**Unit 2: Quadratic Functions**

**UNIT OVERVIEW**

23 days

Quadratic functions are the first family of functions that students will examine in depth in this Algebra 2 curriculum. Students will apply their knowledge of the effects that different transformations have on the graph of a function to the study of quadratic functions. In particular, students will examine the effects of vertical and horizontal shifts, vertical stretches, and reflections over the x-axis and their effects on properties of quadratic functions.

Mastery of the unit will require students to develop their understanding of: (1) properties of quadratic functions based on the parameters *a*, *b* and *c* in the standard form of the quadratic function $f\left(x\right)=ax^{2}+bx+c$, and the parameters *a*, *h* and *k* in the vertex form of the quadratic function $f\left(x\right)=a(x-h)^{2}+k$; (2) different methods for solving quadratic equations; (3) the nature of the roots of a quadratic equation based on the value of the discriminant; (4) the imaginary number *i* and operations involving complex numbers (5) complex conjugates as solutions of quadratic equations with a negative discriminant; (6) the Fundamental Theorem of Algebra in relation to quadratic functions; (7) modeling using quadratic functions; and (9) solving radical equations.

This unit builds on Unit 8 of the Common Core Algebra 1 Curriculum, providing students a deeper conceptual and procedural understanding of quadratic functions. The coverage of material in this unit will depend on the extent to which students studied and acquired quadratic function competencies in Algebra I. Throughout this unit, authors will highlight the connections between activities in this unit and activities in Unit 8 of the Algebra 1 curriculum, detailing the specific Algebra 1 activities that serve as the foundation of Algebra 2 activities.

**Investigation 1** begins with a study of the parent function $f\left(x\right)=x^{2}$, after which students will see how the transformations $f\left(x\right)+k$, $f\left(x+k\right)$, k$f\left(x\right)$ and $f\left(kx\right)$ shift and stretch the graph, and how these transformations are related to writing quadratic functions in standard and vertex forms. Students are also introduced to some key ideas in calculus, such as relative maxima and end behavior, by investigating quadratic functions. Horizontal stretches/compressions are not emphasized and will receive full attention in Unit 6 when graphs of the sine and cosine functions are examined.

**Investigation 2** builds on Unit 8 of the Algebra 1 curriculum and examines the various methods for solving quadratic equations; these include graphing, factoring, completing the square, and the quadratic formula. In this investigation, students will only explore rational and irrational solutions of quadratic equations. Students will conclude that a negative discriminant indicates no real solution. The absolute value function is defined as a piecewise function giving rise to the equivalence to an equation of the form $x^{2}=c$. Absolute value functions and equations are also explored.

**Investigation 3** fosters deep mathematical understanding of complex numbers. It develops the complex numbers as a mathematical structure that retains the field properties of the real numbers but also brings closure to the operation of taking the square root of a negative number. Activities in this investigation develop the definition of the imaginary number *i*; the powers of *i*; the basic operations of addition, subtraction, and multiplication of complex numbers; and the solution of quadratic equations that have complex solutions in the form of complex conjugates, $a+bi$ and $a-bi$. A mid-unit assessment is available to assess understanding of the first 3 investigations.

**Investigation 4** extends the findings about the connection between the zeros of a quadratic function and its factors to illustrate that the Fundamental Theorem of Algebra is true for quadratic functions and lays the foundation for the statement of the theorem for all polynomials. In order to do this, students will verify that the roots of the quadratic equation

$x^{2}+bx+c=0 $satisfy the relationship that the sum of the roots equals the opposite of *b,* and the product of the roots equals *c,* for complex roots as well as real roots. This property, along with the fact that complex roots of quadratic equations come in complex pairs will enable students to create quadratic functions given two real roots or one complex root and a third point on the graph.

**Investigation 5** provides students opportunities to model real-world phenomena using quadratic functions. Students are presented with a variety of tasks (open-ended, prescriptive and non-prescriptive) and algebraically develop quadratic functions and use quadratic regression to model data to explore contextual situations. Student complete activities that enable them to completely experience multiple steps in the mathematical modeling process: understanding the real-world problem, identifying model assumptions, developing a mathematical model, applying the model, and interpreting and validating the mathematical results. The activities in this investigation center on real-life applications and involve the use of graphing and statistical technology.

**Investigation 6** introduces students to real-life applications that can be described by radical functions and radical equations. Students explore radical functions using a variety of representations, solve a variety of radical equations, explore the concept of extraneous solutions, and examine contextual scenarios that can be described by square root functions.

 **Essential Questions**

* How do I describe quadratic functions in terms of transformations?
* How do I describe quadratic functions in terms of properties such as concavity and end behavior?
* How do I solve a quadratic equation?
* How can I determine the nature of the roots of a quadratic equation based on the value of the discriminant?
* What is the definition of the imaginary number *i*?
* What is the structure of the complex numbers?
* What is the Fundamental Theorem of Algebra, and what is its relation to quadratic functions?
* How do I use quadratic functions as mathematical models?
* How do I solve radical equations?

**Enduring Understandings**

* Quadratic functions can be used to model real world relationships and the key points in quadratic functions have meaning in the real world context.
* Dynamic software, graphing calculators, and other technology can be used to explore and deepen our understanding of mathematics.

**Unit Understandings**

* Transform quadratic functions through vertical shifts, horizontal shifts and stretches relate these transformations to the standard and vertex forms of a quadratic.
* Solve quadratic equations by graphing, factoring, completing the square, and the quadratic formula.
* Add, subtract, and multiply complex numbers.
* Use the discriminant to determine the nature of the roots of a quadratic equation.
* Solve radical equations and identify extraneous solutions.
* Model real-world situations with quadratic functions and use the functions to solve problems.

**Unit Contents**

Investigation 1: Transforming Quadratic Functions (3-6 days)

Investigation 2: Methods for Solving Quadratic Equations (4 - 5 days)

Investigation 3: Complex Numbers (3 days)

Investigation 4: Fundamental Theorem of Algebra (3-4 days)

Investigation 5: Modeling with Quadratic Functions (3 days)

Investigation 6: Radical Equations (3 days)

Mid-unit test (1 day)

Performance Task: (1 day)

Review for End-of-Unit Test (1 day)

End-of-Unit Test (1 day)

Teacher Testing Note: For the mid-unit assessment it is perfectly fine to provide the quadratic formula. For the end-of-unit assessment that is a department decision. For STEM-intending students it probably should not be provided.

**Common Core Standards**

*Mathematical Practices #1 and #3* *describe a classroom environment that encourages thinking mathematically and are critical for quality teaching and learning. Practices in bold are to be emphasized in the unit.*

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. **Construct viable arguments and critique the reasoning of others.**

4. **Model with mathematics.**

5. Use appropriate tools strategically.

6. Attend to precision.

7. **Look for and make use of structure**.

8. Look for and express regularity in repeated reasoning.

**Common Core State Standards**

A.SSE.3 Choose and produce an equivalent from of an expression to reveal and explain

properties of the quantity represented by the expression.

A.SSE.3a Factor a quadratic expression to reveal zeros of the function it defines.

A.SSE.3b Complete the square in a quadratic expression to reveal the maximum/minimum

value of the function it defines.

A.REI.A.2 Solve simple rational and radical equations in one variable, and give examples

showing how extraneous solutions may arise.

A.REI.4 Solve quadratic equations in one variable.

A.REI.4b Solve quadratic equations by inspection (e.g., for *x*2 = 49), taking square roots,

completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a\pm bi$ for real numbers *a* and *b*.

BF.A.1 Write a function that describes a relationship between two quantities.

CED.A.1 Create equations and inequalities in one variable and use them to solve problems.

*Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.

CED.A.2 Create equations in two or more variables to represent relationships between

quantities; graph equations on coordinate axes with labels and scales.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key

features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by

hand in simple cases and using technology for more complicated cases.

F.IF.C.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

F.IF.C.7b Graph square root, cube root, and piecewise-defined functions, including step

functions and absolute value functions**.**

N.CN.1 Know there is a complex number *i* such that $i^{2}=\sqrt{-1}$, and every complex

number has the form *a* + *bi* with *a* and *b* real.

N.CN.2 Use the relation i2 = -1 and the commutative, associative, and distributive

properties to add, subtract, and multiply complex numbers.

N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.

N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic

polynomials. (Note that only functions with real coefficients are considered in this investigation.)

N.RN.3 Explain why the sum or product of two rational numbers is rational and the sum of a rational and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

**Assessment Strategies:**

**Performance Task**

In the Unit 2 Performance Task: Modeling Basketball Shots, students develop and explore linear and quadratic models of the trajectories of basketball shots. Students are prompted to develop six models for shots taken by six different players at various positions on a court.  Students are provided information about each shot and a framework for developing models. It is the student’s task to use the information to model the shot, determine characteristics about the ball’s movement, and in some cases, determine whether the shot was successful.

**Other Evidence (Formative and Summative Assessments)**

* Exit slips
* Class work
* Homework assignments
* Math journals
* Unit 2 Mid-unit assessment
* Unit 2 End of unit assessment

**Vocabulary**

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| Absolute value |
| Absolute value |
| Axis of symmetry |
| Binomial |
| Binomial expansion |
| Closed setsClosure of sets under an operation |
| Completing the square |
| Complex Conjugate |
| Complex Number |
| Concave up/down |
| Cost Function |
| Decreasing function |
| Distributive property |
| End behavior |
| Extraneous root |
| Extraneous solution |
| Factor |
| Factor a quadratic over the integers |
| Family of functions |
| Imaginary Number |
| Increasing function |
| Infinity |
| Inside change |
| Mathematical model |
| Mathematical modeling |
| Modeling diagram |
| Monomial |
| Outside change |

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| Parabola |
| Parabolic |
| Parametric equations |
| Parent function |
| Piecewise defined function |
| Profit function |
| Quadratic formula |
| Quadratic Function |
| Quadratic regression |
| Radical equation |
| Radicand |
| Revenue function |
| Root |
| Sets of numbers {Natural numbers (counting numbers), Whole numbers, Integers, Rationals, Irrationals, Real numbers, Complex numbers) |
| Slope |
| Solution |
| Square root equation |
| Standard form of a quadratic function |
| Transformation |
| Vertex |
| Vertex |
| Vertex form of a quadratic function |
| *x*-intercept |
| *y*-intercept |
| Zeros of a function |

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