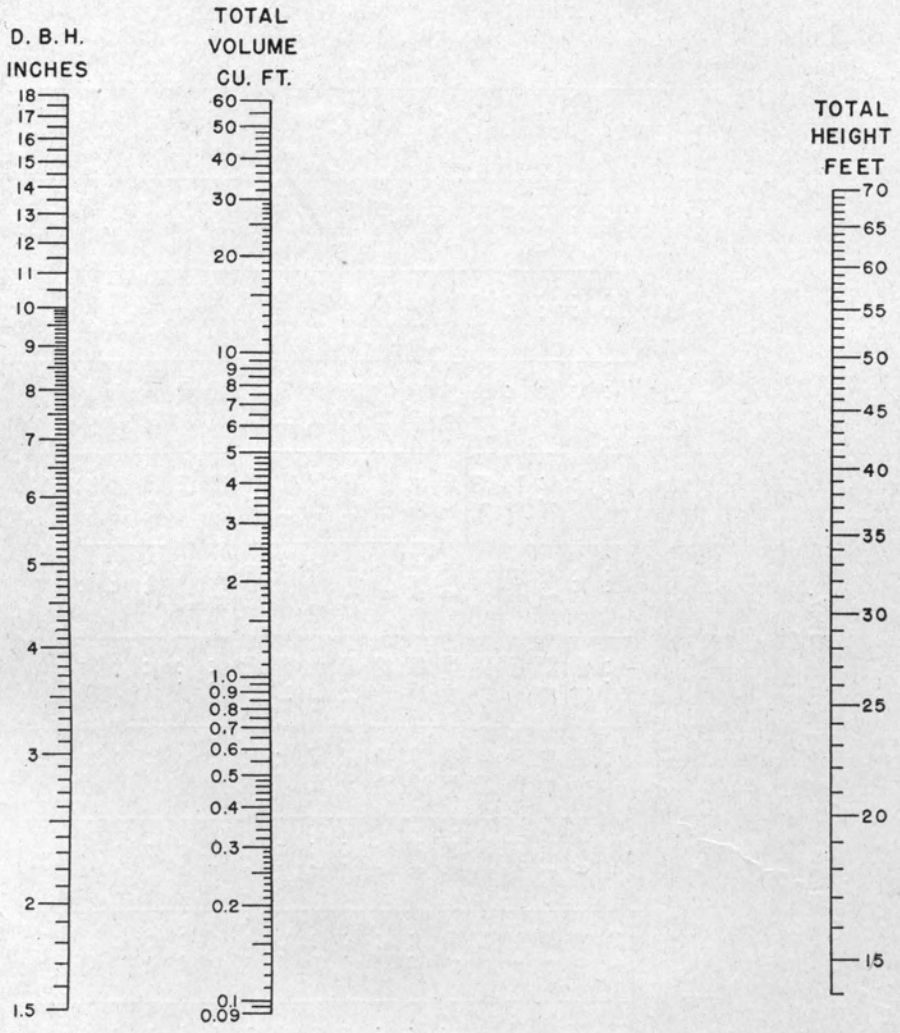


Chart I. Plantation-grown white pine. Volume in cubic feet of the entire stem, less bark. For additional information see Table 1.

TABLE 1. PLANTATION-GROWN WHITE PINE. VOLUME IN CUBIC FEET OF THE ENTIRE STEM, LESS BARK. STUMP, 0.5 FOOT HIGH, MEASURED AS A CYLINDER. VOLUME MEASURED BY PLANIMETER. BASIS: 1,031 TREES; HEIGHTS, 17 TO 57 FEET; DIAMETERS BREAST HIGH, 1.7 TO 15.0 INCHES; AGE, 16 TO 40 YEARS. AVERAGE PERCENTAGE DEVIATION, 4.92 PER CENT; AGGREGATE DIFFERENCE, TABLE 1, 0.11 PER CENT LOW. VALUES READ FROM CHART I. CONNECTICUT, 1947.

Diameter breast high, inches	Total Height in Feet												Number of trees
	15	20	25	30	35	40	45	50	55	60	65	70	
	Volume in Cubic Feet												
2	0.18	0.24	0.29	0.35	...	...	...	...	...	...	...	...	2
3	0.39	0.52	0.64	0.76	0.88	1.00	...	...	...	...	...	...	31
4	0.69	0.90	1.12	1.33	1.54	1.75	1.96	...	...	...	...	...	61
5	...	1.39	1.72	2.04	2.37	2.70	3.01	3.33	...	...	...	...	80
6	...	...	2.44	2.90	3.36	3.82	4.27	4.73	5.19	...	...	...	167
7	...	...	3.29	3.91	4.53	5.16	5.78	6.40	7.00	7.62	...	...	187
8	...	...	...	5.04	5.84	6.64	7.43	8.21	9.00	9.80	...	...	181
9	...	...	...	6.37	7.35	8.35	9.30	10.20	11.20	12.20	13.10	...	156
10	...	...	...	...	9.02	10.20	11.40	12.50	13.70	14.90	16.00	...	98
11	...	...	...	...	10.80	12.20	13.60	15.00	16.50	18.00	19.40	20.80	48
12	...	...	...	...	...	14.30	16.00	17.70	19.30	21.00	22.60	24.30	14
13	...	...	...	...	...	16.70	18.70	20.60	22.50	24.40	26.30	28.30	4
14	...	...	...	...	...	...	21.40	23.70	25.90	28.10	30.30	32.50	...
15	...	...	...	...	...	...	24.30	26.80	29.40	32.00	34.50	37.00	2
16	...	...	...	...	...	...	...	30.20	33.10	36.00	38.90	41.80	...
17	...	...	...	...	...	...	...	34.00	37.20	40.50	43.80	47.00	...
18	...	...	...	...	...	...	...	...	41.50	45.00	48.50	52.00	...
Number of trees	1	5	34	124	218	321	219	90	19	...	...	...	1,031

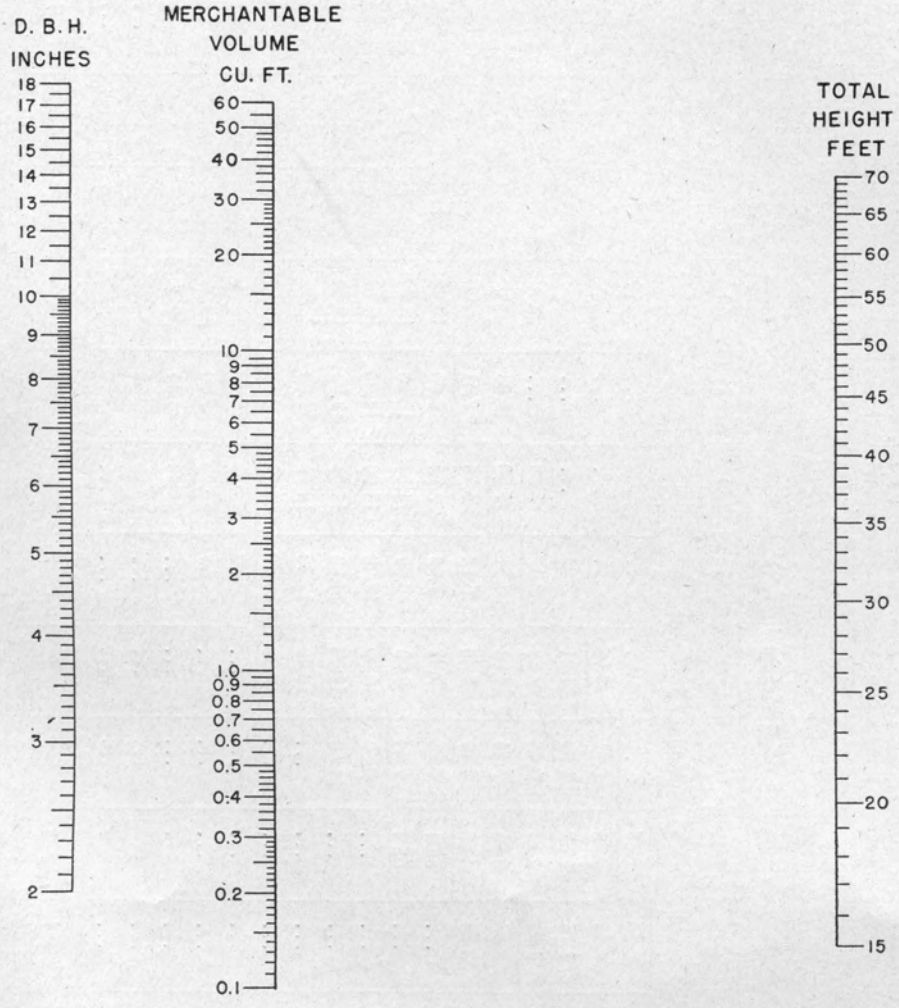


Chart II. Plantation-grown white pine. Volume in cubic feet of the stem, including bark, between a stump height of 0.5 foot and a top diameter of 2.0 inches, outside bark. For additional information see Table 2.

TABLE 2. PLANTATION-GROWN WHITE PINE. VOLUME IN CUBIC FEET OF THE STEM, INCLUDING BARK, BETWEEN A STUMP HEIGHT OF 0.5 FOOT AND A TOP DIAMETER OF 2.0 INCHES, OUTSIDE BARK. VOLUME MEASURED BY PLANIMETER. BASIS: 1,031 TREES; HEIGHTS, 17 TO 57 FEET; DIAMETERS BREAST HIGH, 1.7 TO 15.0 INCHES; AGE, 16 TO 40 YEARS. AVERAGE PERCENTAGE DEVIATION, 5.24 PER CENT; AGGREGATE DIFFERENCE, TABLE 2, 0.24 PER CENT LOW. VALUES READ FROM CHART II. CONNECTICUT, 1947.

Diameter breast high, inches	Total Height in Feet												Number of trees
	15	20	25	30	35	40	45	50	55	60	65	70	
	Volume in Cubic Feet												
2	0.18	0.24	0.30	0.35	...	...	...	...	...	...	...	...	2
3	0.40	0.53	0.66	0.78	0.91	1.03	...	...	...	...	...	...	31
4	0.71	0.93	1.15	1.37	1.59	1.81	2.03	...	...	...	...	...	61
5	...	1.44	1.78	2.13	2.48	2.83	3.17	3.50	...	...	...	...	80
6	...	...	2.57	3.06	3.54	4.02	4.51	5.00	5.48	...	...	...	167
7	...	...	3.46	4.13	4.80	5.46	6.12	6.76	7.41	8.06	...	...	187
8	...	...	...	5.34	6.20	7.05	7.90	8.75	9.60	10.40	...	...	181
9	...	...	...	6.74	7.84	8.90	10.00	11.00	12.10	13.20	14.20	...	156
10	...	...	...	...	9.60	10.90	12.20	13.50	14.80	16.10	17.40	...	98
11	...	...	...	...	11.60	13.20	14.80	16.30	17.90	19.40	21.00	22.60	48
12	...	...	...	...	...	15.40	17.30	19.10	21.00	22.80	24.60	26.40	14
13	...	...	...	...	...	18.20	20.30	22.40	24.50	26.70	28.90	31.00	4
14	...	...	...	...	...	...	23.40	25.80	28.20	30.60	33.10	35.50	...
15	...	...	...	...	...	...	26.60	29.40	32.20	35.00	37.70	40.60	2
16	...	...	...	...	...	...	...	33.30	36.30	39.50	42.60	45.80	...
17	...	...	...	...	...	...	...	37.20	40.80	44.40	48.00	51.50	...
18	...	...	...	...	...	...	...	...	45.50	49.50	53.50	57.50	...
Number of trees	1	5	34	124	218	321	219	90	19	...	...	...	1,031

TABLE 3. PLANTATION-GROWN WHITE PINE. VOLUME IN CUBIC FEET OF THE STEM, INCLUDING BARK, BETWEEN A STUMP HEIGHT OF 0.5 FOOT AND A TOP DIAMETER OF 5.0 INCHES, OUTSIDE BARK. DERIVED FROM DATA INCORPORATED INTO TABLE 2 AS DESCRIBED ON PAGE 6. AVERAGE PERCENTAGE DEVIATION, 6.90 PER CENT; AGGREGATE DIFFERENCE, TABLE 3, 0.51 PER CENT HIGH. CONNECTICUT, 1947.

Diameter breast high, inches	Total Height in Feet										Number of trees
	25	30	35	40	45	50	55	60	65	70	
	Volume in Cubic Feet										
5	...	...	0.96	1.57	2.10	2.54	...	...	...	...	10
6	1.08	1.90	2.56	3.12	3.63	4.14	4.64	...	...	...	30
7	2.49	3.23	3.93	4.62	5.32	5.97	6.68	7.39	...	...	39
8	...	4.49	5.40	6.30	7.21	8.11	9.04	9.90	...	...	36
9	...	5.95	7.14	8.28	9.48	10.50	11.65	12.80	13.80	...	28
10	...	...	9.04	10.40	11.76	13.10	14.40	15.70	16.90	...	22
11	...	...	11.13	12.80	14.40	15.90	17.50	19.00	20.50	22.00	8
12	...	...	...	15.00	16.90	18.70	20.50	22.00	24.00	25.70	3
13	...	...	...	17.80	19.80	21.80	23.90	26.00	28.20	30.30	1
14	...	...	...	...	22.80	25.10	27.50	29.90	32.40	34.80	...
15	...	...	...	...	25.90	28.70	31.50	34.30	36.90	39.80	...
16	...	...	...	...	...	32.60	35.50	38.70	41.80	45.10	...
17	...	...	...	...	...	36.40	40.00	43.60	47.30	50.90	...
18	...	...	...	...	...	...	44.80	48.80	52.90	56.90	...
Number of trees	...	10	38	61	49	17	2	...	...	...	177

first compares the sum of the measured volumes with the sum of the chart volumes for the same trees. Charts I and II and Table 3 give volumes 0.11 per cent low, 0.24 per cent low and 0.51 per cent high, respectively, all within the allowable tolerance (5). The average percentage deviation is an average of the deviations (without regard to sign) of the measured volumes from their corresponding chart volumes, each deviation being expressed as a per cent of the corresponding chart value. The values for Charts I and II and Table 3 are 4.92 per cent, 5.24 per cent and 6.90 per cent, respectively, and are also well within the allowable tolerance.

Board-Foot Volume Table

One board-foot volume table was constructed from the basic data to give the merchantable volume of the stem, without bark, from a stump height of 0.5 foot to a top diameter of 5.0 inches, inside bark.

The method used was that proposed by Meyer (6) in 1944. It is based on the proposition that a straight line relationship is obtained when the volume-diameter ratios of trees within a given height class are plotted over their diameters. Meyer and Kienholz (7) and Meyer (8) found this relationship to be true for a number of species and the present study demonstrated that it is equally true for plantation-grown white pine of the sizes used in the preparation of the table.

The relationship may be expressed mathematically as $V/D = a + b\,D$ where V is the merchantable volume in board feet, D is diameter-breast-high in inches and a and b are coefficients which vary with height class. In order to make use of the above formula in constructing a volume table, it is only necessary to derive mathematical expressions of the way in which a and b vary with height.

Inasmuch as the volume table presented is an additional test of the validity of Meyer's method, the several steps followed in its preparation, together with the necessary graphical checks and a table of derived values, are presented. These values may be obtained either by careful graphics or by solution of equations by the method of least squares. The latter method was used in preparing the white pine board-foot table.

In the derivation of the several equations from the basic data, subscripts and bars have been added to the terms used by Meyer in order to clarify the several steps which were as follows:

(1) The trees were first divided into 10 height classes as shown in Table 4, within each of which the V/D ratio was computed for individual trees by dividing the board-foot volume by actual diameter, D. To test for linearity within a height class, the V/D ratios were plotted over a coded diameter, D', which was equal to D minus 5.5 inches and was used to simplify computations. Since the relationship appeared to be linear, the coordinate values of the points within each height class were assembled for solution by the method of least squares to determine the most probable location of a straight line passing through the points.

Inasmuch as the least squares equation is used several times, a brief explanation of its terms, together with two short-cut formulae, used in

TABLE 4. SHOWING THE VALUES DERIVED FROM THE BASIC DATA AND FROM EQUATIONS USED IN THE DETERMINATION OF BOARD-FOOT VOLUME.

Height class, feet	Number of trees	Average total height, feet	Average V/D or $\overline{V/D}$	Average D′ or $\overline{D}'$, inches	Intercept a	Slope b	Smoothed slope b_1	V/D adjusted to $D'=D_0=3$ and slope b_1 or V_1/D_1	Smoothed intercept a_1	Smoothed V_1/D_1 adjusted to intercept a_1 or V_2/D_2
30.0 to 32.4	36	31.35	.7645	.550	.3018	.8412	.7171	2.5213	.3128	2.4641
32.5 to 34.9	52	33.81	1.1469	.810	.5769	.7036	.7302	2.7461	.5170	2.7076
35.0 to 37.4	111	36.25	1.6686	1.230	.7816	.7211	.7433	2.9842	.7195	2.9494
37.5 to 39.9	121	38.63	2.4288	1.984	.9485	.7461	.7560	3.1969	.9171	3.1851
40.0 to 42.4	199	41.05	3.1209	2.647	1.0474	.7833	.7689	3.3923	1.1179	3.4246
42.5 to 44.9	129	43.53	3.8332	3.264	1.3715	.7542	.7822	3.6267	1.3238	3.6704
45.0 to 47.4	91	45.94	4.6121	3.926	1.4556	.8040	.7951	3.8759	1.5238	3.9091
47.5 to 49.9	52	48.55	5.0171	4.186	2.0297	.7136	.8090	4.0572	1.7404	4.1674
50.0 to 52.4	38	51.18	5.7145	4.370	1.8065	.8943	.8231	4.5869	1.9587	4.4280
52.5 to 54.9	18	53.23	6.5306	4.910	1.6857	.9867	.8340	4.9376	2.1289	4.6309

computing the slope of the curve, is presented at this point. The equation $Y = a + b\,X$, expanded for solution, may be written

$$Y = \overline{Y} + \frac{S\,(\overline{X} - X)\,(\overline{Y} - Y)}{S\,(\overline{X} - X)^2}\,(X - \overline{X}) \qquad (1)$$

in which Y is the dependent variable; X is the independent variable; $\overline{X}$ and $\overline{Y}$ are the means of X and Y, respectively, and are also the coordinates of the mid point of the equation (later designated O); $(\overline{X} - X)$ and $(\overline{Y} - Y)$ are individual deviations of X and Y from their respective means $\overline{X}$ and $\overline{Y}$; $\dfrac{S\,(\overline{X} - X)\,(\overline{Y} - Y)}{S\,(\overline{X} - X)^2}$

is the slope of the equation or b and $\overline{Y} - \overline{X}\ \dfrac{S\,(\overline{X} - X)\,(\overline{Y} - Y)}{S\,(\overline{X} - X)^2}$

is the intercept, a, of the curve with the Y axis.

Two short-cut formulae for the computation of the numerator and denominator of the slope term, b, are

$$S\,(\overline{X} - X)\,(\overline{Y} - Y) = S\,(X \times Y) - \frac{SX \times SY}{N}$$

$$\text{and } S\,(\overline{X} - X)^2 = S\,X^2 - \frac{(S\,X)^2}{N}$$

where N represents the number of items in the series.

With V/D substituted for Y and D' for X, Equation 1 becomes

$$V/D = \overline{V/D} + \frac{S\,(\overline{D'} - D')\,(\overline{V/D} - V/D)}{S\,(\overline{D'} - D')^2}(D' - \overline{D'}) \qquad (2)$$

or

$$V/D = \overline{V/D} + b\,(D' - \overline{D'}) \qquad (3)$$

Equation 2 was solved for each of the 10 height classes to determine intercept a, slope b and mid-point O. The curves of the equations are shown (solid lines) in Figure A, together with their mid-points O. Numerical values for intercept and slope are given in Table 4.

(2) The values for coefficient b for each height class were next plotted over average total height L for that class. Since no marked departure from linearity was evident, the several values of b and L, weighted according to the number of trees in each height class, were assembled for solution by the method of least squares as indicated in Equation 1 and solved to obtain a general equation for b in terms of L. Since this equation smoothed out irregularities among coefficients, its application to a given height class resulted in slope values which were no longer the same as

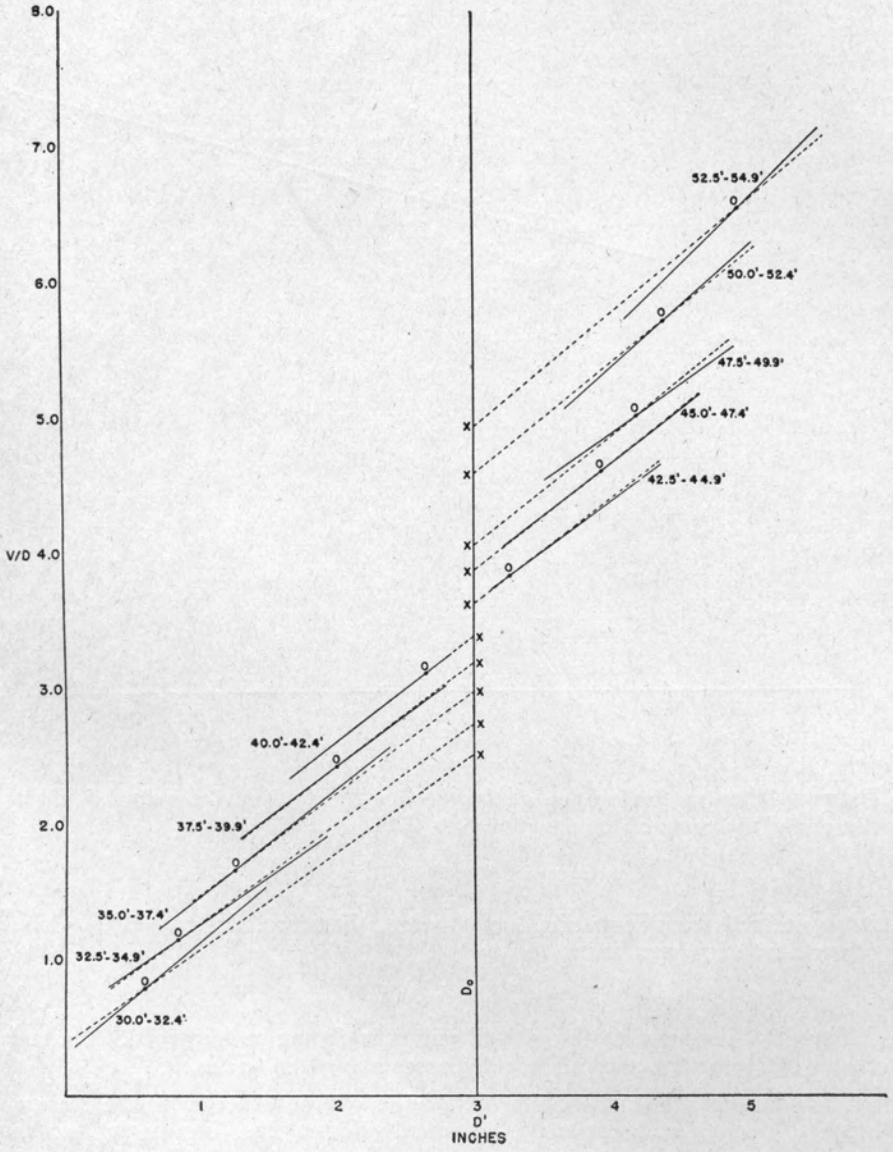


Figure A. Showing (solid lines) the curves of the equation $V/D = a + b\,D'$ and (dotted lines) the curves of the equation $V_1/D_1 = a_0 + b_1\,D'$ for the 10 height classes. Coordinates of points *O* are $\overline{V/D}$ and $\overline{D'}$ and of points *X*, V_1/D_1 and D_0.

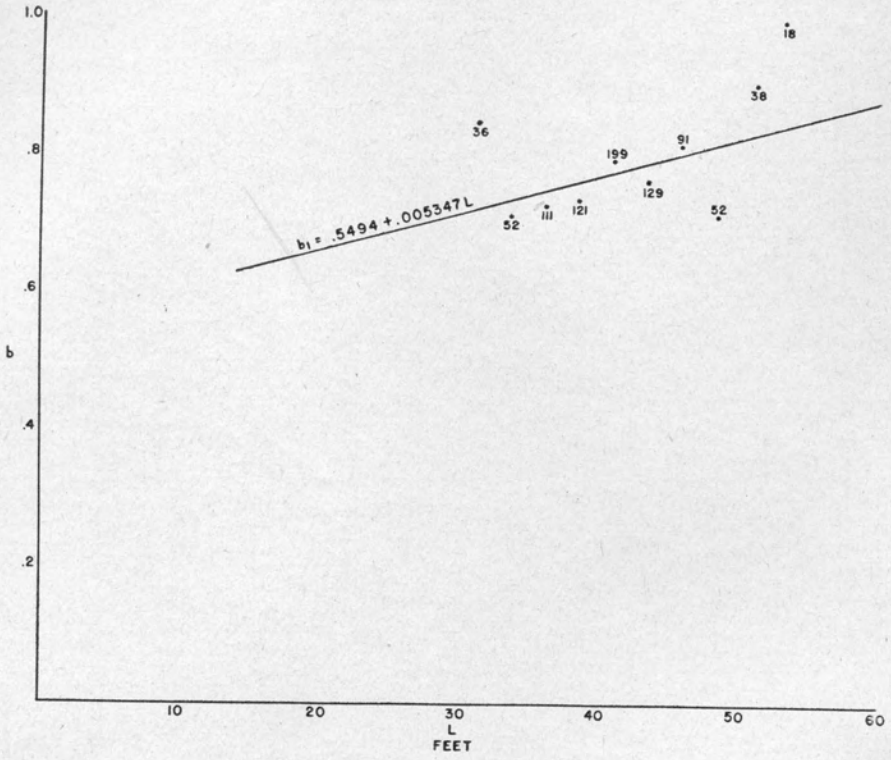


Figure B. Showing the location of the coordinates of *b* and *L* for the 10 height classes and the curve of the equation b_1 = .5494 + .005347 *L*.

those derived from Equation 2 and were, therefore, designated b_1. The general equation was found to be

$$b_1 = .5494 + .005347\ L \tag{4}$$

The fit of its curve to the plotted points is shown in Figure B. Derived values of b_1 for the several height classes are given in Table 4.

(3) Through the mid-points, *O*, of the curves of each of the several height classes, a new curve was assumed (dotted line, Figure A) for an equation obtained by substituting b_1 for b in Equation 3. Since the slope and, consequently, the intercept of the equation was thereby changed, V/D was no longer the same as that derived from Equation 3 and was designated V_1/D_1 to distinguish it from the original. Its equation is

$$V_1/D_1 = \overline{V/D} + b_1\ (D' - \overline{D}') \tag{5}$$

or

$$V_1/D_1 = a_0 + b_1\ D' \tag{6}$$

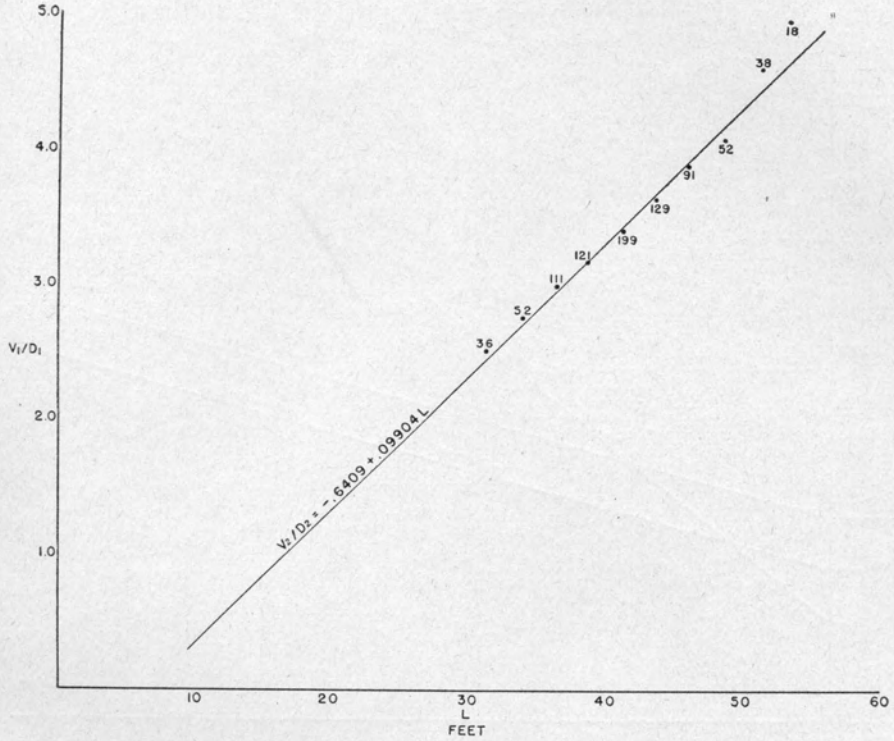


Figure C. Showing the location of the coordinates of V_1/D_1 (in terms of D_0 = 3) and L for the 10 height classes and the curve of the equation V_2/D_2 = —.6409 + .09904 L.

Figure A shows graphically the effect of curving out the b coefficients.

(4) The curve of Equation 5 for all height classes was extended to meet the ordinate of an assigned common diameter D_0 at points X, Figure A. D_0 was arbitrarily chosen as three inches which is approximately the mean of all D' values. By substituting $D_0 = 3$ for D' in Equation 5 a value of V_1/D_1 in terms of D_0 was obtained for each height class (see Table 4).

(5) Values of V_1/D_1 in terms of D_0 for each height class were next plotted over average total height, L, for that class. A relationship closely approximating linearity was evident. The several values of V_1/D_1 and L, weighted according to the number of trees in each height class, were assembled for solution by the method of least squares as indicated in Equation 1 and solved to obtain a general equation for V_1/D_1 in terms of L. Since this equation smoothed out irregularities among V_1/D_1 ratios, its application to a given height class resulted in V_1/D_1 values which were no longer the same as those derived from Equation 5 and were designated V_2/D_2. The general equation was found to be

$$V_2/D_2 = -.6409 + .09904\ L \qquad (7)$$

The fit of the curve is shown in Figure C.

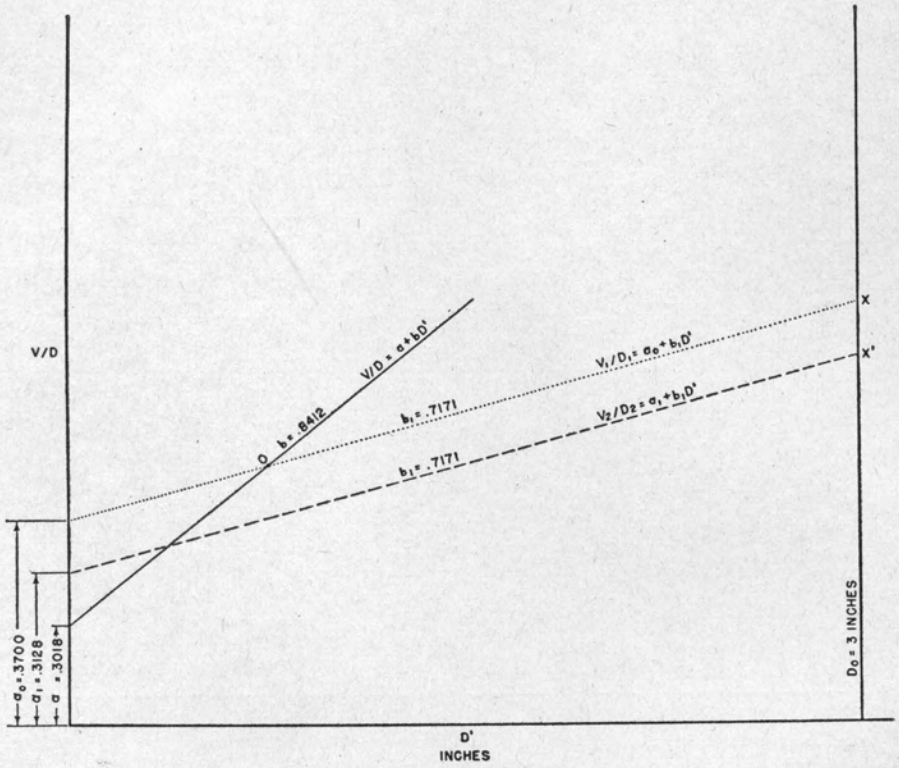


Figure D. Showing (to an exaggerated scale) for the 30.0 to 32.4 foot height class the relative positions of the curves of the equations $V/D = a + b\ D'$ (solid line), $V_1/D_1 = a_0 + b_1\ D'$ (dotted line) and $V_2/D_2 = a_1 + b_1\ D'$ (dashed line). Coordinates of point O are $\overline{V/D} = .7645$ and $\overline{D'} = .55$; of point X, $V_1/D_1 = 2.5213$ and $D' = 3.0$; of point X', $V_2/D_2 = 2.4641$ and $D' = 3.0$.

In effect Equation 4 harmonizes the slopes of the curves as shown by the dotted lines in Figure A and Equation 7 harmonizes the spacings between these curves, giving each a new intercept, a_1. The general equation for the curves with the new intercepts is

$$V_2/D_2 = a_1 + b_1\ D' \qquad (8)$$

When $D' = D_0 = 3$, Equation 8 may be written $a_1 = V_2/D_2 - 3b_1$ and by substituting values for V_2/D_2 and b_1 from Equations 4 and 7, respectively,

$$a_1 = -.6409 + .09904L - 3(.5494 + .005347\ L) = -2.2892 + .083\ L \qquad (9)$$

Applying Equations 3, 4, 5, 7, 9 and 8 to the 30.0 to 32.4 foot height class the following values were obtained. For purposes of illustration they are shown drawn to an exaggerated scale in Figure D.

TABLE 5. PLANTATION-GROWN WHITE PINE. VOLUME IN BOARD FEET (INTERNATIONAL LOG RULE, ¼ INCH SAW KERF) OF THE STEM, WITHOUT BARK, BETWEEN A STUMP HEIGHT OF 0.5 FOOT AND A TOP DIAMETER OF 5.0 INCHES, INSIDE BARK. BASIS: 847 TREES; HEIGHTS, 27 TO 57 FEET; DIAMETERS BREAST HIGH, 5.5 TO 15.0 INCHES; AGE, 18 TO 40 YEARS. AVERAGE PERCENTAGE DEVIATION FOR TREES 6.5 INCHES D.B.H. AND OVER, 8.7 PER CENT; AGGREGATE DIFFERENCE, TABLE 5, 0.33 PER CENT LOW. CONNECTICUT, 1947.

Diameter breast high, inches	Total Height in Feet									Number of trees
	25	30	35	40	45	50	55	60	65	
	Volume in Board Feet									
6	1	3	6	8	11	14	16	19	21	145
7	6	9	12	15	18	22	25	28	31	187
8	12	16	20	24	27	31	35	39	43	179
9	...	24	29	33	38	42	47	52	56	161
10	...	34	39	45	50	55	61	66	71	106
11	...	...	51	58	64	70	76	82	88	52
12	...	...	65	72	79	86	93	100	107	9
13	...	...	...	88	96	104	112	120	128	6
14	...	...	...	105	114	123	132	141	150	...
15	...	...	...	124	134	144	154	164	174	2
16	...	...	...	145	156	167	178	189	200	...
17	...	...	...	167	179	191	203	216	228	...
Number of trees	...	36	163	320	220	90	18	...	...	847

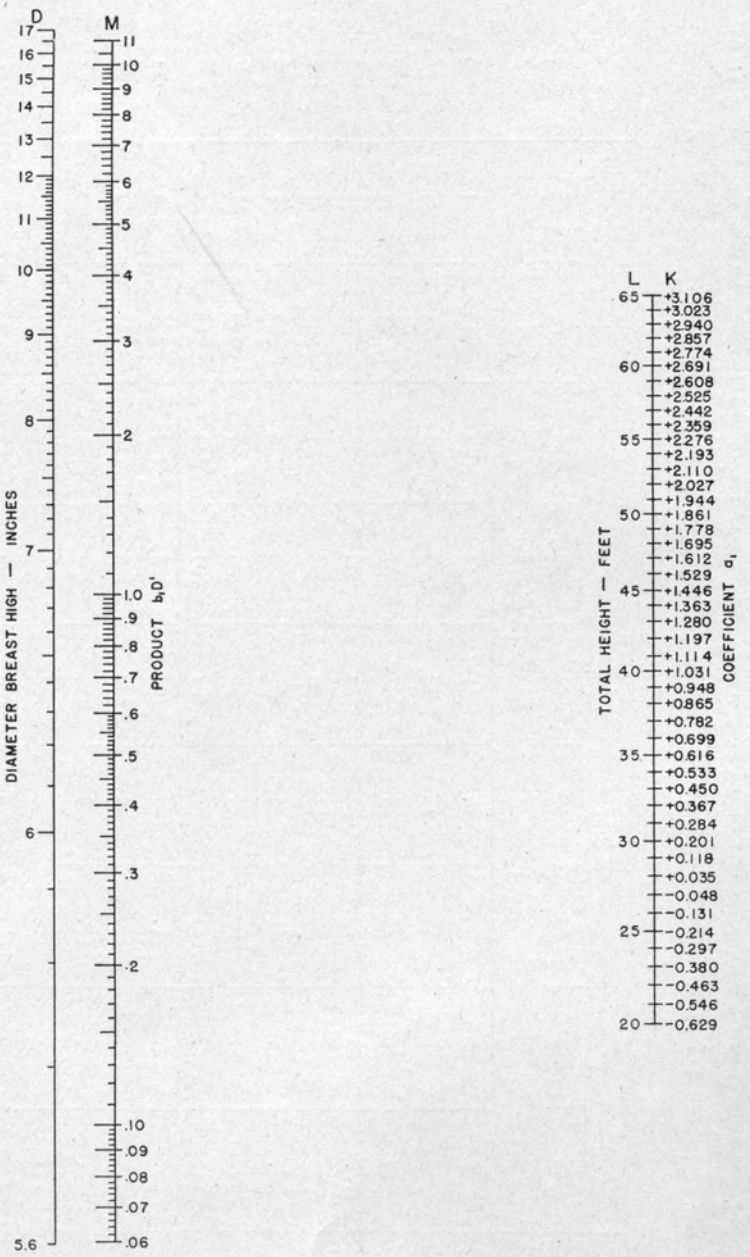


Chart III. Plantation-grown white pine. Volume in board feet of the stem from a stump height of 0.5 foot to a top diameter of 5.0 inches, inside bark. For additional information see Table 5. To compute volume, read coefficient a_1 on axis K for the desired height L, span axes D to L and read product b_1D' on axis M. Add a_1 to b_1D' and multiply their sum by diameter-breast-high of the tree to obtain its volume.

Equation 3: $V/D = .7645 + .8412(D' - .55) = .3018 + .8412\ D'$
whereby $a = .3018$ and $b = .8412$

Equation 4: $b_1 = .5494 + .005347 \times 31.35 = .7171$

Equation 5: $V_1/D_1 = .7645 + .7171(D' - .55) = .3700 + .7171\ D'$
whereby, when $a_0 = .3700$ and $D' = D_0 = 3$, $V_1/D_1 = 2.5213$ (Point X, Figure D)

Equation 7: $V_2/D_2 = -.6409 + .09904 \times 31.35 = 2.4641$

Equation 9: $a_1 = -2.2892 + .083 \times 31.35 = .3128$

Equation 8: $V_2/D_2 = .3128 + .7171\ D'$
whereby when $D' = D_0 = 3$, $V_2/D_2 = 2.4641$ (Point X', Figure D)

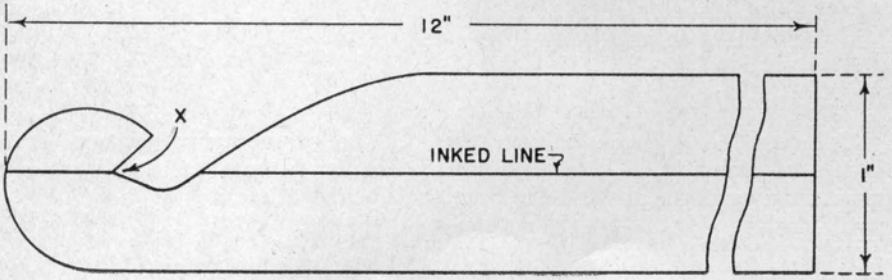
Similar values for the other height classes are entered in Table 4.

Having determined values for intercept a_1 and slope, b_1, based on the data from all height classes the general equation $V_2/D_2 = a_1 + b_1D'$ was solved to give the smoothed volume diameter ratios for each diameter-breast-high and height class required in the completed volume table. Finally, each ratio was multiplied by its respective diameter D to obtain the board-foot volume listed in Table 5. Alinement Chart III was prepared to facilitate computations in cases where volumes are needed for intermediate diameters and heights.

Two tests were made of the accuracy of the above equations in fitting the basic data. The aggregate difference showed Table 5 to be 0.33 per cent low, and the average percentage deviation for trees 6.5 inches and over was found to be 8.7 per cent. Both figures are well within the limits usually accepted for board-foot volume tables.

Reading the Alinement Chart

A straightedge is necessary for reading the alinement chart. One made of a strip of transparent plastic with a fine line scratched on the under side throws no shadow and makes reading accurate and rapid. The dimensions of the strip should be about 1 x 12 x 1/32 inches. To construct, (see sketch below) scribe a very fine, straight line down the middle. Remove any burr with the thumb nail, and rub India ink into the line to make it more visible. Near one end of the straightedge cut a "V" notch



as shown, being careful that the inked line bisects the angle at X formed by the sides of the notch. Sand all corners and edges with emery paper. A needle mounted in a cork with the point exposed completes the equipment needed.

The long axis of the needle should always be held perpendicular to the surface of the chart and the inked line should always be on the under side of the straightedge next to the chart.

APPLICATION OF THE VOLUME TABLES

Since the basic data were from trees grown in Connecticut, the tables should be tested for applicability when used in other regions. To do so, obtain the necessary diameter and height measurements on 20 to 25 trees, selected to give a good range of diameters and heights. Compute the actual volumes, using the same units (cubic or board feet) and the same limits of utilization as in the table. Express the deviation in volume for each tree as a percentage of the tabular volume of a tree of the same dimensions and compute the average of these deviations.

Compare the total volume of the measured trees with the total of the tabular volumes of these trees to arrive at the aggregate difference.

If the average percentage deviation of the local trees is not appreciably greater than that of the table, and if their aggregate difference is not more than two and one-half times the average percentage deviation of the table divided by the square root of the number of trees used in the test, correction for locality is unnecessary.

If the volumes of the local trees differ consistently from the tabular values, the table should be corrected. If the table is to be used for limits of utilization other than those used in its construction, it must be corrected to give volume adjusted to the new limits (9).

A table for local use, reading in terms of diameter only may be made as follows:

1. Obtain sufficient heights in the field to plot a height-on-diameter curve. From this read heights corresponding to one-inch diameter classes and tabulate, as in column 2 of the table below.

2. From the chart, read the volumes for the several paired diameter-height values and enter these in column 3 of the table.

DBH in Inches	Total Height in Feet	Total Volume in Cubic Feet[1]
(1)	(2)	(3)
2.0	14.4	.17
3.0	17.7	.46
4.0	20.3	.91
5.0	22.7	1.57
etc.	etc.	etc.

Columns 1 and 3 now constitute a local table reading in terms of full inches of diameter. These values may be curved to discover errors in chart reading or to provide for fractional-inch diameters.

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[1] Read from Chart I.

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