**Unit 4: Investigation 4 (3 Days)**

**Operations on Rational Expressions**

**Common Core State Standards**

A.SSE.1b Interpret complicated expressions by viewing one or more parts as a single entity.

A.APR.6Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for more complicated examples, a computer algebra system.

A.APR.7(+)Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

**Overview**

In Investigations 2 and 3 students have had to add rational expressions with like denominators. This investigation extends addition of rational expressions to ones with unlike denominators and completes a discussion of operations on rational expressions by also including reducing, and division and multiplication of rational expressions. Students should be encouraged to note the parallel to operations on common fractions and to see that some practical uses of their work will be the ability to solve equations that have rational expressions and to represent verbal situations that need a quotient of expressions. Focus on equivalent expressions as well as simplifying expressions and what that means, for at times students will need an equivalent expression that is not necessarily “simpler.”

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**

* Translate verbal descriptions of relationships that need rational expressions.
* Use technology to solve problems that utilize rational expressions or rational functions.

**Assessment Strategies: How Will They Show What They Know?**

* **Exit slip 4.4.1** asks students to reduce rational expressions and to combine rational expressions.
* **Exit slip 4.4.2** asks students to multiply and divide rational expressions.
* **Journal Prompt 1** asksstudents toexplain to a partner how you can add two rational expressions with unlike denominators. It also asks what properties are being used to guarantee the expressions are equivalent.
* **Journal Prompt 2** asks students toexplain why it makes sense to factor the numerators and denominators first when simplifying a product of rational expressions.
* **Journal Prompt 3** asks students why 5y/5 = y but (y + 5)/y does not equal 5. Students are also asked why (x + 2)/(x - 2) cannot be further simplified and to write a rational expression that can be simplified to 1.
* **Activity sheet 4.4.1 Rational Expressions** **I** has students write some algebraic expressions from real word contexts.
* **Activity sheet 4.4.2 Rational Expressions** **II** has some expressions that need to be reduced, some expressions with like denominators and then some with unlike denominators that need to be combined.
* **Activity sheet 4.4.3 Rational Expressions** **III** contains rational expressions that need to be added, multiplied and divided mostly in real world contexts.

**Launch Notes**

You could show the following 4 minute clip [www.youtube.com/watch?V=5BVSRj\_ZEuM](http://www.youtube.com/watch?V=5BVSRj_ZEuM). It was made by Derek Grimes and you may have to go to Youtube’s search and type in Derek Grimes’s Rational Functions. It has as a great opener. The need to be able to solve word problems and to use fractions and rational expressions to express quantities involving fractions should be evident as they watch the clip. However the clip does not show any rational expressions explicitly. Another clip listed in the resources could be substituted and used another day. It lists careers that use rational expressions and the salaries of those careers.

**Teaching Strategies**

**Activity sheet 4.4.1 Rational Expressions** **I** has some real world applications that need rational expressions. By the end of the activity students should be able to state that a rational expression is a fraction but that often the numerator or denominator or both contain an algebraic expression that is not just a number. They may not yet catch that the expressions must be polynomials.

**Activity sheet 4.4.2 Rational Expressions** **II** has some expressions that need to be reduced, some expressions with like denominators and then some with unlike denominators. Use of the Identity Property for Multiplication and the Distributive Property once expressions have been rewritten as a/b = a (1/b) should be encouraged. Many of the expressions come from applications for finding the perimeter of common polygons. The activities also require that students find the domain of an expression. You may want to pose a situation such as drug company’s claim that it takes about $60 million to $1.2 billion dollars to research and test a new drug. For example, Claritin 10 mg costs $0.71 a pill to make, has a large body of consumers who will purchase it, and required less testing and research than for example a chemotherapy drug, so its fixed costs probably come closer to the $60 million. What is the average cost per pill? How does the manufacturing cost compare to the retail cost per pill to a consumer? One can easily assume at least 150 million pills will be bought in one year. The price for 100 pills is about $215 at a pharmacy. You may want to selectively do a few problems on the activity sheet in whole class mode and then have students work in groups.

Students must also be aware of the domains of expressions, restrictions due to the nature of an application if they are working in a context, and the implications for equivalent expressions. Students should also be exposed to obtaining an expression and then using that expression to define a rational function that can then be graphed and a minimum or maximum approximated with technology. For example: Create a function f(x) that expresses the sum of the two numbers as a function of one of the numbers, x when you know that the product of the two positive numbers is 9. Graph f and using your grapher, find which value of x will make that sum as small as possible. Make a table of values in auto mode starting .5 units smaller than your guess and let the step be 0.1. Do you think you may have found the value of x exactly? Using calculus you will be able to be sure. **Exit Slip 4.4.1** can be used any time after Activity 4.4.2 has been completed.

**Group Activity** Sheet 4.4.2 contains expressions that need to be reduced or combined where the expressions have both like and unlike denominators. Students will also need to determine the domain of an expression. Some are in a context.

**Journal Prompt 1** Explain to your partner how you can add two rational expressions with unlike denominators. What property (ies) is/are being used to guarantee the expressions are equivalent? Students might say that they need to find a common denominator, ideally the least, to avoid extra reducing later. Then each rational expression needs to be multiplied by one, the multiplicative identity. Then because the distributive property can now permit you to factor out the common denominators and tells you to add the numerators, the new rational expression is the sum, though it may need to be reduced using the Multiplicative Identity.

**Activity Sheet 4.4.3** **Rational Expressions** **III** contains rational expressions that need to be added, multiplied and divided. One motivation could be to use squares, rectangles, triangles, parallelograms and represent the sides and altitudes with rational expressions and to find expressions for the area of these polygons. Students may need a review of multiplying and dividing numerical fractions before beginning this activity. **Exit Slip 4.4.2** can be assigned any time after Activity Sheet 4.4.3 has been discussed.

**Journal Prompt 2** Explain why it makes sense to factor the numerators and denominators first when simplifying a product of rational expressions**.** Studentsmight saymultiplying polynomials takes time and then to reduce the answer would require factoring again so may as well work with the factors to begin with.

**Differentiated Instruction (For Learners Needing More Help).** Have students express the area of rectangles, triangles, squares and parallelograms with concrete measures for the bases and altitudes before proceeding to expressions with variables. You may want to discuss area again and contrast it with the perimeter of these polygons.

**Differentiated Instruction (Enrichment)** Have students make up some problems involving probability expressions. See Activity 4.1.1 and 4.1.3

**Journal Prompt 3** Explain why 5y/5 = y but (y + 5)/y does not equal 5 and why (x + 2)/(x - 2) cannot be further simplified. Now write a rational expression that can be simplified to 1**.** Students might say 5y/5 = (5/5)y = 1 y = y but (y + 5)/y= y/y +5/y = 1 + 5/y. In the first expression we can use the Multiplicative Identity to make the expression very simple but the second expression involves addition. In

(x + 2)/(x - 2) the expressions are neither identical nor are they opposites so there is no way to get the Multiplicative Identity involved. Lastly one possible expression a student might write is (x + 2)/(x + 2).

**Closure Notes**

In preparation for tomorrow’s movie clip launch, have students express work ratios such as if you take 3 hours to rake the leaves of your lawn, how much will you complete in 30 minutes, 1 hour? Suppose a friend helps and (s)he can rake the lawn alone in 4 hours. What is her/his ratio? If you work together what is an expression for the amount of lawn you would complete in 1 hour? How can you recognize when the lawn is done? This will set the stage for needing to solve equations with fractions and in particular ones with fractions that contain algebraic expressions in a numerator and/or denominator, the subject of the last investigation in this unit.

**Vocabulary**

Average cost

Area

Definition of Division

Difference

Domain of an expression

Fraction

Like and unlike denominators

Multiplicative Identity

Percent markup

Perimeter

Product

Quotient

Rational Expression

Sum

Surface area

**Resources and Materials**

**All activity sheets 4.4.1 – 4.4.3 should be completed.**

Activity sheet 4.4.1 Rational Expressions I

Activity sheet 4.4.2 Rational Expressions II

Activity sheet 4.4.3 Rational Expressions III

Graphers

Video clip [www.youtube.com/watch?V=5BVSRj\_ZEuM](http://www.youtube.com/watch?V=5BVSRj_ZEuM). Where rational expressions are used

<https://www.youtube.com/watch?v=fG-zjUR9mM8> Careers needing rational expressions