**Activity 8.4.2: Finding the Inverse of a Two by Two Matrix**

Part I

We can use algebra to find the inverse of a general 2×2 matrix if it has an inverse. Since we are considering a general matrix, all the matrix entries have to be represented by letters. We want to find the inverse of a matrix $A=\left[\begin{matrix}a&b\\c&d\end{matrix}\right]$ if it has an inverse. If *A* has an inverse matrix, *B*, we will want to determine the entries of $B=\left[\begin{matrix}e&f\\g&h\end{matrix}\right]$.

1. Therefore, what must be the product matrix $AB$? Fill in the blank matrix.

 $\left[\begin{matrix}a&b\\c&d\end{matrix}\right]\left[\begin{matrix}e&f\\g&h\end{matrix}\right]=$

1. Multiplying the two matrices will give us four equations since corresponding cell entries must be equal. Write the four equations in the space below:

To find the inverse matrix *B* we will need to use these four equations to determine the values of *e, f, g,* and *h*. We will also wish to see under what conditions matrix *B* is defined and under what conditions matrix *A* does not have an inverse.

The example below shows how to use the elimination method to do this. Since there are four equations, there are six different pairs of equations. Two of the pairs lead to an efficient application of the elimination method to solve and determine the entries of *B*. Here is an example: Use these two equations:

$$ae+bg=1$$

$$ce+dg=0$$

Multiply the first equation by *d* and the second equation by *b*.

$$\left(ae+bg\right)d=\left(1\right)d$$

$$\left(ce+dg\right)b=\left(0\right)b$$

$$ade+bdg=d$$

$$cbe+bdg=0$$

Subtract the second equation from the first.

$$ade-cbe=d$$

Solve the result for *e*. This requires that $ad-bc\ne 0$

$$e\left(ad-bc\right)=d$$

$$If ad-bc\ne 0, then$$

$$e=\frac{d}{ad-bc}$$

1. Use the same pair of equations to solve for *g*.
2. A. Find another pair of equations that can be used to efficiently solve for *f* and *h* and then find *f* using the method above.

B. Now find *h*.

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1. $So if ad-bc\ne 0, then \left[\begin{matrix}e&f\\g&h\end{matrix}\right]=$
2. Look at activity 4.1.1 and find the inverse of matrix $\left[\begin{matrix}2&4\\3&1\end{matrix}\right]$ using the above equation

Part II

1. Find the inverse of the following matrices. If a matrix does not have an inverse, write the word “singular” next to it.

$$\left[\begin{matrix}-1/10&4/10\\3/10&-2/10\end{matrix}\right]$$

$$\left[\begin{matrix}1&0\\0&1\end{matrix}\right]$$

$$\left[\begin{matrix}12&0\\0&12\end{matrix}\right]$$

$$\left[\begin{matrix}1&2\\3&4\end{matrix}\right]$$

$$\left[\begin{matrix}1&2\\2&4\end{matrix}\right]$$